

Optimizing Production Overtime Period and Backorder Quantity in Joint Production and Maintenance Scheduling

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Abstract

The paper develops a mathematical model for integrated production and maintenance scheduling to optimize backorder quantity. It is assumed that at the end of each uptime, deterministic preventive maintenance is scheduled to incur backlog to reduce inventory carrying cost. The overall incurred cost of the proposed mathematical model is derived and used as a basis for optimal determination of backorder quantity in joint production-inventory model. The proposed model is extended emphasis on increase in production rate accomplished through production overtime period. An optimal overtime production period (T_P') is determined to minimize backordering cost per unit time in the joint production-inventory model. Wide variety of situations and effects are discussed for the extended production-inventory model. The model is validated with illustrating numerical examples, and sensitivity analysis to show the effectiveness and the robustness of the proposed integrated model.

Keywords

Joint production-inventory model, Overtime production rate and Integrated preventive maintenance.

1. Introduction

In a manufacturing environment, both raw materials stored in inventory and machines in production are subjected to deterioration and process failure. Continuous usage of manufacturing machines without maintenance reduces machine reliability which results in out-of-state quality products with an upsurge in the cost of nonconformance (Sana and Chaudhuri 2010; Dhouib 2012). Regular maintenance of manufacturing machines will lessen nonvalue-added process (Chelbi and Ait-Kadi 2004) and restore manufacturing machine to produce statistically in-control quality products (Dhouib 2012).

Maintenance is a set of activities performed during regular production without interrupting and some maintenance activities performed during production downtime (Sana and Chaudhuri 2010) namely servicing (aligning and verifying sensor operation, replacing filters), lubricating (flushing and topping of fluids), fastening, and inspections (leaks and cracks). These maintenance activities are performed to reduce fatigue, corrosion, wear, and tear of manufacturing equipment's to extend lifetime of manufacturing machines. Maintenance can be performed when part breakdown (corrective maintenance) or determined by observed condition (condition based maintenance) or component replaced preventively (preventive maintenance) (Poppe et al. 2017). In order to compete effectively in global market modern manufacturing system, optimize production inventory total cost in supply chain by coordinating (Poppe et al. 2017) production scheduling with maintenance planning based on several process capabilities (Chelbi and Ait-Kadi 2004) such as production rate, lot-sizing, mean time between failure, mean time to repair and backed with safety stock of finished goods to buffer against demand during maintenance period.

Preventive maintenance strategies are widely used in manufacturing industries like Logging, Textile, Steel manufacturing, Discrete part manufacturing, Chemical processing plant and many more. This paper considers regular

preventive maintenance of manufacturing machines carried out at each window downtime to maintain manufacturing machines operates smoothly. To reduce inventory holding cost, maintenance period is scheduled to occur backlog. Higher backordering level for the consecutive cycle will negatively impact total cost due to backlog caused by maintenance schedule. This paper is focused on optimizing backordering level in integrated production-maintenance scheduling model. In a manufacturing environment, Economic Production Quantity (EPQ) model have been analyzed and extended by several researchers (Mungan et al. 2012). Many researches illustrated the benefits of integrating production and PM scheduling model (Aghezzaf et al. 2007; Gouiaa-Mtibaa et al. 2018; Pan et al. 2010; Yildirim and Nezami 2014; Yulan et al. 2008) for minimizing total cost associated with production and simultaneously improving machine reliability considering manufacturing system deterioration (Ahmadi and Newby 2011; Rivera-Gomez et al. 2013), scheduling maintenance time (Cassady and Kutanoğlu 2003), periodic nature of PM (Fitouhi and Nourelnfath 2012), backorder quantity, and decisional variable on operational parameters (Yildirim and Nezami 2014; Helu et al. 2011).

The objective of scheduling joint Production and PM activities is to simultaneously improve machine reliability and reduce machine downtime (Liao and Chen 2003; Ji et al. 2007; Fitouhi and Nourelnfath 2012). Liao and Chen (2003) analyzed the issue of scheduling PM for non-resumable jobs with an objective of minimizing maximum tardiness of PM. Ji et al. (2007) determined an integrated production-PM schedule to minimize the make-span from PM perspective. Cyclic PM predetermines fixed time interval to carry out recurring optimal maintenance activities (Pan et al. 2010) and non-cyclic PM (Fitouhi and Nourelnfath 2012) relaxes the restriction of periodic PM (stochastic machine breakdown) to maximize machine availability for production and profit margin. determined optimal scheduled for production and noncyclic PM for randomly failing manufacturing system with an aim to minimize the cost associated with production and inventory. Most of the integrated production and PM scheduling model available in the literature focused on determining optimal interval and the optimal number of PM from the manufacturing system perspective of increasing system reliability, improving operational parameters and quality output. However, the impact of overtime production period has not been analyzed in the integrated production and PM model. To fill the gap and contribute to the body of research the proposed joint production and maintenance model for optimizing production overtime period to minimize backordering cost per unit is demonstrated in this literature.

