# Mathematical Modelling of Multi-Product Ordering in Three-Echelon Supply Chain Networks

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## Abstract

This paper proposes a mixed integer linear programming model for a multi-product ordering in a three-echelon supply chain network, where multiple manufacturers supply multiple warehouses with multiple products, which in turn distribute the products to the multiple retailers involved. The model considers practical production constraints such as production capacity, backorder allowances, and economically-viable minimum order quantities. Numerical computations show that the model can efficiently solve small-sized problem instances.

## Keywords
Supply chain network, Vendor managed inventory, Order quantity, Backorder

## 1. Introduction

Production systems are under tremendous pressure and face numerous new challenges, such as fluctuation in demand and preferences and rapid changes in technology. They often are part of a larger network, a supply chain network (SCN), integrating material, financial, and information flows.

Supply chain management (SCM) allows for supplying customers in a more efficient and cost-effective way, since a broad range of planning, implementing, and controlling activities required throughout product flow, from the supply of materials to the final product, can be addressed collectively by all parties involved (Zadeh et al., 2014). Decision making in SCM can be categorized into three levels based on the planning horizon (Chopra and Meindl, 2007): strategic, tactical, and operational. Strategic decisions involve long term decisions such as customer segments, product development, logistics infrastructure (manufacturing, warehousing, vendors, etc.), and purchasing; while tactical decisions involve transportation modes, transportation planning, and inventory handling and refer to a time horizon of a few months. Lastly, at the operational level, daily decisions such as on-hand inventory levels, order quantities, and resources allocation are made. This work addresses operational decisions in a supply chain network.

Recently, both practitioners and scientists have actively contributed to increasing supply chain integration, which not only enhances customers' service level but also decreases costs throughout the whole system. One way of achieving such integration is to define a vendor-managed inventory (VMI) policy under which the supply chain performance can be improved by decreasing inventory-related costs and increasing customer service. VMI is a streamlined approach to inventory management and order fulfilment, in which a downstream body shares its demand information with suppliers and in return benefits from a lower inventory level and a more reliable flow of products. Indeed, in VMI, suppliers decide on the order quantities and their replenishment policies bearing the risk of unfulfilled demands. Figure 1 illustrates the impact of a VMI policy on the decisions made in an SCN. Such a policy can prevent stocking undesired inventories and thus, can lead to an overall cost reduction.

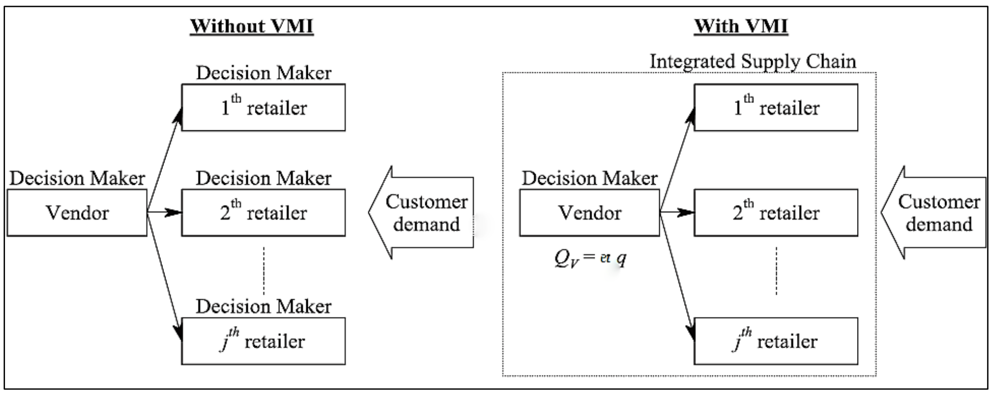


Figure 1. Decision making process with and without VMI policy (Sadeghi et al., 2016).

First VMI's research efforts date back to 1990s, when it has been introduced as an important strategy to decrease *bullwhip effect* in supply chains (Holmstrom, 1998; Waller et al., 1999). Since then, several inventory models under VMI policy have been developed and proposed. For example, Zhang et al. (2007) proposed an integrated cost model for a single vendor that purchases and processes raw materials at a constant production rate and then delivers finished products to multiple buyers, each with a constant demand rate. Buyers may be replenished more than once in a production cycle, possibly having different ordering cycles. The model proposed minimizes the sum of the total costs incurred by the vendor and all the buyers; while finding investment and replenishment decisions for both the vendor and the buyers. Another example is the nonlinear inventory model devised by Darwish and Odah (2010) that minimizes average total cost in a two-echelon SCN involving a single vendor and multiple retailers. The model is solved by finding all Karush–Kuhn–Tucker (KKT) points. The proposed model has been examined in two problem instances; the first one considers five retailers which was used for the sensitivity analysis, the second one, with 30 retailers, was utilized to compare the VMI model with an optimization model without VMI. The VMI model in addition to providing feasible order quantities reduces the total supply chain's cost by 3%; though, it does not necessarily provide the minimum cost for each individual body in the supply chain.

