

A Clustering Algorithm for Location Routing Problem with Outsourced Delivery

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Abstract

In this paper, the location routing problem (LRP) with outsourced delivery is studied. The objective of LRP is to determine the best combination of depot locations and vehicle routes that minimizes the total costs. In the classical LRP, all products are delivered with non-outsourced delivery, which is the delivery mode where all customers are included in the delivery routes. Non-outsourced delivery is efficient if customers are densely located. However, if customers are sparsely located, outsourced delivery is more efficient. In outsourced delivery, products are delivered to customers directly by third-party delivery services. In this study, the outsourced delivery mode is added to the LRP model so that suitable transportation modes are selected. Introducing this model makes the LRP more complicated. To solve the problem efficiently, a clustering-based heuristic is proposed. Test instances based on LRP benchmark instances were solved by using the algorithm. The results of our computational experiments showed the proposed algorithm can find a 20% better solution than MIP within an hour for problems with a hundred customers.

Keywords

Location routing problem, Outsourced delivery, Clustering and Heuristics

1. Introduction

It is essential for logistics businesses to solve the location routing problem (LRP) to reduce delivery costs. The LRP is the combined problem of the facility location problem (FLP) and the vehicle routing problem (VRP). The objective of the LRP is to determine the best combination of depot locations and vehicle routes that minimizes the total costs, which include the fixed costs of the depot, fixed costs of the vehicle, and variable costs of the vehicle. Maranzana (1964) was aware of the need to decide the location of the facility and the dispatch route simultaneously. Since a heuristic algorithm for the LRP was first proposed by Jacobsen and Madsen (1980), many algorithms for the LRP were proposed. Tuzun and Burke (1999) applied a two-phase tabu search. Chan & Baker (2005) combined the minimum spanning forest and a modified Clarke-Wright procedure. To solve the capacitated LRP, Prins et al. (2007) used a Lagrangian relaxation of the assignment constraints and a granular tabu search, and Escobar et al. (2014) proposed algorithm that included a granular tabu search within a variable neighborhood search. Ponboon et al. (2016) tried to get exact solutions for the LRP with delivery time windows by using a branch and price algorithm. Farham et al. (2018) also applied a column generation approach for the LRP with delivery time windows.

For logistics, it is essential to solve the LRP, therefore, there are many variations. One of the variations is multi echelon LRP. Goodarzi and Zegordi (2016) tried to solve the two-echelon LRP with cross-docking. Rahmani et al. (2016) proposed the two-echelon LRP with pickup and delivery. A heterogeneous fleet with two-echelon capacitated

LRP for joint delivery was proposed by Zhao et al. (2018). Other variations are the open LRP (Vincent and Lin 2015), dynamic LRP (Gao et. al. 2016), and LRP with lateral transshipment.

There are also many studies on the applications of the LRP. Asefi et al. (2017) applied the LRP to the municipal solid waste process. Rabbani et al. (2019) solved the stochastic multi-period industrial hazardous waste LRP. Hiassat et al. (2017) used a genetic algorithm approach for the location-inventory-routing problem with perishable products. Soysal et al. (2018) considered a green inventory routing problem for perishable products. Yang and Sun (2015) integrated the decision of battery swap station location into the LRP. Çetinkaya et al. (2018) added arc time windows for terror regions. In real business, Hadian et al. (2019) tried to reduce the differences between the vehicles' travelled distances.

From the viewpoint of algorithms, metaheuristics, especially genetic algorithms (Lopes et al. 2016, Hiassat et al. 2017, and Wang et al. 2018), simulated annealing (Vincent and Lin 2015, Asefi et al. 2017, and Javad and Karimi 2017), and large or variable neighborhood search (Zhang et. al. 2015, Schiffer and Walther 2018, and Asefi et al. 2019), are used.

A survey of research on the LRP was published by Nagy and Salhi (2007). Prodhon and Prins (2014) summarized the research of general LRP studies from the viewpoint of algorithms after Nagy and Salhi (2007). The latest survey on LRP is Drexl and Schneider (2015), which includes many variants of LRP, such as multi-echelon LRPs, periodic LRPs, and LRPs with pickup and delivery.

As described above, various LRP models and solutions have been proposed. However, outsourced delivery has not been considered. In this study, outsourced delivery is introduced to the LRP to further improve delivery efficiency, and a heuristic for the LRP with outsourced delivery is proposed. The remainder of the paper is organized as follows. Section 2 formally describes the problem, Section 3 explains the proposed algorithm, Section 4 provides the computational results, and Section 5 concludes the paper.