2. Problem description

In order to have profitability in growing economic market, manufacturing companies implement various production & maintenance strategies, lean six sigma principles to optimize their supply chain. In modern manufacturing, it is possible to schedule a deterministic maintenance period to occur backlog in order to reduce inventory holding cost. For any manufacturing activity for instance in case of furniture manufacturing, the various operation involved are illustrated in Figure 1. The manufacturing process begins with Kiln wood of raw materials outsourced from an external supplier, an input item subjected to various operation as presented in Figure 1. At the end of packing and transportation manufactured furniture are transferred to the warehouse before being shipped to retailers or customer. To overcome the effect of obsolescence of manufactured goods, cost associated with handling staff and information technology required for warehouse, utilities and insurance, warehouse security and risk associated with furniture being damaged as it is moved in and out of the warehouse, deterministic preventive maintenance is scheduling along with the production process to incur backlog to overcome the effect of inventory holding cost.

2.1 Assumption

Following assumptions are considered to develop the expected total cost for joint production and maintenance scheduling model.

- A single item is produced in single machine.
- Demand rate (D) is constant and known for each cycle period.
- Holding cost of inventory is greater than backordering cost.
- Unsatisfied demand is back ordered.
- Production rate (P) is higher than Demand rate (D).
- The maintenance period is deterministic.
- Instantaneous replenishment inventory is followed for raw material inventory cycle.
- Machines undergo preventive maintenance at the end of uptime.
- No defects and shortages occur in raw materials.
- Raw materials and finished goods are imperishable with time.
- Demand during the maintenance period is met from deterministic produced safety stock.

- After completion of Preventive maintenance T_M , Production T_P for the next cycle begins instantaneously when inventory is completely depleted else the production starts till inventory is being completed.

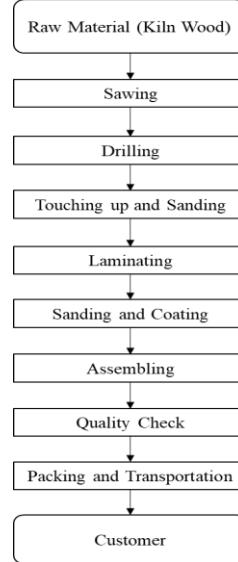


Figure 1. Furniture manufacturing process

To avoid holding cost and maintenance cost of raw materials it is assumed that instantaneous replenishment cycle inventory with known lead time period is followed. To overcome the effect of back ordering as inventory level falls below zero, raw materials for the consecutive cycle is brought in an appropriate quantity to meet the lost demand in the subsequent production cycle. Production time for the consecutive cycle impacts total cost due to the extended maintenance schedule. Production time (T_P) cannot be too long to overcome backlog and meet the same demand as previous production cycle as the maintenance cost of the machine increases at the same time T_P cannot be shorter or same as of previous cycle which increases total inventory backlog cost. Determining the trade-off in production time (T_P) in the consecutive cycle have positive impact in total cost. ***The objective is to find an optimal backordering quantity by formulating an economic model to minimize backordering cost in an integrated production-maintenance scheduling model.***

2.2 Notation

Following notations will apply throughout this research to develop the integrated model,

P	Production rate (units/year)
P'	Production rate for overtime production period (units/year)
D	Demand rate (units/year)
Π	Backordering (\$/units/year)
H	Finish product holding cost (\$/units/year)
h	Raw material holding cost (\$/units/year)
I_m	Maximum inventory (units)
b	Backordering level (units)
b'	Backordering level for overtime production period (units)
x	Production overtime period (cycle/year)
A	Ordering cost (\$/order)
T_P	Production time during shortage (year)
T'_P	Production time for overtime production period (year)
T_M	Maintenance time period (year)
M_T	Maintenance cost (\$/cycle)
M_F	Manufacturing cost (\$/unit)
M_f	Overtime manufacturing cost (\$/unit)
T_C	Total cycle time (year)

C_A	Ordering cost (\$/unit)
C_H	Average inventory holding cost (\$/unit)
C_M	Manufacturing cost (\$/unit)
C_P	Preventive maintenance cost (\$/unit)
C_B	Backordering cost (\$/unit)
Q	Raw material (unit/year)

