A Review of Adomian Decomposition Method and Applied to Deferential Equations

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Abstract

Adomian decomposition method is a numerical method that introduced by George Adomian to solve stochastic equations. This method is able to solve equations without linearization, discretization, perturbation or other restrictive assumptions. This method can also be used to solve differential equations with integer or fractional order, ordinary or partial, with initial value or boundary problems, with variable or constant coefficients, linear or nonlinear, homogeneous or nonhomogeneous. Thus, the purpose of this paper is to review the application of the Adomian decomposition method to find solutions for various equations. For example ordinary and partial differential equations, also with fractional order. There is also a review of the Adomian decomposition method developed with Laplace transform. Heat and Black-Scholes equations can also be easily solved by this decomposition method. The results show that Adomian decomposition method is an effective and easy algorithm to solve various differential equations.

Keywords:
Adomian decomposition method (ADM), differential equation, Laplace transform, heat equation, Black-Scholes.

1. Introduction

The Adomian decomposition method was first introduced by George Adomian to solve the system of stochastic equations (Adomian, 1980). This decomposition method can be an effective procedure for obtaining analytical solutions without linearization or weak nonlinear assumptions, perturbation theory or restrictive assumptions on stochastic cases (Adomian, 1988). This method can be used to solve integral, differential and integral-differential equations. Differential equations that can be solved by this method can have order of integers or fractional numbers, ordinary or partial, with initial value or boundary problems, with variable or constants coefficients, linear or nonlinear, homogeneous or nonhomogeneous (Bellman and Adomian, 1985; Ray and Bera, 2005; Nhawu et al., 2016; Al awadah, 2016). This decomposition method is also a powerful and useful technique for solving heat, waves (Biazar and Amirtaimoori, 2005; Jafari and Daftardar-gejji, 2006), Fokker-Plank (Tatari et al., 2007) and Black-Scholes equation for pricing option (Bohner and Zheng, 2009; Biazar and Goldoust, 2013). Using the Caputo derivative, the Adomian decomposition method can solve the fractional Korteweg de Vries (Momani, 2005) and Burger equation (Gepreel, 2012). The numerical scheme of Laplace transform based on the modified Adomian decomposition method can be used to solve nonlinear differential equations. The main advantage of this technique is that solutions are expressed as infinite series that converge quickly to the exact solution (Khuri, 2001; Naghipour & Manafian, 2015).

The aim of this paper is to review the application and algorithm of the Adomian decomposition method to find solutions ordinary, partial differential equations, also those with fractional order. There is also a review of the Adomian decomposition method developed with Laplace transform. Heat and Black-Scholes equations can also be easily solved by this decomposition method.

Here is the systematics of writing this paper. Section 2 is the basic theory of the Adomian decomposition method. Section 3 is the solution of ordinary differential equations using the Adomian decomposition method, partial differential equations in Section 4 and fractional order in Section 5. Then, Section 6 presents the algorithm Adomian decomposition method with Laplace transform. Section 7 applies the Adomian decomposition method to solve heat equations and Black-Scholes for option pricing in Section 8. Finally, the conclusion is presented in Section 9.

2. Basic Theory Adomian Decomposition Method

In this section, we describe the Adomian decomposition method (ADM) that refer to (Adomian, 1988; Al awawdah, 2016). Given the general equation

$$Mu + Nu + Ru = g, \tag{1}$$

where u is the unknown function, M is the linear term which is easily invertible, so that assumes the inverse of M is M^{-1}, N is the nonlinear terms and R is the reminder of the linear operator. Solving for Mu, equation (1) can be written

$$Mu = g - Nu - Ru. \tag{2}$$

Applying M^{-1} to (2)

$$M^{-1}Mu = M^{-1}(g - Nu - Ru)$$

thus obtained

$$u = \Phi + M^{-1}g - M^{-1}(Nu + Ru) \tag{3}$$

where Φ is initial or boundary condition or both, according to the problem given the equation. This decomposition method assumes that solution u can be decomposed into infinite series

$$u = \sum_{n=0}^{\infty} u_n \tag{4}$$

or equivalent with

$$u = u_0 + u_i + u_u + \cdots,$$

and the nonlinear term Nu can be decomposed into infinite series

$$Nu = \sum_{n=0}^{\infty} A_n \tag{5}$$

where $A_n = A_n(u_0, u_1, \cdots, u_n)$ is Adomian polynomials which is defined by