2. Mathematical formulation

In this section, we explain the model for the LRP with outsourced delivery. In this model, the sets are defined as follows:

D: Set of customers, F: Set of potential facilities,

S: Set of sites; this set includes potential facilities and customers, V: Set of vehicles

The parameters are defined as follows:

DQ_d : Quantity required by customer d , TWR_d : Ready time of customer d , TWD_d : Due time of customer d

TS_d : Service time of customer d , CFS_f : Fixed costs of facility f , LX_s : X-coordinate of site s ,

LY_s : Y-coordinate of site s , VW_v : Capacity of vehicle v , TVW_v : Working time of vehicle v ,

CVS_v : Fixed costs of vehicle v , CVV_v : Variable costs of vehicle v ,

$CC_{d,f}$: Outsourced delivery costs from facility f to customer d , $TT_{s1,s2}$: Travel time from site $s1$ to site $s2$

The decision variables are defined as follows:

y_f : Binary variable for facility selection (1 if facility f is open and 0 otherwise)

$z_{f,v}$: Binary variable for vehicle assignment (1 if vehicle v is assigned to facility f and 0 otherwise)

$x_{v,s1,s2}$: Binary variable for delivery route selection (1 if vehicle v travels from site $s1$ to site $s2$ and 0 otherwise)

$u_{d,v}$: Binary variable for customer (1 if vehicle v delivers to customer d and 0 otherwise)

$qv_{d,v}$: Quantity delivered to customer d by vehicle v

$qd_{d,f}$: Quantity delivered to customer d by outsourced delivery from facility f

$ts_{f,v}$: Start time of vehicle v in facility f

$tl_{v,s}$: Departure time of vehicle v from site s

$ta_{v,s}$: Arrival time of vehicle v arrives at site s

The objective function is as follows:

$$\min \sum_{f \in F} (CFS_f \cdot y_f) + \sum_{f \in F} \sum_{v \in V} (CVS_v \cdot z_{f,v}) + \sum_{v \in V} \sum_{s1 \in S} \sum_{s2 \in S} (CVV_v \cdot TT_{s1,s2} \cdot x_{v,s1,s2}) + \sum_{d \in D} \sum_{f \in F} (CC_{d,f} \cdot qd_{d,f}) \quad (1)$$

The first term means the fixed costs of the facility, the second term means the fixed costs of the vehicle, the third term means the variable costs of the vehicle, and last term means the outsourced delivery costs.

The constraints are as follows:

$$\sum_{v \in V} qv_{d,v} + \sum_{f \in F} qd_{d,f} = DQ_d \quad (d \in D) \quad (2)$$

$$qv_{d,v} \leq M \cdot u_{d,v} \quad (d \in D, v \in V) \quad (3)$$

$$qd_{d,f} \leq M \cdot y_f \quad (d \in D, f \in F) \quad (4)$$

$$\sum_{v \in V} u_{d,v} \leq 1 \quad (d \in D) \quad (5)$$

$$\sum_{f \in F} z_{f,v} \leq 1 \quad (v \in V) \quad (6)$$

$$z_{f,v} \leq y_f \quad (v \in V, f \in F) \quad (7)$$

$$u_{d,v} \leq \sum_{f \in F} z_{f,v} \quad (d \in D, v \in V) \quad (8)$$

$$\sum_{d \in D} qv_{d,v} \leq VW_v \quad (v \in V) \quad (9)$$

$$x_{v,d,s} \leq u_{d,v} \quad (v \in V, d \in D, s \in S) \quad (10)$$

$$x_{v,s,d} \leq u_{d,v} \quad (v \in V, d \in D, s \in S) \quad (11)$$

$$z_{f,v} = \sum_{d \in D} x_{v,f,d} \quad (f \in F, v \in V) \quad (12)$$

$$z_{f,v} = \sum_{d \in D} x_{v,d,f} \quad (f \in F, v \in V) \quad (13)$$

$$\sum_{s1 \in S} x_{v,s1,s} = \sum_{s2 \in S} x_{v,s,s2} \quad (v \in V, s \in S) \quad (14)$$

$$qv_{d,v} \leq M \cdot \sum_{s \in S} x_{v,s,d} \quad (d \in D, v \in V) \quad (15)$$

$$x_{v,s1,s2} \leq \sum_{f \in F} z_{f,v} \quad (v \in V, s1 \in S, s2 \in S) \quad (16)$$

$$TWR_d \cdot u_{d,v} \leq ta_{v,d} \quad (d \in D, v \in V) \quad (17)$$

$$ta_{v,d} \leq TWD_d + M \cdot (1 - u_{d,v}) \quad (d \in D, v \in V) \quad (18)$$

$$ta_{v,d} + TS_d \cdot u_{d,v} \leq tl_{v,d} \quad (v \in V, d \in D) \quad (19)$$

$$tl_{v,s1} + TT_{v,s1,s2} \leq ta_{v,s2} + M \cdot (1 - x_{v,s1,s2}) \quad (v \in V, s1 \in S, s2 \in S) \quad (20)$$