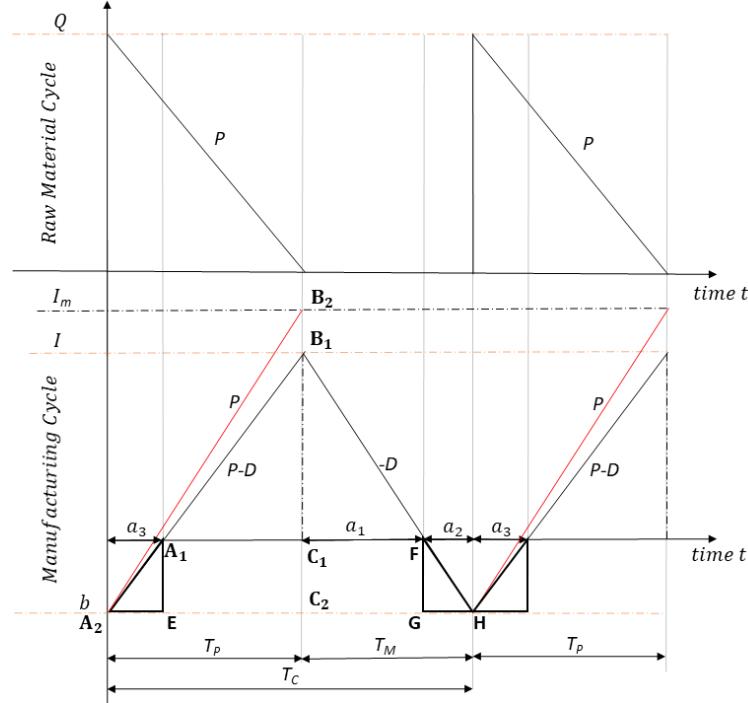


Figure 2. Inventory Diagram

During the maintenance period, demand is satisfied from deterministically produced safety stock to prevent stock-outs in order to avoid abruptness in supply during the maintenance period. The production stops at the end of uptime T_p and inventory is depleted during downtime or preventive maintenance period T_M which composed of two parts a_1 and a_2 . The demand is met from physical inventory during a_1 period and backlog occurs through a_2 period where the demand is not satisfied (Figure 3.2).

3. Mathematical formulation

Total cycle time (T_c) of production cycle is sum of production uptime (T_p) and deterministic maintenance downtime period (T_M).

$$T_c = T_p + T_M \quad (3.1)$$

Where, downtime T_M is sum of a_1 and a_2 and. Then we can express a_1 as,

$$a_1 = T_M - a_2 \quad (3.2)$$

$$b = Da_2 \Rightarrow a_2 = \frac{b}{D} \quad (3.3)$$

From ΔFGH of Figure 2, a_2 can also be expressed as

Substituting a_2 from equation (3.3) in equation (3.2) and solving a_1 yields,

$$a_1 = T_M - a_2 = T_M - \frac{b}{D} \quad (3.4)$$

From $\Delta A_1 EA_2$ backordered level b can also be expressed as

$$b = (P - D)a_3$$

$$a_3 = \frac{b}{(P - D)} \quad (3.5)$$

From $\Delta A_2 B_1 C_2$ production time period T_p can be expressed as,

$$T_p = \frac{I + b}{(P - D)} \quad (3.6)$$

From $\Delta B_1 C_1 D$, maximum on hand inventory I can be expressed as

$$I = Da_1 \quad (3.7)$$

Substituting (3.7) and (3.4) in equation (3.6) and solving for T_p yields,

$$T_p = \frac{D T_M}{(P - D)} \quad (3.8)$$

From (3.8) T_p is a constant variable as Demand, Production rate and Maintenance time period are constant.

3.1 Total cost

The first step in formulation of total expected cost per unit time for production inventory system is to identify individual components of the cost. The total expected cost per unit time in production inventory composed of raw material ordering cost (C_A), inventory holding cost (C_H), manufacturing cost (C_M), maintenance cost (C_P) and backordering cost (C_B). Therefore, the objective function is obtained by combining aforementioned costs and expressed as,

$$\text{Total cost } (TC) = C_A + C_H + C_M + C_P + C_B \quad (3.9)$$

3.1.1 Ordering cost (C_A)

Cumulative ordering cost is demand over number of production cycle P. Let A is the amount of raw material ordering cost per cycle.

$$C_A = A \frac{D}{Q} = \frac{A D}{P T_p} \quad (3.10)$$

3.1.2 Average inventory holding cost (C_H)

Finished goods inventory

Average inventory per cycle \bar{I} is the area of on-hand inventory I during uptime (T_p) over the time (a_3) and downtime period T_M over time period ($a_1 + a_2$). Amount of inventory carrying cost per cycle is H, then C_H is expressed as

$$C_H = H \bar{I} + h \bar{t} \quad (3.11)$$

Where average inventory \bar{I} is expressed as

$$\bar{I} = \frac{I}{2T_C} (T_p - a_3 + a_1) \quad (3.12)$$

Replacing I , a_1 and a_3 from equation (3.7), (3.4) and (3.5) respectively in equation (3.12),

$$\bar{I} = \frac{Da_1}{2(T_p + T_M)} [T_p - \frac{b}{(P - D)} + \frac{T_M D - b}{D}]$$

