Sumudu Decomposition Method for Black-Scholes Option Pricing Equation

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Abstract

The Black-Scholes partial differential equation is a very well-known model for pricing European call options. The purpose of this paper is to solve the Black-Scholes equation using the Sumudu decomposition method. This method is a combination of the Adomian decomposition method and Sumudu transform which is able to effectively and easily solve differential equations without discretization, linearization or pertrubation. Two numerical examples are presented in this paper. The results show that the Sumudu decomposition method is an efficient and reliable method for solving the Black-Scholes equation.

Keywords:
Pricing option, Black-Scholes equation, Sumudu decomposition method (SDM).

1. Introduction

Option is the right held by the holder to buy or sell an underlying asset at a certain price in a certain period. Option is a financial derivative product, so its value is derived from or depends on the price of the underlying asset (Hull, 2012). Black-Scholes partial differential equation is a very well-known equation for pricing European call options where the underlying assets are non-dividend stocks (Black & Scholes, 1973). Black-Scholes partial differential equation with boundary conditions for pricing European call option is defined as (Wilmott et al., 1995)

$$\frac{\partial C}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 C}{\partial S^2}+rS\frac{\partial C}{\partial S}-rC=0, \tag{1}$$

where $C=C(S,t)$ is the option price when the asset price S and time t, r is the risk free interest rate and σ is the asset price volatility. Based on equation (1), it is known that $C(0,t)=0$, $C(S,t)\sim S$ so $S\to\infty$ and $C(S,T)=\max\{S-E,\ 0\}$, where T is the maturity time and E is the exercise price. We set

$$S=Ee^x,\quad t=T-\frac{2\tau}{\sigma^2},\ \text{and}\ C(S,t)=Ev(x,\tau). \tag{2}$$

Therefore, equation (1) can be written as a simpler equation resembling a diffusion equation, i.e.

$$\frac{\partial v}{\partial \tau} = \frac{\partial^2 v}{\partial x^2} + (k-1)\frac{\partial v}{\partial x} - kv , \tag{3}$$

with $k = \frac{2r}{\sigma^2}$ and initial condition $v(x,0) = \max\{e^x - 1,\ 0\}$.

Various methods are used to solve Black-Scholes partial differential equations, including homotopy perturbation methods (Gulkac, 2010), finite difference methods (Cen & Le, 2011), variation iteration methods, homotopy analysis methods (Allahviranloo & Behzadi, 2013), projected differential transformation methods (Edeki et al., 2015) and the Adomian decomposition method (Biazar & Goldoust, 2013; Gonzalez-Gaxiola, 2016).

The Adomian decomposition method was first introduced by George Adomian to solve the system of stochastic equations (Adomian, 1980). This method is able to find solutions without linearization, discretization, transformation and perturbation. This decomposition method can be used to solve integral, differential and integral-differential equations. Differential equations that can be solved by this method can have integers or fractional order, ordinary or partial, with initial or boundary value problems, with variable or constants coefficients, linear or nonlinear, homogeneous or nonhomogeneous (Adomian, 1988; Duan et al., 2012, Al Awadah, 2016). The basic concept of this method assumes that solutions are decomposed into infinite series, nonliear forms (if any) are decomposed into Adomian polynomials and iterative algorithms are built to determine solutions recursively. Adomian decomposition method is a powerful technique and is very useful for solving heat equations (Biazar & Amirtaimoori, 2005), waves (Luo et al., 2006) and Fokker-Planck (Tatari et al., 2007). The Adomian decomposition method can be combined with Laplace (Khuri, 2001) or Sumudu transform (Khan et al., 2008; Kumar et al., 2012). The exact and approach solution of Klein-Gordon differential equations (Ramadan & Al-Luhaibi, 2014), waves (Ramadan & Al-Luhaibi, 2016) and Volterra integro-differential equations (Akinola et al., 2016) can be easily obtained using Sumudu decomposition method.

Based on the background of the problem and previous studies that have been presented, thus the purpose in this paper is the application of Sumudu decomposition method to solve the Black-Scholes equation.

2. Sumudu Decomposition Method

This section describes the basic theory and algorithm of the Sumudu decomposition method (SDM) used to solve partial differential equations. Consider partial differential equations as follows

$$L_t w(x,t) + Nw(x,t) + Rw(x,t) = g(x) \text{ and initial condition } w(x,0) = f(x) \tag{4}$$

where $L_t = \dfrac{\partial}{\partial t}$ is a partial derivative operator, N is a nonlinear operator, R is a linear operator, $g(x)$ is a function that shows the homogeneity of differential equations and $w(x,t)$ is a function to be determined. Equation (4) can be rewritten $L_t w(x,t)$ as the subject

$$L_t w(x,t) = g(x) - Nw(x,t) - Rw(x,t) . \tag{5}$$

In 1993, Watugala first introduced a new integral transform called Sumudu transform. Just like Laplace transform, Sumudu transform can be used to solve ordinary and partial differential equations (Belgacem & Karaballi, 2006). Before using the combination of the Adomian decomposition method with Sumudu transform, it was first presented the theory and properties of Sumudu transform.