Here, M is a large positive value. Eq.(2) shows that an order is delivered by a 3PL provider's own fleet and/or outsourced delivery. Eq. (3) guarantees that the order assignment flag will be 1 when the customer is delivered to by the vehicle. Eq. (4) shows that the facility is open when the item is shipped by outsourced delivery from the facility. Eqs. (5) and (6) show the upper limit of the assignment. Eqs.(7) and (8) show the relationships between the open facility, vehicle, and customer. Eq. (9) shows the capacity constraints of the vehicle. Eqs. (10) to (16) show the relationships between the origin and destination sites in the vehicle routing. Eqs. (17) to (20) show the constraints of the delivery time window, service time, and travel time.

3. Clustering-Based Heuristic for Location Routing Problem with Outsourced Delivery

3.1 Overview of clustering-based heuristic for LRP with outsourced delivery

A mixed integer programming (MIP) solver is a powerful solver to solve combinatorial optimization problems. However, an MIP solver needs a very long calculation time. For example, it may take more than a week to solve an LRP including five potential depots and 100 customers. In the real-world, it is necessary to solve LRPs within one hour. Therefore, we propose a clustering-based heuristic. The algorithm divides the original LRP into smaller

problems to make it easier to solve without reducing the optimization potential as far as possible. Each cluster is solved by MIP independently. The reason for applying MIP is that the target problem including delivery mode selection is more complicated than the classical LRP. Therefore, global optimization is more important. Some delivery routes of the original LRP are invalidated according to the solutions of the cluster.

Figure 1 shows an overview of the proposed algorithm. The first step is the clustering step. In this step, customers are clustered by their location, and each cluster is solved by the MIP independently. The next step is solving the original LRP whose routes are partially invalid. The routes between customers that are not selected in the cluster are invalidated. The locations of the facility, routes to/from the facility, and routes between clusters are determined in this step. The delivery modes are also selected in this step. The last step is the local search step. A solution has been improved by removal and insertion heuristics.

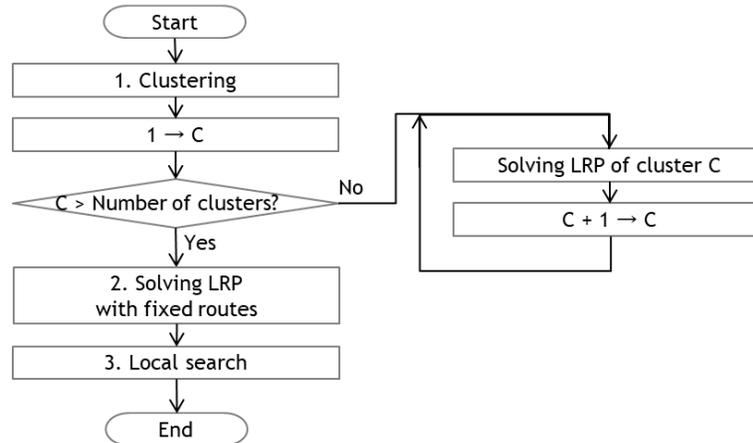


Figure 1 Overview of the proposed algorithm

In previous studies, k-means method (Gao et al. 2016) or metaheuristics were used. These algorithms are necessary to adjust the parameters for each problem. However, the criteria for parameter selection are not clarified. In this study, a novel clustering algorithm whose parameters are decided by the features of the problem is proposed.

3.2 Clustering algorithm

The purpose of clustering is to divide problems into smaller problems to make them easier to solve, without reducing the optimization potential as much as possible. From this viewpoint, the study clusters customers as follows.

$$t_{ij} \leq mt \quad (i, j \in C) \quad (21)$$

Where, t_{ij} is the travel time from site i to site j , and mt is the clustering criterion, which is the maximum travel time in a cluster.

Figure 2 shows the impact of the value of the clustering criteria. There are 50 customers in the problem. Four cases are shown, and they are clustered by different clustering criteria. The values of the clustering criterion are 6/8/10/13 for case (a),(b),(c), and (d), respectively. The open triangles represent customers that are not clustered. The solid triangles represent customers that are clustered, and customers that have the same color belong to the same cluster. Case (a) has six clusters. The clustering criterion is small; therefore, only nearby customers can belong to the cluster. Case (b) has eight clusters, and the size of the cluster bigger than that of case (a). Case (c) has a big cluster and a small cluster. Case (d) has only one big cluster. These cases show that the value of the clustering criteria has a large impact and selecting the value properly is important.