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{n}\lambda^i u_i\right)\right]_{\lambda=0}; \qquad n = 0, 1, 2, \cdots. \tag{6}$$

If A_n is described, then obtaining

$$\begin{aligned}
A_0 &= \frac{1}{0!}\frac{d^0}{d\lambda^0}\left[N\left(\sum_{i=0}^{0}\lambda^i u_i\right)\right]_{\lambda=0} = N(u_0),\\
A_1 &= \frac{1}{1!}\frac{d^1}{d\lambda^1}\left[N\left(\sum_{i=0}^{1}\lambda^i u_i\right)\right]_{\lambda=0} = u_1 N'(u_0),\\
A_2 &= \frac{1}{2!}\frac{d^2}{d\lambda^2}\left[N\left(\sum_{i=0}^{2}\lambda^i u_i\right)\right]_{\lambda=0} = \frac{u_1^{\;2}}{2!}N'(u_0) + u_2 N(u_0),\\
&\vdots
\end{aligned}$$

Substituting both of infinite series (4) and (5) to (3)

$$\sum_{n=0}^{\infty} u_n = \Phi + M^{-1}g - M^{-1}\left(\sum_{n=0}^{\infty} A_n + \sum_{n=0}^{\infty} Ru_n\right) \tag{7}$$

Based on (7), recursive relation of the solution as follows

$$\begin{aligned}
u_0 &= \Phi + M^{-1}g,\\
u_{n+1} &= -M^{-1}(A_n + Ru_n); \qquad n = 0, 1, 2, \cdots.
\end{aligned} \tag{8}$$

The ADM can be an effective procedure for obtaining analytical solutions without linearization or weak nonlinear assumptions, perturbation theory or restrictive assumptions on stochastic cases. This method can be used to solve integral, differential and integral-differential equations. Differential equations that can be solved by this method can have order of integers or fractional numbers, ordinary or partial, with initial value or boundary problems, with variable or constants coefficients, linear or nonlinear, homogeneous or nonhomogeneous.

3. Ordinary Differential Equation

Ordinary differential equation (ODE) is differential equation that contain a function of one independent variable and its derivatives. Dehghan and Tatari (2006) applied the Adomian decomposition method to find solutions to ODE that arise from problems of variation. In addition, Nhawu et al. (2016) used this decomposition method to solve the problem of modeling population growth has the law of exponential change through ODE that are linear and first order. Consider a nonlinear first order initial value of the ordinary differential equation

$$Mu(x) + Nu(x) + Ru(x) = g(x), \qquad u(0) = c. \tag{9}$$

In this case $M = d/dx$ is the differential operator and inverse of M is integral operator $M^{-1} = \int_0^x [\cdot]dx,$ where N is the nonlinear, R is the remainder linear term and g is a given function. Solving for Mu, equation (9) can be written

$$Mu(x) = g(x) - Nu(x) - Ru(x). \tag{10}$$

Applying M^{-1} to (10)

$$M^{-1}Mu(x) = M^{-1}\left(g(x) - Nu(x) - Ru(x)\right)$$
$$\int_0^x \frac{du}{dx}\,dx = M^{-1}g(x) - M^{-1}(Nu(x) + Ru(x))$$
$$u(x) - u(0) = M^{-1}g(x) - M^{-1}(Nu(x) + Ru(x))$$

thus obtained

$$u(x) = u(0) + M^{-1}g(x) - M^{-1}(Nu(x) + Ru(x)). \tag{11}$$

ADM assumes that solution u can be decomposed into infinite series (4) and the nonlinear term Nu can be decomposed into infinite series (5), so substituting both of infinite series to (11)

$$\sum_{n=0}^{\infty} u_n = u(0) + M^{-1}g(x) - M^{-1}\left(\sum_{n=0}^{\infty} A_n + \sum_{n=0}^{\infty} Ru_n\right). \tag{12}$$