Substituting T_p from (3.8) and solving the above expression yields,

$$\bar{I} = \frac{(D T_M - b)^2}{2 D T_M} \quad (3.13)$$

Raw materials inventory

Similarly, latter part of equation (3.11) is the amount of inventory carrying cost for raw material per cycle is h , then average inventory for raw materials \bar{t} is expressed as expressed as,

$$\bar{t} = \frac{Q T_p}{2T_C} \quad (3.13.1)$$

Raw material cycle inventory from Figure 2 and Q expressed as,

$$Q = P T_p \quad (3.13.2)$$

Replacing T_p and Q from equation (3.8) and (3.13.2) in equation (3.13.1) yields,

$$\bar{t} = \frac{D^2 T_M}{2(P - D)} \quad (3.13.3)$$

Substituting equation (3.13) and (3.13.3) in equation (3.11) yields,

$$C_H = \frac{H(D T_M - b)^2}{2 D T_M} + \frac{h D^2 T_M}{2(P - D)} \quad (3.13.4)$$

3.1.3 Manufacturing cost (C_M)

Manufacturing cost is the product of total demand and manufacturing cost per item M_F then C_M is expressed as,

$$C_M = M_F D \quad (3.14)$$

3.1.4 Preventive maintenance cost (C_P)

Regular maintenance of manufacturing machines is carried out at each window downtime. Total number of downtime is demand number of production cycle with maintenance cost per cycle of M_T .

$$C_P = M_T \frac{D}{Q} = \frac{M_T D}{P T_P} \quad (3.15)$$

3.1.5 Backordering cost (C_B)

Average backordering \bar{B} is the shortage during a_2 over the time period T_C with shortage per cycle of π .

$$C_B = \pi \bar{B} \quad (3.16)$$

Where \bar{B} is expressed as,

$$\bar{B} = \frac{a_2 b}{2(T_P + T_M)} \quad (3.16.1)$$

Substituting equation (3.3) and (3.8) in equation (3.16.1) yields,

$$\bar{B} = \frac{(P - D) b^2}{2 D P T_M} \quad (3.16.2)$$

Substituting equation (3.16.2) in (3.16) yields,

$$C_B = \frac{\pi (P - D) b^2}{2 D P T_M} \quad (3.16.3)$$

Finally, equation (3.9) after substituting equation (3.10), (3.13.4), (3.14), (3.15) and (3.16.3) gives equation (3.17)

$$T_C = \frac{A D}{P T_P} + \frac{H (D T_M - b)^2}{2 D T_M} + \frac{h D^2 T_M}{2(P - D)} + M_F D + \frac{M_T D}{P T_P} + \frac{\pi (P - D) b^2}{2 D P T_M} \quad (3.17)$$

3.2 Solution Methodology – Base Model

The objective of model is to minimize the backordering cost in total expected cost composed of raw material ordering cost, average inventory holding cost, manufacturing cost, preventive maintenance cost and backordering cost with function of $TC(b)$. First derivate of function $TC'(b)$ gives stationary point in b and second derivate $TC''(b)$ determines the nature of stationary point whether its convexity or concavity.

$$\frac{dTC(b)}{db} = \frac{H(b - D T_M)}{D T_M} + \frac{\pi (P - D) b}{D (P T_M)} = 0 \quad (3.18)$$

Solving for b from Equation (3.18) yields stationary point for backlog quantity and it is given by

$$b^* = \frac{T_M P D H}{P(H + \pi) - \pi D} \quad (3.19)$$

Second derivate of the Total cost with respect to b can be written as

$$\frac{d^2TC(b)}{db^2} = \frac{H}{D T_M} + \frac{\pi (P - D)}{D P T_M} > 0. \quad (3.20)$$

Since $P > D$ according to assumption, $\frac{d^2TC}{db^2} > 0$ for any values of other parameters. Therefore, b obtained in equation (3.19) is optimal backlog quantity that minimizes total cost.