Definition 1 (Belgacem & Karaballi, 2006) Based on the set of functions

$$A = \left\{ f(t) \mid \exists M, \tau_1, \tau_2 > 0, |f(t)| < Me^{\frac{|t|}{\tau_j}}, \text{jika } t \in (-1)^j \times [0,\infty) \right\},$$

Sumudu transform is defined as

$$G(u) = \mathbf{S}\left[f(t)\right] = \int_0^\infty e^{-t} f(ut)\,dt = \lim_{b\to\infty}\int_0^b e^{-t} f(ut)\,dt, \qquad u \in (-\tau_1, \tau_2)$$

where the limit value exists and finite. The inverse transformation is not denoted as

$$\mathbf{S}^{-1}\left[G(u)\right] = f(t), \;\; t \geq 0.$$

Based on Definition 1, for $f(t) = t^n$ where $t \geq 0$, the Sumudu transform from $f(t)$ is

$$\mathbf{S}\left[t^n\right] = n!u^n\,. \tag{6}$$

Theorem 1 (Belgacem & Karaballi, 2006) Let *f*, f', ..., $f^{(n)}$ be a continuous function at $(0,\infty)$ and exponentially limited, then the Sumudu transformation from $f^{(n)}$ is

$$\mathbf{S}\left[f^{(n)}(t)\right] = \frac{G(u)}{u^n} - \frac{f(0)}{u^n} - \cdots - \frac{f^{(n-1)}(0)}{u} = \frac{G(u)}{u^n} - \sum_{k=0}^{n-1}\frac{f^{(k)}(0)}{u^{n-k}}.$$

Use the Sumudu transform in equation (5), so that it is obtained

$$\mathbf{S}\left[L_t w(x,t)\right] = \mathbf{S}g(x) - \mathbf{S}\left[Nw(x,t)\right] - \mathbf{S}\left[Rw(x,t)\right]$$

or equivalent with

$$\frac{w(x,u)}{u} - \frac{w(x,0)}{u} = \mathbf{S}g(x) - \mathbf{S}\left[Nw(x,t)\right] - \mathbf{S}\left[Rw(x,t)\right]. \tag{7}$$

Substitution of initial conditions into equation (7)

$$w(x,u) = f(x) + u\mathbf{S}g(x) - u\mathbf{S}\left[Nw(x,t)\right] - u\mathbf{S}\left[Rw(x,t)\right]. \tag{8}$$

Use the inverse of Sumudu transform in equation (8)

$$w(x,t) = f(x) + \mathbf{S}^{-1}\left[u\mathbf{S}g(x)\right] - \mathbf{S}^{-1}\left[u\mathbf{S}\left[Nw(x,t)\right] + u\mathbf{S}\left[Rw(x,t)\right]\right]. \tag{9}$$

The Adomian decomposition method assumes that the function *w* can be decomposed into an infinite series

$$w = \sum_{n=0}^{\infty} w_n \tag{10}$$

where w_n can be determined recursively. This method also assumes that *Nw* nonlinear operators can be decomposed with infinite polynomial series

$$Nw = \sum_{n=0}^{\infty} A_n \tag{11}$$

where $A_n = A_n(w_0, w_1, \ldots, w_n)$ is Adomian polynomials which is defined by

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{n}\lambda^i w_i\right)\right]_{\lambda=0}; \qquad n = 0, 1, 2, \ldots .$$

with λ is the parameter, the A_n Adomian polynomial can be described as follows

$$\begin{aligned}
A_0 &= \frac{1}{0!}\frac{d^0}{d\lambda^0}\left[N\left(\sum_{i=0}^{0}\lambda^i w_i\right)\right]_{\lambda=0} = N(w_0),\\
A_1 &= \frac{1}{1!}\frac{d^1}{d\lambda^1}\left[N\left(\sum_{i=0}^{1}\lambda^i w_i\right)\right]_{\lambda=0} = w_1 N'(w_0),\\
A_2 &= \frac{1}{2!}\frac{d^2}{d\lambda^2}\left[N\left(\sum_{i=0}^{2}\lambda^i w_i\right)\right]_{\lambda=0} = \frac{w_1^{\,2}}{2!}N'(w_0) + w_2 N(w_0),\\
&\vdots
\end{aligned}$$