Based on (12) and substituting the initial value, recursive relation of the solution as follows

$$\begin{aligned} u_0 &= c + \int_0^x g(x)\,dx, \\ u_{n+1} &= -\int_0^x \left(A_n + Ru_n\right)dx; \qquad n = 0, 1, 2, \cdots. \end{aligned} \tag{13}$$

4. Partial Differential Equation

If a differential equation contains a function with more than one independent variable then it is called partial differential equation (PDE). Bougoffa and Rach (2013) present a new approach of ADM using Fourier series to solve nonlocal initial-boundary value problems for linear and nonlinear parabolic and hyperbolic PDE which is transformed into local Dirichlet initial-boundary value problems. Pourgholi and Saeedi (2015) use ADM for solving inverse nonlocal initial-boundary problems in PDE, where the problems is mildly ill-posed using the Tikhonov regularization methods to deal with noisy input data. In his study, Al awawdah (2016) use ADM for obtaining solution of PDE and consider different types of its inverse, boundary conditions, coefficient and source identifications, also try to solve the heat conduction inverse problem in special cases. Consider the nonlinear partial differential equation initial value problem

$$M_t u(x,t) + Nu(x,t) + Ru(x,t) = g(x,t), \qquad u(x,0) = f(x). \tag{14}$$

In this case $M_t = \partial / \partial t$ is the differential operator and inverse of M_t is integral operator $M_t^{-1} = \int_0^t [\cdot]dt,$ where N is the nonlinear, R is the remainder linear term and g is a given function. Solving for $M_t u$ equation (14) can be written

$$M_t u(x,t) = g(x,t) - Nu(x,t) - Ru(x,t). \tag{15}$$

Applying M_t^{-1} to (15)

$$M_t^{-1}M_t u(x,t) = M_t^{-1}\left(g(x,t) - Nu(x,t) - Ru(x,t)\right)$$
$$\int_0^t \frac{\partial u}{\partial t}\,dt = M_t^{-1}g(x,t) - M_t^{-1}(Nu(x,t) + Ru(x,t))$$
$$u(x,t) - u(x,0) = M_t^{-1}g(x,t) - M_t^{-1}(Nu(x,t) + Ru(x,t))$$

thus obtained

$$u(x,t) = u(x,0) + M_t^{-1} g(x,t) - M_t^{-1} (Nu(x,t) + Ru(x,t)). \tag{16}$$

ADM assumes that solution u can be decomposed into infinite series (4) and the nonlinear term Nu can be decomposed into infinite series (5), so substituting both of infinite series to (16)

$$\sum_{n=0}^{\infty} u_n = u(x,0) + M_t^{-1} g(x,t) - M_t^{-1} \left(\sum_{n=0}^{\infty} A_n + \sum_{n=0}^{\infty} Ru_n \right). \tag{17}$$

Based on (17) and substituting the initial value, recursive relation of the solution as follows

$$\begin{aligned} u_0 &= f(x) + \int_0^t g(x,t)\, dt, \\ u_{n+1} &= -\int_0^t \left(A_n + Ru_n \right) dt; \qquad n = 0, 1, 2, \cdots. \end{aligned} \tag{18}$$

5. Fractional Differential Equation

If an ordinary or partial differential equation contains a fractional order then it is called a fractional differential equation (FDE). Ray and Bera (2005) apply ADM for solving a nonlinear fractional ordinary differential equations with Riemann-Liouville derivatives. Gepreel (2012) use ADM to construct the approximate solution for partial differential equations with time and space fractional derivative in the sense of Caputo, for example wave and Burger equation. Yavuz (2018) solve some initial boundary value problems of fractional partial differential equations by using conformable fractional Adomian decomposition method. Before applying ADM to solve FDE, here is notation, basic definition and main properties of fractional calculus theory.