3.3 Numerical analysis

Consider a rectangle wooden table whose annual demand and production rate are 2500 (units/year) and 3250 (units/year) with manufacturing cost of 60\$/unit. Each unit of table consume 100 board foot (bd. Ft.) of wood. Holding cost and cost per bd. Ft. of wood is 0.05\$/year and 5\$. Production rate, demand rate and other relevant costs in terms of bd. Ft. in Table 1. From Equation (3.19), computed value of backlog $b = 505$ (bd. Ft /year) and from equation (3.17) total cost $TC(b) = \$186,670.31$ with individual cost as in Table 2,

After performing ceiling and flooring operator function on backlog quantity $[b^*] = 600$, $TC(b) = \$186,670.33$ and $\lfloor b^* \rfloor = 500$, $TC(b) = \$186,670.31$. Backlog quantity of 505 (bd. Ft /year) which is 5 (tables/year) yields the optimal total cost. Overall expected cost corresponding to different backlog quantity is presented in Figure 3. The result clearly illustrate the trade-offs that exists between maintenance period and backlog quantity. Figure 3. specifically shows the uniqueness of optimal solution.

3.4 Sensitivity analysis

A set of experiments have also considered to measure the sensitivity of the proposed optimal backlog quantity with respect to the model parameters. The goal is to demonstrate the efficiency and the robustness of the proposed model and to study the behavior of the optimal backlog policy with varying parameters, Demand and Preventive maintenance time period with total cost and backlog.

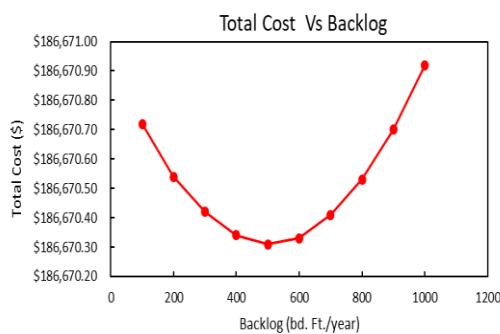


Figure 3. Overall total cost with respect to backlog quantity

Table 1. Parameter

$P = 325000$ (bd. Ft /year)
$D = 250000$ (bd. Ft /year)
$A = 30$ (\$ /order)
$\Pi = 0.0020$ (\$/bd. Ft /year)
$H = 0.0025$ (\$/bd. Ft /year)
$h = 0.05$ (\$/bd. Ft./year)
$T_M = 0.00239$ (year)
$M_F = 0.6$ (\$/bd. Ft)
$M_f = 1$ (\$/bd. Ft)

Table 2.
Individual cost

C_A - \$2,891.08
C_H - \$49.91
C_M - \$150,000.00
C_P - \$33,729.23
C_B - \$0.10

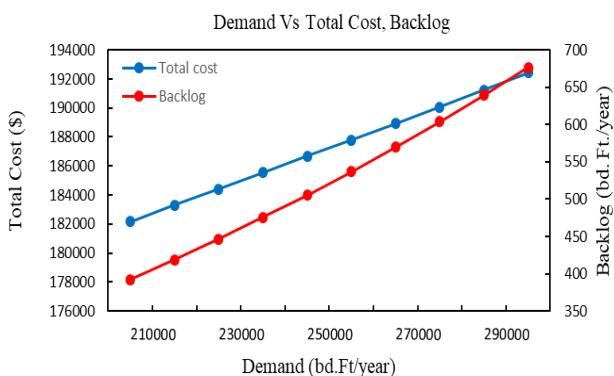


Figure 4. Overall total cost and backlog quantity with varying demand

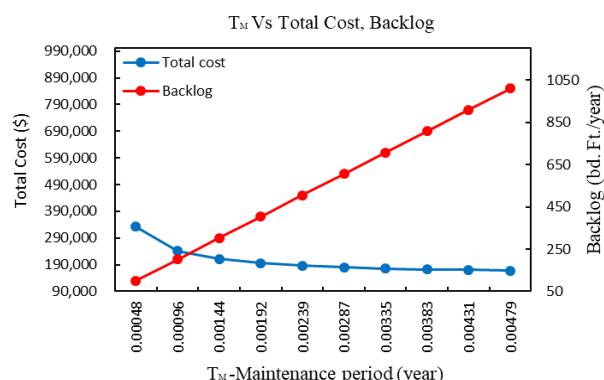


Figure 5. Overall total cost and backlog quantity with maintenance period variation

3.4.1 Influence of demand parameter

When demand of the product increases, total cost and backlog quantity increases (Figure 4). As the demand rate approaches to production rate, total cost significantly influenced by backlog quantity.

3.4.2 Influence of preventive maintenance time period parameter

A variation of maintenance time period has conflicting trend with total cost as observed with demand. The total cost is proportional to maintenance time period. But for the backordering quantity, amount of backlog decreases with increase in maintenance period (Figure 5).