Recently, inventory models under VMI policy have been extended to cope with three-echelon supply chains. Kadadevaramath et al. (2012) propose a mathematical model for a three-echelon SCN (supplier, manufacturer, and distributors) involving the production and distribution of a single product in one planning period that minimizes the total supply chain operating costs, that is purchasing, manufacturing, and transportation costs. The main problem features include demand uncertainty and limited production and supply capacity. Solutions were found by resorting to a particle swarm optimization (PSO) algorithm (in four variants) and a genetic algorithm (GA), for 20 instances obtained by considering one problem instance with two manufacturers and six distributors in 20 different scenarios. Later, Cárdenas-Barrón and Treviño-Garza (2014) proposed an integer linear programming (ILP) model for the three-echelon SCN considered by Kadadevaramath et al. (2012), but extending it, since they consider multiple products and multiple planning periods. They were able to solve optimally the instances generated by Kadadevaramath et al. (2012). However, none of the studies allows for a backorder or considers shipment decisions.

Sadeghi et al. (2013) and Sadeghi et al. (2014) extended the work of Darwish and Odah (2010) by including a central warehouse to where the product is shipped to before being distributed to the retailers. Since the warehouse storage capacity is limited, the product may also be, temporarily, stored at the vendors. Sadeghi et al. (2013) also consider several vendors, and that the annual number of orders received by the warehouse is limited. The problem is formulated as an integer nonlinear programming (INLP) model that while minimizing the total inventory costs of the network (vendors and warehouse) determines the number of orders and orders' quantity received by both the vendors and the warehouse. The authors developed a PSO algorithm, and a GA to obtain approximate solutions for the problem. The authors designed a set of 10 problem instances of different sizes, in which two to five vendors supply up to 20 retailers via a central warehouse. Sadeghi et al. (2014) consider a bi-objective three-echelon SCN involving a single-product, single-manufacturer, single-warehouse, and multi-retailer SCN. The manufacturer produces the product using several machines that work in series. The authors propose an INLP model for which solutions are found by a hybrid bat algorithm. The objectives include minimizing the total supply network costs and maximizing the production machines reliability, which are then aggregated through linear scalarization. Sadeghi et al. (2014) designed a set of 10 problem instance of different sizes, in which a manufacturer with four up to 14 machines produce and distribute a product to three up to 19 retailers via a central warehouse.

More recently, Chan et al. (2016) modelled a three-echelon SCN considering multiple time periods, multiple products, and uncertain demands. Since the main focuses were on enhancing the network's responsiveness (retailers' demand satisfaction within a certain time frame) the main decisions are associated with truck selection and loading sub-problem. The authors consider a multi-objective problem version in which, on the one hand, the costs are minimized (production, transportation, storage, and risk penalty costs) and, on the other hand, the percentage number of times that retailers do not receive their orders in the desired time frame is minimized. Solutions were obtained by using a non-dominated sorting genetic algorithm (NSGA-II) and a heuristic method to determine manufacturer-warehouse allocations, warehouse-retailers allocations, quantities sent from the manufacturers to the warehouses and from them to the retailers, and the trucks of different types needed to transport such quantities. A single instance with 24 time-periods, three manufacturers, five warehouses, eight retailers, 20 products, and 15 truck types was considered.

In this paper, we propose a mixed integer linear programming (MILP) model for a constrained multi-product order quantity under VMI policy in a three-echelon SCN with multiple manufacturers, multiple distributors, and multiple retailers. On the one hand, it extends the model developed by Sadeghi et al. (2013) and Sadeghi et al. (2014) by involving multi-product orders and multiple distributors within the SCN and also by allowing backorder, enforcing production capacity, and imposing upper and lower limits on order quantities; and on the other hand, it extends the model developed by Chan et al. (2016) by allowing backorder, enforcing production capacity, and imposing upper and lower limits on order quantities. A set of small sized-instances is solved optimally as the model proposed was implemented in and solved by the Gurobi® software package. Section 2 defines the specific problem being addressed; while Section 3 formulates the proposed MILP model. Computational experiments and numerical results are reported in Section 4 and some conclusions are drawn in Section 5.