Substituting both of infinite series (10) and (11) to (9)

$$\sum_{n=0}^{\infty} w_n = f(x) + \mathbf{S}^{-1}[u\mathbf{S}g(x)] - \mathbf{S}^{-1}\left[u\mathbf{S}\left[\sum_{n=0}^{\infty} A_n\right] + u\mathbf{S}\left[R\sum_{n=0}^{\infty} w_n\right]\right] \tag{12}$$

thus a recursive relation is obtained from the solution as follows

$$\begin{aligned}
w_0 &= f(x) + \mathbf{S}^{-1}[u\mathbf{S}g(x)],\\
w_{n+1} &= -\mathbf{S}^{-1}[u\mathbf{S}[A_n] + u\mathbf{S}[Rw_n]], n = 0,1,2,\ldots
\end{aligned} \tag{13}$$

Therefore, the solution to the approach is

$$w \approx \sum_{n=0}^{k} w_n, \text{ where } \lim_{k\to\infty}\sum_{n=0}^{k} w_n = w. \tag{14}$$

The SDM can be an able and poweful procedure for obtaining analytical or numerical solutions without linearization or weak nonlinear assumptions or perturbation theory. This method can be used to solve integral, differential and integral-differential equations. Differential equations that can be solved by this method can have order of integers or fractional numbers, ordinary or partial, with initial value or boundary problems, with variable or constants coefficients, linear or nonlinear, homogeneous or nonhomogeneous, even system equations. The SDM builds an iterative algorithm to determine recursive solutions that are effective and easy to use in solving various equations, and produce numerical solutions that converge quickly and accurately in infinite series.

3. Numerical Example

Two numerical examples of the solution of the Black-Scholes equation with the SDM are presented in this paper.

Example 1. The Black-Scholes equation is given as follows

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2} + (k-1)\frac{\partial v}{\partial x} - kv \tag{15}$$

with initial condition $v(x,0) = \max\{e^x - 1,\ 0\}$.

The solution of the Black-Scholes equation (15) using the SDM is as follows

$$v_0 = \max\{e^x - 1,\ 0\},$$
$$v_{n+1} = \mathbf{S}^{-1}\left[u\mathbf{S}\left[\frac{\partial^2 v_n}{\partial x^2} + (k-1)\frac{\partial v_n}{\partial x} - kv_n\right]\right], n = 0,1,2,\ldots \tag{16}$$

If the recursive solution is described, then it is obtained

$$\begin{aligned} v_1 &= \mathbf{S}^{-1}\left[u\mathbf{S}\left[\frac{\partial^2 v_0}{\partial x^2} + (k-1)\frac{\partial v_0}{\partial x} - kv_0\right]\right] \\ &= \mathbf{S}^{-1}\left[u\mathbf{S}\left[k\max\{e^x,0\} - k\max\{e^x-1,0\}\right]\right] \\ &= \mathbf{S}^{-1}\left[ku\max\{e^x,0\} - ku\max\{e^x-1,0\}\right] \\ &= kt\max\{e^x,0\} - kt\max\{e^x-1,0\} \end{aligned}$$

because $\frac{\partial v_1}{\partial x} = 0$, so

$$\begin{aligned} v_2 &= \mathbf{S}^{-1}\left[u\mathbf{S}\left[\frac{\partial^2 v_1}{\partial x^2} + (k-1)\frac{\partial v_1}{\partial x} - kv_1\right]\right] \\ &= \mathbf{S}^{-1}\left[u\mathbf{S}\left[-k^2t\max\{e^x,0\} + k^2t\max\{e^x-1,0\}\right]\right] \\ &= \mathbf{S}^{-1}\left[(ku)^2\max\{e^x,0\} - (ku)^2\max\{e^x-1,0\}\right] \\ &= -\frac{1}{2}(kt)^2\max\{e^x,0\} + \frac{1}{2}(kt)^2\max\{e^x-1,0\} \end{aligned}$$

$$\begin{aligned} v_3 &= \mathbf{S}^{-1}\left[u\mathbf{S}\left[\frac{\partial^2 v_2}{\partial x^2} + (k-1)\frac{\partial v_2}{\partial x} - kv_2\right]\right] \\ &= \mathbf{S}^{-1}\left[u\mathbf{S}\left[\frac{1}{2}k^3t^2\max\{e^x,0\} - \frac{1}{2}k^3t^2\max\{e^x-1,0\}\right]\right] \\ &= \mathbf{S}^{-1}\left[(ku)^3\max\{e^x,0\} - (ku)^3\max\{e^x-1,0\}\right] \\ &= \frac{1}{6}(kt)^3\max\{e^x,0\} - \frac{1}{6}(kt)^3\max\{e^x-1,0\} \\ &\vdots \end{aligned}$$