4. Mathematical modeling for overtime production period

From raw material cycle from Figure 2, quantity of raw material Q is expressed as $Q = PT_p$ and production time period T_p is constant from equation (3.8) which makes Q also constant. In real world manufacturing environment efficiency of the manufacturing machine declines over time regardless of machine maintenance (Poppe et al. 2017). To overcome this effect it is assumed that production time per cycle is increased with linear manufacturing cost of M_f for the overtime production period during T'_p . As a consequence production rate increases to P' and total cost function is expressed as $TC(x)$ for the cycle time period ($T_c' = T_p' + T_M$) where x is overtime production period. With increase in production rate, finished goods inventory level increases and as a result backlog level falls into two cases. Case

A where increase in production rate without backlog (Figure 6) and case B increase in production rate with backorder (Figure 7).

4.1 Case A: Mathematical formulation for increase in production rate without backlog

From raw material inventory cycle from Figure 6, Q can be expressed as,

$$Q = P'T'_P \quad (4.1)$$

Production time T'_P in a cycle is extended with an overtime period x added to the old production time period T_P which can be written as,

$$T'_P = T_P + x \quad (4.2)$$

Substituting equation (4.2) in (4.1) and solving for P' yields,

$$P' = \frac{Q}{T'_P} = \frac{Q}{T_P + x} \quad (4.3)$$

Total cycle time (T'_C) of production cycle is sum of production uptime (T'_P), deterministic maintenance downtime period (T_M) and idle time period (i_d), thus

$$T'_C = T'_P + T_M + i_d \quad (4.4)$$

From $\Delta A'_1 B'_1 C'_1$ and $\Delta B'_1 C'_1 F'$ from Figure 6, the maximum on hand inventory I' can be expressed in following ways,

$$I' = (P' - D)T'_P \quad (4.5)$$

$$I' = D(T_M + i_d) \quad (4.6)$$

Substituting equation (4.3) in (4.5) and equating with (4.6) for solving i_d yields,

$$i_d = \frac{Q - D(T_P + x + T_M)}{D} \quad (4.7)$$

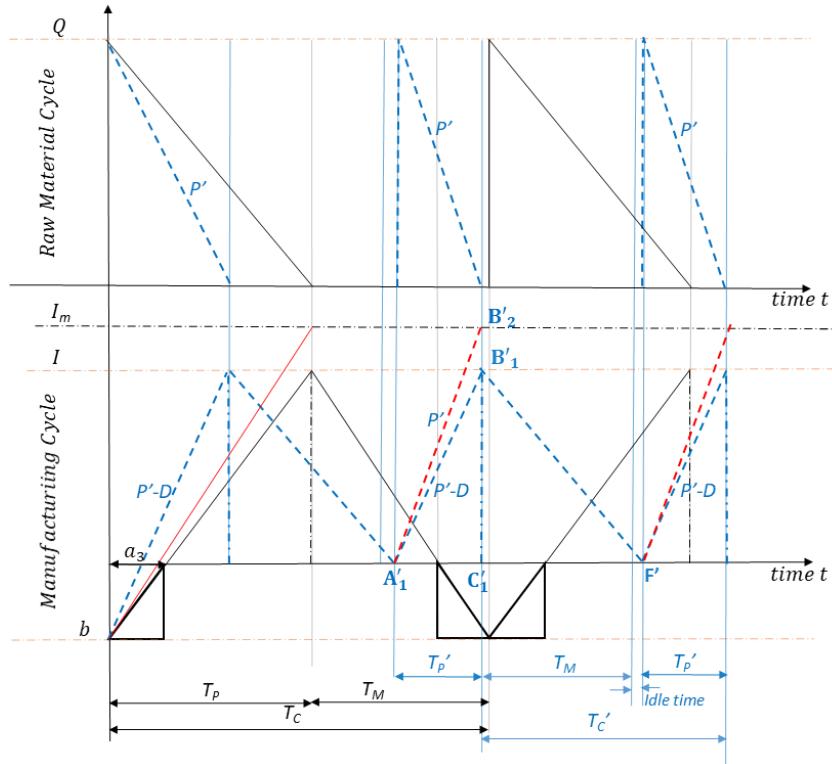


Figure 6. Inventory diagram for increase in production rate without backlog

4.1.1 Total cost

Total cost as derived in section 3.1, expected total cost in production inventory for increase in production rate without backlog from Figure 6 is expressed as,

$$\text{Total cost } (TC) = C_A + C_H + C_M + C_P$$

$$TC(x) = (A + M_T) \frac{D}{Q} + \frac{H(Q - D(T_P + x))}{2} + \frac{h D(T_P + x)}{2} + D(M_F + xM_f) \quad (4.8)$$

4.2 Solution methodology – Case A

The objective of model is to minimize backordering cost in the total expected cost expressed in equation (4.8). First derivate of function $TC'(x)$ gives stationary point in x and second derivate $TC''(x)$ determines the nature of stationary point whether its convexity or concavity.

$$\frac{dTC(x)}{dx} = -\frac{HD}{2} + \frac{h D}{2} + DM_f = \text{constant} \quad (4.9)$$

First derivate of the of equation (4.8) results in a constant function expressed in equation (4.9). The global minimum for $TC(x)$ is when overtime production $x = 0$, yields the same total cost obtained in equation (3.17).