## 2. Problem Description

The problem described in this section is mainly motivated by a real case, where a holding company within the food industry produces a wide variety of dairy products in its manufacturing plants across the country and distributes them to a wide network of retailers through a set of distribution centres (warehouses). Due to economic reasons, the holding company assumes its manufacturers, warehouses, and retailers to compete among each other and thus, each decides independently on its own costs, prices, and capacities. Nevertheless, decisions for the whole SCN are made under a central inventory management system to gain advantages over competitors. Although the main motivation of the work is a specific food industry supply chain, the problem description and model proposed are general and can be used in any other supply chain with similar specifications.

We consider a three-echelon SCN with a set of manufacturers, a set of distribution centres (warehouses), and a set of retailers; each facility has a known and fixed location. Each manufacturer can produce any of the products, which are then shipped to any of the warehouses, from where the retailers are supplied. Figure 2 depicts a schematic view of such a three-echelon supply chain network. Each retailer has a known constant demand rate for each product within the planning period and can be supplied by one or more warehouses. To supply the retailers, the warehouses order the products from the one or more manufacturers, each of which can produce any or all products.

Although any manufacturer can produce any product, production quantities have lower and upper limits. The lower limits are per product and reflect economic viability; while upper limits are imposed both on each product, reflecting the availability of specific resources or production mix, and on the total production, reflecting production capacity limits (in our case through production time availability). In addition, since products are produced at a constant rate and delivered all at once to the warehouses, for inventory costs purposes we consider the average inventory over the planning period.

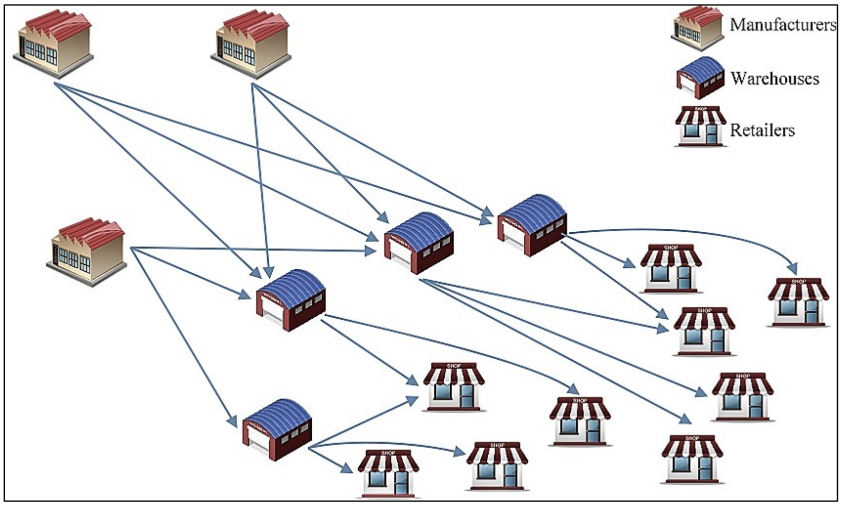


Figure 2. A schematic view of a three-echelon supply chain network

As said before, warehouses buy the products from the manufacturers and since, typically, there is a time difference between their arrival from the manufacturers and their delivery to the retailers, they need to be temporarily stored. Warehouse ordering decisions are bound by storage capacity and budget availability. In here, we consider the total quantity of all products received by a warehouse for the storage capacity and the average inventory over the period for calculating the inventory costs. In addition, warehouses have to comply with a minimum order quantity imposed by manufactures. Retailers place orders for the different products to one or more warehouses and receive them all at once. Since demand occurs at a constant rate throughout the planning period, retailers also have to, temporarily, store the products; again, for cost purposes, we consider the average inventory. In addition, retailers have to comply with a minimum order quantity imposed by warehouses. Backorder is allowed at both the manufacturers and the warehouses; however, it is limited and implies a cost.

Products are transported between manufacturers and warehouses and between warehouses and retailers in vehicles with limited capacity and each manufacturer and warehouse has a fixed and known number of vehicles, all of the same type. Finally, we assume that vehicles do single origin single destination trips only and there is no preexisting backorder.