Definition 1 (Mathai & Haubold, 2017) The Riemann-Liouville fractional derivative of y with order $\alpha > 0$ is defined

$$D^{\alpha} y(x) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dx^n} \int_0^x (x-t)^{n-\alpha-1} y(t)\, dt, \quad n-1 < \alpha \le n.$$

Definition 2 (Mathai & Haubold, 2017) The Caputo fractional derivative of y with order $\alpha > 0$ is defined as

$${}_C D^{\alpha} y(x) = \frac{1}{\Gamma(n-\alpha)} \int_0^x (x-t)^{n-\alpha-1} y^{(n)}(t)\, dt, \quad n-1 < \alpha \le n.$$

Theorem 3 (Vance, 2014) The Caputo fractional derivative with order $\alpha > 0$ and $n-1 < \alpha \le n$ of $y(x) = t^{\beta}$ for $\beta \ge 0$ is

$$D^{\alpha} x^{\beta} = \begin{cases} \dfrac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)} x^{\beta-\alpha} & \beta > n-1 \\ 0 & \beta \le n-1. \end{cases}$$

Note that the relation between Riemann-Liouville and Caputo fractional differential operator is given as follows

$$J^{\alpha} D_x^{\alpha} y(x) = D_x^{-\alpha} D_x^{\alpha} y(x) = y(x) - \sum_{k=0}^{n-1} \frac{x^k}{k!} y^{(k)}(0), \qquad n-1 < \alpha \le n. \tag{19}$$

Consider a nonlinear fractional partial differential equation (FPDE)

$$D_t^\alpha u(x,t) + Nu(x,t) + Ru(x,t) = g(x,t), \qquad u(x,0) = f(x). \tag{20}$$

In this case $D_t^\alpha = \partial^\alpha / \partial t^\alpha$ is the fractional differential operator and inverse of D_t^α is fractional integral operator $J^\alpha = D_t^{-\alpha}$ where N is the nonlinear, R is the remainder linear term and g is a given function. Solving for $D_t^\alpha u$, equation (20) can be written

$$D_t^\alpha u(x,t) = g(x,t) - Nu(x,t) - Ru(x,t). \tag{21}$$

Apply J^α to (21) by using (19)

$$\begin{aligned} J^\alpha D_t^\alpha u(x,t) &= J^\alpha \left(g(x,t) - Nu(x,t) - Ru(x,t) \right) \\ u(x,t) - u(x,0) &= J^\alpha g(x,t) - J^\alpha (Nu(x,t) + Ru(x,t)) \end{aligned}$$

thus obtained

$$u(x,t) = u(x,0) + J^\alpha g(x,t) - J^\alpha (Nu(x,t) + Ru(x,t)) \tag{22}$$

ADM assumes that solution u can be decomposed into infinite series (4) and the nonlinear term Nu can be decomposed into infinite series (5), so substituting both of infinite series to (22)

$$\sum_{n=0}^{\infty} u_n = u(x,0) + J^\alpha g(x,t) - J^\alpha \left(\sum_{n=0}^{\infty} A_n + \sum_{n=0}^{\infty} Ru_n \right). \tag{23}$$

Based on (23) and substituting the initial value, recursive relation of the solution as follows

$$\begin{aligned} u_0 &= f(x) + J^\alpha g(x,t), \\ u_{n+1} &= -J^\alpha \left(A_n + Ru_n \right); \qquad n = 0, 1, 2, \cdots. \end{aligned} \tag{24}$$

6. Laplace Adomian Decomposition Method

In this section, we describe the algorithm of Laplace Adomian decomposition method (LADM). Khuri (2001) introduced a numerical Laplace transform algorithm with is based on the decomposition method for the approximate solution of nonlinear differential equations. Hussain and Khan (2010) presented the modified Laplace decomposition method in their work. Naghipour and Manafian (2015) applied the modified LADM for solving the Burgers' equation. Haq et al. (2018) used LADM to find numerical solution of fractional order smoking model. Consider the partial differential equation

$$M_t u(x,t) + Nu(x,t) + Ru(x,t) = g(x,t), \qquad u(x,0) = f(x). \tag{25}$$

In this case $M_t = \partial / \partial t$ is the differential operator, where N is the nonlinear, R is the remainder linear term and g is a given function. Solving for $M_t u,$ equation (25) can be written

$$M_t u(x,t) = g(x,t) - Nu(x,t) - Ru(x,t). \tag{26}$$

The Laplace transform is the transformation of the integral function of a real variable t to the function of a complex variable s, introduced by Pierre-Simon Laplace. Laplace transform can be used to solve differential equations by turning them into algebraic equations (Potter, 2019; Schiff, 1999). Before using the ADM combined with Laplace transform, here are some basic definition and theory of Laplace transform.