Therefore the solution to the Black-Scholes equation (15) is

$$v(x,t) = \sum_{n=0}^{\infty} v_n(x,t) = \max\{e^x - 1,0\}e^{-kt} + \max\{e^x,0\}(1 - e^{-kt})\ . \tag{17}$$

Example 2. The generalization of the Black-Scholes equation is given as follows (Chen & Le, 2011; Edeki et al., 2015)

$$\frac{\partial v}{\partial t} + 0.08(2+\sin x)^2 x^2 \frac{\partial^2 v}{\partial x^2} + 0.06x\frac{\partial v}{\partial x} - 0.06v = 0 \tag{18}$$

with initial condition $v(x,0) = \max\{x - 25e^{-0.06},\ 0\}$.

The solution of the Black-Scholes equation (17) using the SDM is as follows

$$
\begin{aligned}
v_0 &= v(x,0) = \max\{x - 25e^{-0.06},\ 0\}, \\
v_{n+1} &= \mathbf{S}^{-1}\left[u\mathbf{S}\left[-0.08(2+\sin x)^2 x^2 \frac{\partial^2 v_n}{\partial x^2} - 0.06x\frac{\partial v_n}{\partial x} + 0.06v_n\right]\right], n = 0,1,2,\ldots
\end{aligned}
\tag{19}
$$

If the recursive solution is described, then it is obtained

$$
\begin{aligned}
v_1 &= \mathbf{S}^{-1}\left[u\mathbf{S}\left[-0.08(2+\sin x)^2 x^2 \frac{\partial^2 v_0}{\partial x^2} - 0.06x\frac{\partial v_0}{\partial x} + 0.06v_0\right]\right] \\
&= \mathbf{S}^{-1}\left[u\mathbf{S}\left[-0.06x + 0.06\max\{x - 25e^{-0.06}, 0\}\right]\right] \\
&= \mathbf{S}^{-1}\left[-0.06ux + 0.06u\max\{x - 25e^{-0.06}, 0\}\right] \\
&= -0.06tx + 0.06t\max\{x - 25e^{-0.06}, 0\}
\end{aligned}
$$

because $\frac{\partial v_1}{\partial x} = 0$, so

$$
\begin{aligned}
v_2 &= \mathbf{S}^{-1}\left[u\mathbf{S}\left[-0.08(2+\sin x)^2 x^2 \frac{\partial^2 v_1}{\partial x^2} - 0.06x\frac{\partial v_1}{\partial x} + 0.06v_1\right]\right] \\
&= \mathbf{S}^{-1}\left[u\mathbf{S}\left[-(0.06)^2 tx + (0.06)^2 t\max\{x - 25e^{-0.06}, 0\}\right]\right] \\
&= \mathbf{S}^{-1}\left[-(0.06u)^2 x + (0.06u)^2 \max\{x - 25e^{-0.06}, 0\}\right] \\
&= -\frac{1}{2}(0.06t)^2 x + \frac{1}{2}(0.06t)^2 \max\{x - 25e^{-0.06}, 0\}
\end{aligned}
$$

$$
\begin{aligned}
v_3 &= \mathbf{S}^{-1}\left[u\mathbf{S}\left[-0.08(2+\sin x)^2 x^2 \frac{\partial^2 v_2}{\partial x^2} - 0.06x\frac{\partial v_2}{\partial x} + 0.06v_2\right]\right] \\
&= \mathbf{S}^{-1}\left[u\mathbf{S}\left[-\frac{1}{2}(0.06)^3 t^2 x + \frac{1}{2}(0.06)^3 t^2 \max\{x - 25e^{-0.06}, 0\}\right]\right] \\
&= \mathbf{S}^{-1}\left[-(0.06u)^3 x + (0.06u)^3 \max\{x - 25e^{-0.06}, 0\}\right] \\
&= -\frac{1}{6}(0.06t)^3 x + \frac{1}{6}(0.06t)^3 \max\{x - 25e^{-0.06}, 0\} \\
&\vdots
\end{aligned}
$$

Therefore the solution to the Black-Scholes equation (17) is

$$
v(x,t) = \sum_{n=0}^{\infty} v_n(x,t) = \max\{x - 25e^{-0.06}, 0\}e^{0.06t} + x\left(1 - e^{0.06t}\right). \tag{20}
$$

4. Conclusion

The Sumudu decomposition method (SDM) is a combination of the Adomian decomposition method and Sumudu transform. This method is able to solve ordinary and partial differential equations. Based on the two examples given, the SDM is effectively and easily used to solve the Black-Scholes equation.

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