## 3. Problem Formulation

This section proposes a mixed linear mathematical programming model to find the quantities of each product that should be i) ordered by each retailer from each warehouse and by each warehouse from each manufacture and ii) backordered by each manufacturer regarding each warehouse order and by each warehouse regarding each retailer order and the number of vehicles each manufacturer and each warehouse need in its fleet to transport the products, at minimum total cost. The costs considered include ordering costs, transportation costs, purchasing costs, holding costs, and backorder costs. Let us first introduce the notation and decision variables used in the MILP model.

**Sets and indices:**

$M$ Set of the manufacturers, indexed by $m \in M$;

$W$ Set of the warehouses, indexed by $w \in W$;

$R$ Set of the retailers, indexed by $r \in R$;

$P$ Set of the product types, indexed by $p \in P$.

**Parameters:**

$D_{rp}$ Demand of retailer $r \in R$ for product $p \in P$;

$S_m$ Fixed ordering cost from manufacturer $m \in M$;

$S'_w$ Fixed ordering cost from warehouse $w \in W$;

$T_{mw}$ Fixed transport cost per vehicle dispatched from manufacturer $m \in M$ to warehouse $w \in W$;

$T'_{wr}$ Fixed transport cost per vehicle dispatched from warehouse $w \in W$ to retailer $r \in R$;

$C_{mp}$ Marginal purchase cost of product $p \in P$ from manufacturer $m \in M$;

$C'_{wp}$ Marginal purchase cost of product $p \in P$ from warehouse $w \in W$;

$\pi_p$ Marginal backorder cost of product $p \in P$ at the manufacturers;

$\pi'_p$ Marginal backorder cost of product $p \in P$ at the warehouses;

$H_{mp}$ Marginal holding cost of product $p \in P$ at manufacturer $m \in M$;

$H'_{wp}$ Marginal holding cost of product $p \in P$ at warehouse $w \in W$;

$H''_{rp}$ Marginal holding cost of product $p \in P$ at retailer $r \in R$;

$L_m$ Minimum order quantity from manufacturer $m \in M$, regardless of the warehouse;

$L'_w$ Minimum order quantity from warehouse $w \in W$, regardless of the retailer;

$\mathcal{L}_{mp}$ Minimum total quantity of product $p \in P$ that needs to be ordered from manufacturer $m \in M$;

$\mathcal{U}_{mp}$ Maximum total quantity of product $p \in P$ that can be ordered from manufacturer $m \in M$;

$\mathcal{T}_m$ Production time available (in minutes) for manufacturer $m \in M$ within the planning period;

$\beta_r$ Per product percentage backorder allowance limit by retailer $r \in R$, regardless of the product;

$UB_{mp}$ Maximum backorder units of product $p \in P$ at manufacturer $m \in M$;

$\tau_p$ Production time, per unit, of product $p \in P$, in minutes;

$\omega_p$ Weight, per unit, of product $p \in P$, in Kg;

$\nu_p$ Volume, per unit, of product $p \in P$, in $m^3$;

$\mathcal{C}_w$ Storage capacity of warehouse $w \in W$ in $m^3$;

$\mathcal{B}_w$ Budget available at warehouse $w \in W$ to purchase product(s);

$N_m$ Available vehicles at manufacturer $m \in M$;

$N'_w$ Available vehicles at warehouse $w \in W$;

$VC$ Manufacturers' vehicle capacity (in Kg);

$VC'$ Warehouses' vehicle capacity (in Kg);

$\mathcal{M}$ A sufficiently large positive number.

**Decision Variables:**

$Q^p_{mw}$ Quantity of product $p \in P$ purchased by warehouse $w \in W$ from manufacturer $m \in M$;

$Q'^p_{wr}$ Quantity of product $p \in P$ purchased by retailer $r \in R$ from warehouse $w \in W$;

$B^p_{mw}$ Backordered quantity of product $p \in P$ ordered by warehouse $w \in W$ from manufacturer $m \in M$;

$B'^p_{wr}$ Backordered quantity of product $p \in P$ ordered by retailer $r \in R$ from warehouse $w \in W$;

$V_{mw}$ Number of vehicles required to transport the products supplied by manufacturer $m \in M$ to warehouse $w \in W$;

$V'_{wr}$ Number of vehicles required to transport the products supplied by warehouse $w \in W$ to retailer $r \in R$;