Definition 3 (Schiff, 1999) Suppose that *f* is a real or complex function of $t > 0$ and *s* is a real or complex parameter. Laplace transform is defined

$$F(s) = L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt = \lim_{b\to\infty} \int_0^b e^{-st} f(t)\,dt,$$

where the limit value exist and finite. If $L[f(t)] = F(s),$ then the inverse of Laplace transform is denoted as

$$L^{-1}[F(s)] = f(t), \quad t \geq 0.$$

Based on Definition 3, for $f(t) = t^n$ where $t \geq 0,$ Laplace transform $f(t)$ is $L[t^n] = \frac{n!}{s^{n+1}}, s > 0,$ and Laplace transform for $f^{(n)}(t)$ is

$$L[f^{(n)}(t)] = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - \cdots - f^{(n-1)}(0) = s^n F(s) - \sum_{k=1}^{n} s^{n-k} f^{(k-1)}(0).$$

The technique consists first applying Laplace transform to (26)

$$L[M_t u(x,t)] = L[g(x,t) - Nu(x,t) - Ru(x,t)]$$
$$L\left[\frac{\partial u}{\partial t}\right] = L[g(x,t)] - L[Nu(x,t) + Ru(x,t)]$$
$$su(x,s) - u(x,0) = L[g(x,t)] - L[Nu(x,t) + Ru(x,t)].$$

Inhomogeneous case, $g(x,t) = 0,$ thus obtained

$$u(x,s) = \frac{u(x,0)}{s} - \frac{1}{s} L[Nu(x,t) + Ru(x,t)]. \tag{27}$$

Then, apply inverse of Laplace transform to (27)

$$u(x,t) = u(x,0) - L^{-1}\left[\frac{1}{s} L[Nu(x,t) + Ru(x,t)]\right]. \tag{28}$$

ADM assumes that solution *u* can be decomposed into infinite series (4) and the nonlinear term *Nu* can be decomposed into infinite series (5), so substituting both of infinite series to (28)

$$\sum_{n=0}^{\infty} u_n = u(x,0) - L^{-1}\left[\frac{1}{s} L\left[\sum_{n=0}^{\infty} A_n + \sum_{n=0}^{\infty} Ru_n\right]\right]. \tag{29}$$

Based on (29) and substituting the initial value, recursive relation of the solution as follows

$$\begin{aligned} u_0 &= f(x), \\ u_{n+1} &= -L^{-1}\left[\frac{1}{s} L[A_n + Ru_n]\right]; \qquad n = 0, 1, 2, \cdots. \end{aligned} \tag{30}$$

7. Heat Equation

In this section, we apply ADM to solve heat equation that refer to (Biazar and Amirtaimoori, 2005; Pamuk, 2005) which governs on numerous scientific, engineering experimentations and mathematical biology. Jafari and Daftardar-Gejji (2006) used ADM to obtain solutions of linear or nonlinear fractional diffusion equations. Given the general form of heat equation as follows

$$\frac{\partial p}{\partial t} = A(x,y,z,t)\frac{\partial^2 p}{\partial x^2} + B(x,y,z,t)\frac{\partial^2 p}{\partial y^2} + C(x,y,z,t)\frac{\partial^2 p}{\partial z^2} + D(x,y,z,t) \tag{31}$$

with the initial condition $P(x,y,z,0) = f(x,y,z)$.

For solving this equation by ADM, the equation should be in canonical form which can be derived by rewriting (31) as follows

$$M_t p = D + Np \tag{32}$$

where $M_t = \frac{\partial}{\partial t}$ and inverse of M_t is integral operator $M_t^{-1} = \int_0^t [\cdot] dt$, where $N = A\frac{\partial^2}{\partial x^2} + B\frac{\partial^2}{\partial y^2} + C\frac{\partial^2}{\partial z^2}$ is the nonlinear term. Applying M_t^{-1} to (32)