4.3 Case B: Mathematical formulation for increase in production rate with backlog

From $\Delta J' B'_1 C'_1$ and $\Delta H' I' J'$ from Figure 7, total inventory can be expressed as,

$$I' + b' = (P' - D)T'_P \quad (4.10)$$

Total cycle time (T'_C) of production cycle is sum of production uptime (T'_P) and downtime period (T_M).

$$T'_C = T'_P + T_M \quad (4.11)$$

Where, downtime T_M is sum of a'_1 and a'_2 . Then we can express a'_1 as,

$$a'_1 = T_M - a'_2 \quad (4.12)$$

From $\Delta F' G' H'$ of Figure 4.2, a'_2 can also be expressed as,

$$b' = Da'_2 \Rightarrow a'_2 = \frac{b'}{D} \quad (4.13)$$

Substituting a'_2 from equation (4.13) in equation (4.12) and solving a'_1 yields,

$$a'_1 = T_M - a'_2 = T_M - \frac{b'}{D} \quad (4.14)$$

From Figure 7, $\Delta H' I' J'$ backordered level a'_3 can also be expressed as,

$$a'_3 = \frac{b'}{(P' - D)} \quad (4.15)$$

4.3.1 Total cost

Using P' from equation (4.3) and equation from (4.10),(4.13),(4.14),(4.15) expected total cost for increase in production rate with backlog can be expressed by $TC(x)$ as performed in section 3.1 using Figure 7 as,

$$\text{Total cost } (TC) = C_A + C_H + C_M + C_P + C_B$$

$$TC(x) = \frac{D(A + M_T)}{Q} + D(M_F + xM_f) + \frac{H(D T_M - b')}{2(T_P + x + T_M)} \left(T'_P - \frac{b'(T_P + x)}{(Q - D(T_P + x))} + \frac{D T_M - b'}{D} \right) + \frac{h Q (T_P + x)}{2(T_P + x + T_M)} + \frac{\pi b'^2}{2D(T_P + x + T_M)} \quad (4.17)$$

4.4 Solution methodology – Case B

The objective of model is to minimize backordering cost in total expected cost which is composed of ordering cost of raw material, average inventory holding cost, manufacturing cost, preventive maintenance cost and backordering cost with function of $TC(x)$. The stationary point of overtime production period level x is obtained from equating first order partial derivate of equation (4.18) $\frac{\partial TC}{\partial x} = 0$ and the second order partial derivate $\frac{\partial^2 TC}{\partial x^2}$ determines the nature of stationary point whether its convexity or concavity.

$$\frac{\partial TC}{\partial x} = 2DM_f(T_P + x + T_M)^2 - H b'(D T_M - b') \left(\frac{1}{D} + \frac{(Q T_M + D(T_P + x)^2)}{(Q - D(T_P + x))^2} \right) + \frac{D h Q T_M - \pi b'^2}{D} \quad (4.18)$$

Owing to complexity nature of partial first order condition derivate of total cost with respect to overtime period x , closed form expression for the function x is non-determinant. An open solution approach for value of x is followed and expressed in the following flowchart to compute the optimal value of overtime production period by plugin numerical operating value,