$X_{mw}$ Binary auxiliary variable taking the value 1 if manufacturer $m \in M$ supplies, at least partially, warehouse $w \in W$ and 0 otherwise;

$Y_{wr}$ Binary auxiliary variable taking the value 1 if warehouse $w \in W$ supplies, at least partially, retailer $r \in R$ and 0 otherwise;

$Z_{mp}$ Binary auxiliary variable taking the value 1 if manufacturer $m \in M$ produces product $p \in P$ and 0 otherwise.

$$\begin{aligned} MinZ = & \sum_{m\in M}\sum_{w\in W} X_{mw}\, S_m + \sum_{w\in W}\sum_{r\in R} Y_{wr}\, S'_w + \sum_{m\in M}\sum_{w\in W} T_{mw}\, V_{mw} + \sum_{w\in W}\sum_{r\in R} T'_{wr}\, V'_{wr} \\ & + \sum_{m\in M}\sum_{w\in W}\sum_{p\in P} C_{mp}\, Q^p_{mw} + \sum_{w\in W}\sum_{r\in R}\sum_{p\in P} C'_{wp}\, Q'^p_{wr} + \sum_{m\in M}\sum_{w\in W}\sum_{p\in P} \pi_p\, B^p_{mw} \\ & + \sum_{w\in W}\sum_{r\in R}\sum_{p\in P} \pi'_p\, B'^p_{wr} + \sum_{m\in M}\sum_{w\in W}\sum_{p\in P} H_{mp}\frac{Q^p_{mw}}{2} + \sum_{m\in M}\sum_{w\in W}\sum_{p\in P} H'_{wp}\frac{Q^p_{mw}}{2} \\ & + \sum_{w\in W}\sum_{r\in R}\sum_{p\in P} H''_{rp}\frac{Q'^p_{wr}}{2}, \end{aligned} \tag{1}$$

Subject to

$$\sum_{w\in W}\left(Q'^p_{wr} + B'^p_{wr}\right) = D_{rp}, \qquad \forall r \in R, p \in P, \tag{2}$$

$$\sum_{r\in R}\left(Q'^p_{wr} + B'^p_{wr}\right) = \sum_{m\in M}\left(Q^p_{mw} + B^p_{mw}\right), \qquad \forall w \in W, p \in P, \tag{3}$$

$$\sum_{r\in R} Q'^p_{wr} \le \sum_{m\in M} Q^p_{mw}, \qquad \forall w \in W, p \in P, \tag{4}$$

$$\sum_{w\in W} B'^p_{wr} \le \beta_r\, D_{rp}, \qquad \forall r \in R, p \in P, \tag{5}$$

$$\sum_{w\in W} B^p_{mw} \le UB_{mp}, \qquad \forall m \in M, p \in P, \tag{6}$$

$$\sum_{p\in P} Q^p_{mw} \ge L_m\, X_{mw}, \qquad \forall m \in M, w \in W, \tag{7}$$

$$\sum_{p\in P} Q'^p_{wr} \ge L'_w\, Y_{wr}, \qquad \forall w \in W, r \in R, \tag{8}$$

$$\sum_{p\in P} Q^p_{mw} \le \mathcal{M}\, X_{mw}, \qquad \forall m \in M, w \in W, \tag{9}$$

$$\sum_{p\in P} Q'^p_{wr} \le \mathcal{M}\, Y_{wr}, \qquad \forall w \in W, r \in R, \tag{10}$$

$$\sum_{w\in W} Q^p_{mw} \ge \mathcal{L}_{mp}\, Z_{mp}, \qquad \forall m \in M, p \in P, \tag{11}$$

$$\sum_{w\in W} Q^p_{mw} \le \mathcal{U}_{mp}\, Z_{mp}, \qquad \forall m \in M, p \in P, \tag{12}$$

$$\sum_{w\in W}\sum_{p\in P}\left(Q^p_{mw}\, \tau_p\right) \le \mathcal{T}_m, \qquad \forall m \in M, \tag{13}$$

$$\sum_{m\in M}\sum_{p\in P}\left(Q^p_{mw}\, C_{mp}\right) \le \mathcal{B}_w, \qquad \forall w \in W, \tag{14}$$

$$\sum_{m\in M}\sum_{p\in P}\left(Q_{mw}^{p}\,\nu_p\right)\le \mathcal{C}_w, \qquad \forall w\in W, \tag{15}$$