$$\begin{aligned} M_t^{-1} M_t p &= M_t^{-1}\left(D + Np\right) \\ \int_0^t \frac{\partial p}{\partial t} dt &= M_t^{-1} D + M_t^{-1} Np \\ p(x,y,z,t) - p(x,y,z,0) &= M_t^{-1} D + M_t^{-1} Np \end{aligned}$$

thus obtained

$$p(x,y,z,t) = p(x,y,z,0) + M_t^{-1} D + M_t^{-1} Np. \tag{33}$$

ADM assumes that solution *u* can be decomposed into infinite series (4) and the nonlinear term *Nu* can be decomposed into infinite series (5), so substituting both of infinite series to (33)

$$\sum_{n=0}^{\infty} p_n = p(x,y,z,0) + M_t^{-1} D + M_t^{-1}\left(\sum_{n=0}^{\infty} A_n\right). \tag{34}$$

Based on (34) and substituting the initial value, recursive relation of the solution as follows

$$\begin{aligned} p_0 &= f(x,y,z) + \int_0^t D(x,y,z,t)\, dt, \\ p_{n+1} &= \int_0^t A_n\, dt; \qquad n = 0,1,2,\cdots. \end{aligned} \tag{35}$$

8. Black-Scholes Equation

The Black-Scholes equation with boundary condition for a European option pricing problem is the most well-known model for pricing financial derivatives. Bohner and Zheng (2009) presented a theoretical analysis solution of the Black-Scholes equation is solved by using the Adomian approximate decomposition technique. Similar research was carried out by Biazar and Goldoust (2013). Ghandehari and Ranjbar (2014) applied ADM to solve the fractional Black-

Scholes equation. González et al. (2017) used ADM to obtain solution of the nonlinear equation the generalized Black-Scholes model that considers the volatility as a non-constant function. Consider the Black-Scholes equation

$$\frac{\partial v}{\partial \tau} = \frac{\partial^2 v}{\partial x^2} + (k-1)\frac{\partial v}{\partial x} - kv \tag{36}$$

with the initial condition $v(x,0) = \max\{e^x - 1, 0\}$.

For solving this equation by ADM, the equation should be in canonical form which can be derived by rewriting (36) as follows

$$M_\tau v = Nv \tag{37}$$

where $M_\tau = \frac{\partial}{\partial t}$ and inverse of M_τ is integral operator $M_\tau^{-1} = \int_0^\tau [\cdot] d\tau$, where $N = \frac{\partial^2}{\partial x^2} + (k-1)\frac{\partial}{\partial x} - k$ is the nonlinear term. Applying M_τ^{-1} to (37)

$$\begin{aligned} M_\tau^{-1} M_\tau p &= M_\tau^{-1} Nv \\ \int_0^\tau \frac{\partial v}{\partial \tau} d\tau &= M_\tau^{-1} Nv \\ v(x,\tau) - v(x,0) &= M_\tau^{-1} Nv \end{aligned}$$

thus obtained

$$v(x,\tau) = v(x,0) + M_\tau^{-1} Nv \tag{38}$$

ADM assumes that solution *u* can be decomposed into infinite series (4) and the nonlinear term *Nu* can be decomposed into infinite series (5), so substituting both of infinite series to (38)

$$\sum_{n=0}^{\infty} v_n = v(x,0) + M_\tau^{-1}\left(\sum_{n=0}^{\infty} A_n\right). \tag{39}$$

Based on (39) and substituting the initial value, recursive relation of the solution as follows

$$\begin{aligned} v_0 &= \max\{e^x - 1, 0\}, \\ v_{n+1} &= \int_0^\tau A_n d\tau; \qquad n = 0, 1, 2, \cdots. \end{aligned} \tag{40}$$

9. Conclusion

The Adomian decomposition method (or combined by Laplace transform) can be an effective, straightforward and powerful technique for solving ordinary or partial differential equation, which is providing generally a rapidly convergent series solution, also fractional differential equations. This method provides several advantages such as easily computable and be able to obtain analytical or approximate solutions that can be accepted without perturbation, linearization and discretization, or resulting massive computation. The ADM assumes the nonlinear term can be decomposed into infinite series which loads Adomian polynomials. This decomposition method is also capable, successful and efficient to solve heat and Black-Scholes equation.

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