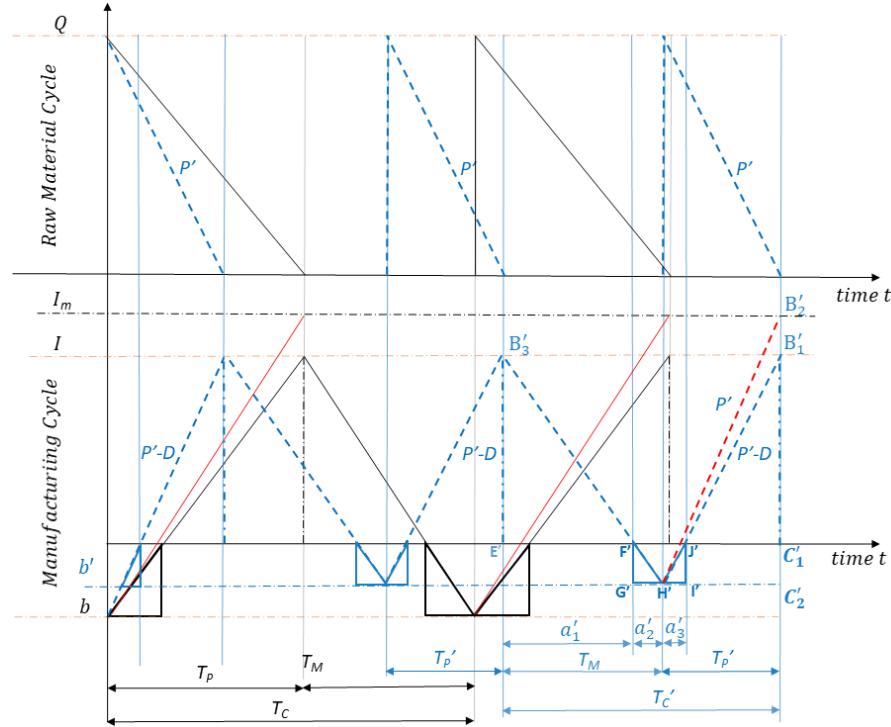
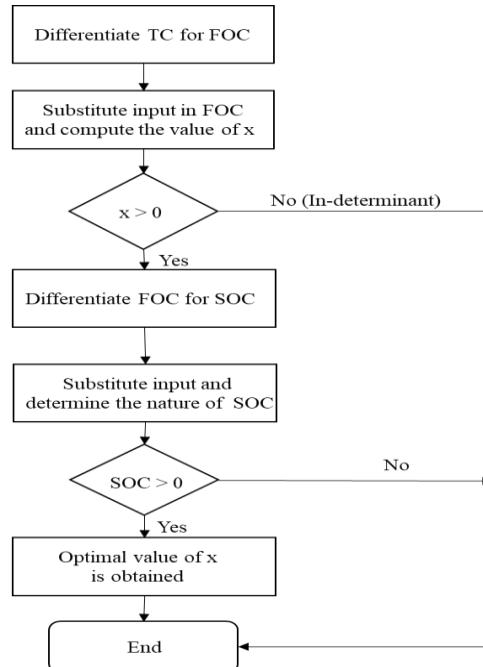


Figure 7. Inventory diagram for increase in production rate with backlog

4.5 Numerical analysis



Where,

TC – Total cost of expression $TC(x)$
FOC- First order condition ($\partial TC / \partial x$)
SOC- Second order condition ($\partial^2 TC / \partial x^2$)

Figure 8. Methodology for solving x

Table 4. Individual cost

$C_A = \$2,891.08$
$C_H = \$7.653$
$C_M = \$150,605.00$
$C_P = \$33,729.2$
$C_B = \$0.079$

Following the illustrated method from Figure 8 and considering data from Table.1 an optimal backlog value obtained with equation (3.19) is substituted in equation (4.17) and solving for x yields four different values of x ,

$$x = -0.0103721 \pm 0.00071i, 0.00236952, 0.00214975$$

$x = -0.0103721 \pm 0.00071i$ is neglected as overtime production period will be zero or positive. Substituting the value of $x = 0.00236952$ and 0.00214975 in SOC equation to determine the nature of $\frac{\partial^2 TC}{\partial x^2}$. For, $x = 0.00236952$, $\frac{\partial^2 TC}{\partial x^2} < 0$ and when $x = 0.00214975$, $\frac{\partial^2 TC}{\partial x^2} > 0$. From the nature of $\frac{\partial^2 TC}{\partial x^2}$, $x = 0.00214975$ yields the optimal overtime production period for convexity function. An optimal overtime production of 0.00241975 per year approximately 5 hours/production cycle with total cost of \$187,286 obtained from equation (4.17).

The backordering cost reduces from base model from 0.098 (\$/unit) to extended overtime production of 0.079 (\$/unit). Though the backordering and holding cost reduces, total cost increases from base model of \$186,670.31 with 505.36 (bd. Ft) backlog to \$187,286 with zero backlog in overtime production period. The increase in total cost is due to linear overtime manufacturing cost per unit. Backordering cost and backlog quantity is a major concern to finished products in furniture manufacturing industry where frequent transportation in and out of inventory possibly damage the product. Reducing backlog with increase in overtime production period could prevent the damage to the product by reducing cost of rework. This model can be further extended to determine total cost for uncertain demand.

5. Conclusion

A mathematical model has been presented to find an optimal backordering quantity by formulating an economic model to minimize backordering cost that incorporates maintenance period in production inventory model. The proposed model is extended emphasis on increases in production rate accomplished through production overtime to determine optimal overtime production period (T_P') which minimize the production-inventory backordering cost per unit time. Numerical analysis was performed with base case example of a manufacturing cell with realistic production and cost parameters in furniture manufacturing industry. Sensitivity analysis based on several parameters have also been carried out to show the effectiveness and the robustness of the proposed model.

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