$$\frac{\sum_{p\in P}\left(Q_{mw}^{p}\,\omega_p\right)}{VC}\le V_{mw}, \qquad \forall m\in M, w\in W, \tag{16}$$

$$\sum_{w\in W}V_{mw}\le N_m, \qquad \forall m\in M, \tag{17}$$

$$\frac{\sum_{p\in P}\left(Q_{wr}^{\prime p}\,\omega_p\right)}{VC'}\le V'_{wr}, \qquad \forall w\in W, r\in R, \tag{18}$$

$$\sum_{r\in R}V'_{wr}\le N'_w, \qquad \forall w\in W, \tag{19}$$

$$Q_{mw}^{p}, Q_{wr}^{\prime p}, B_{mw}^{p}, B_{wr}^{\prime p}, V_{mw}, V'_{wr}\in \mathcal{Z}_0^{+}, \qquad \forall m\in M, w\in W, r\in R, p\in P, \tag{20}$$

$$X_{mw}, Y_{wr}, Z_{mp}\in\{0,1\}, \qquad \forall m\in M, w\in W, r\in R, p\in P. \tag{21}$$

The objective function of the MILP model (1) minimizes the total costs, which comprise i) the fixed costs of ordering from both the manufacturers and the warehouses; ii) the costs of dispatching vehicles from both the manufacturers and the warehouses; iii) the costs of buying the products from both the manufacturers and the warehouses; iv) the backorder costs for both the manufacturers and the warehouses; v) the holding costs incurred at the manufacturers, the warehouses, and the retailers.

Constraints (2) ensure demand satisfaction for all products, either in the current period or at a later period, in the latter case, it is backordered; while constraints (3) ensure that for each warehouse and each product the total quantity, committed and backordered, is the same as the product total quantity received and backordered at the manufacturers. In addition, warehouses cannot supply to the retailers more product than received from the manufacturers, as stated in (4). Service level is ensured by constraints (5) and (6), which limit the backordered quantities as desired.

Minimum order quantities are ensured regarding both the manufactures and the warehouses by constraints (7) and (8), respectively, after ensuring the correctness of the auxiliary variables in constraints (9) and (10), respectively. Three production limits are enforced. On the one hand, a minimum (economic viability) and a maximum (e.g., specific resources availability or production mix) quantity is required for each product, as stated in constraints (11) and (12), respectively. On the other hand, the total production capacity must be satisfied, as stated in (13). Warehouse limits regarding purchasing budget and storage capacity are imposed in (14) and (15), respectively.

The number of required vehicles by each manufacturer to supply each warehouse is determined in (16) the total number of vehicles needed by each manufacturer is ensured to be no more than available ones in (17). Similarly, constraints (18) and (19) ensure the correct number of vehicles for each warehouse. Finally, constraints (20) define the decision variables as non-negative integers and constraints (21) define the auxiliary variables as binary.

## 4. Computational Experiments

To illustrate the efficiency of the proposed MILP model a set of seven small-sized numerical problem instances were designed. For two of the problem instances (ES1 and ES4), we generated all parameters (i.e., demands, costs, capacities, allowances, and number of available vehicles) randomly. In these two instances, all products can, potentially, be produced by all the manufacturers. Instances ES2 and ES5 have the same characteristics of ES1 and ES4, respectively; however, the values of the cost parameters are 20% higher than those of instances ES1 and ES4, respectively. In contrast, a 20% reduction in the values of the parameters is imposed when generating instances ES3 and ES6, which have the same characteristics of instances ES1 and ES4, respectively. Finally, in instance ES7, which has the same characteristics and parameters as instance ES4, manufacturers are no longer able to produce all products. In fact, each manufacturer can only produce a subset of the products considered.

For ES1, demand of each retailer for each product was randomly generated by a uniform distribution between 400 and 500, marginal purchase costs from the manufacturers were generated randomly by a uniform distribution between 18 and 26, and they were, on average, increased by 5 units for each product purchased from the warehouses; the marginal backorder costs for the manufacturers and warehouses, for each product, are to be approximately 2 and 3 times the purchasing costs of that product, respectively; the holding costs during the planning period are on average 3.3%, 6.7%, and 10% of the purchasing cost of each product for manufacturers, warehouses and retailers, respectively; and the fixed ordering costs were set to 1000 for each retailer and 500 for each manufacturer. The same methodology was used to generate instance ES4, while the demand of each retailer for each product was generated randomly between 200 and 300, and the purchasing cost from the manufacturers were generated randomly between 18 and 33.

The MILP model was implemented in and solved by Gurobi® software package (Gurobi, 2018), and it was run on a 3.20 GHz Intel® Core™ i7-8700 PC with 24 GB RAM. Table 1 reports the characteristics of the problem instances (number of manufacturers, warehouses, retailers, and products), the optimal supply cost ($Z^*$), and the computational time in seconds ($Time$) required to find it. The following columns provide the number of nodes and the number of iterations required to solve the instances.

Table 1. Results for the small-sized problem instances.

| **Instances** | **Characteristics** | $Z^*$ | $Time$ | **Nodes** | **Iterations** |
|---|---|---|---|---|---|
| ES1 | 3*3*6*3 | 475,217 | 0.79 | 6,169 | 48,332 |
| ES2 | 3*3*6*3 | 570,261 | 1.04 | 10,770 | 68,752 |
| ES3 | 3*3*6*3 | 381,574 | 1.32 | 12,222 | 83,165 |
| ES4 | 3*5*10*4 | 690,469 | 12.85 | 52,268 | 1,439,579 |
| ES5 | 3*5*10*4 | 834,343 | 7.83 | 30,814 | 1,121,098 |
| ES6 | 3*5*10*4 | 548,522 | 6.51 | 21,181 | 681,990 |
| ES7 | 3*5*10*4 | 811,903 | 35.07 | 188,546 | 7,059,209 |

Clearly, as the size of the problem increases the time to solve the model also increases. However, from our experiments, it seems that the number of products that manufacturers can produce plays an important role in the required computational time to solve the model. This can be seen from the comparison of the computational times required to find an optimal solution for instances ES7 and ES4; the former being almost a third of the latter. Note that, in instance ES4 each manufacturer can produce any of the four products, while in instance ES7 this figure comes down to three.

Figure 3 illustrates the five main categories of the network's supply costs in instances ES1 to ES7, i.e., total fixed ordering costs (TFC), total transport costs (TTC), total purchasing costs (TPC), total backordering costs (TBC), and total holding costs (THC). On average, 78.4% of the network's supply costs are due to the purchasing costs, followed by 15.9%, 2.2%, 1.8%, and 1.7% of the costs due to holding costs, backorder costs, ordering costs, and transportation costs, respectively.

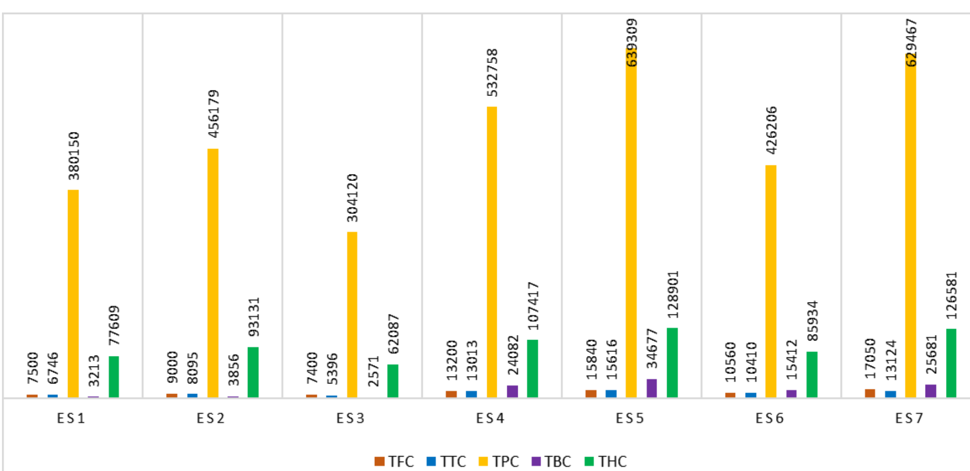

Figure 3. Detailed cost categories in the problem instances

## 5. Conclusion

This paper introduces a mixed integer linear programming model for the multi-product ordering under the vendor managed inventory policy in a three-echelon SCN. Its novelty comes from the simultaneous optimization of order quantities and number of shipments. Furthermore, the problem considered has features not studied yet in the optimization of such a problem in a three-echelon SCN, such as the allowance of backorder. The computational results obtained show that the proposed modelling approach, in addition to being novel, is capable of quickly finding optimal solutions for small-sized problem instances.

In the near future, we will further our computational experiments and also develop heuristic approaches so that real-sized problem instances can be tackled.

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