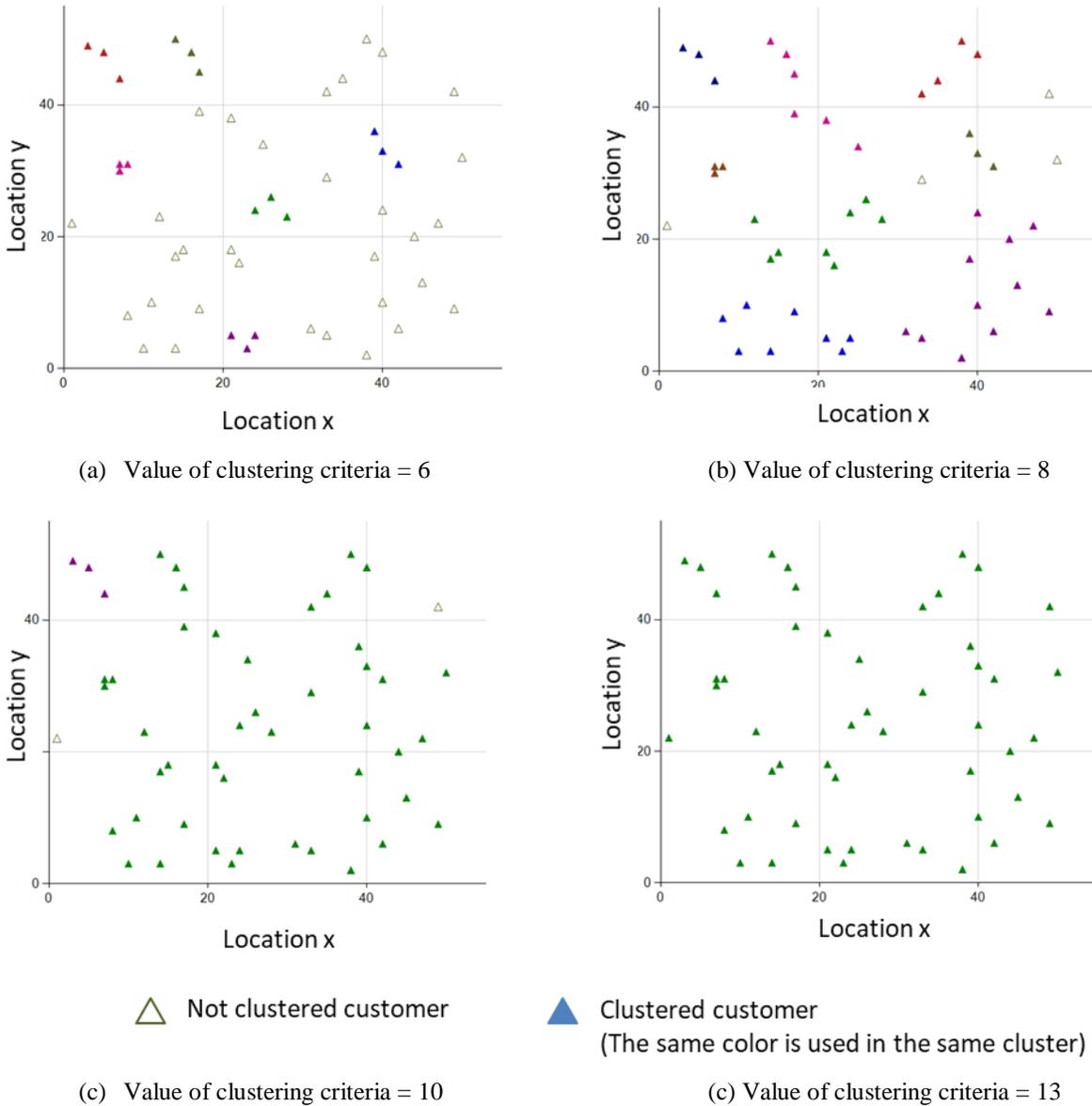


Figure 2 Impact of the value of clustering criteria

4. Computational Experiments

4.1 Conditions

We used a Gurobi optimizer 8.0.1 running on an Intel Core i7-8700 3.2 GHz PC. We used the benchmark problems proposed by Prins et. al. (2007) to decide the locations of the facility and the customer. The delivery time windows were generated randomly in accordance with a uniform distribution from 0 to 500. The lengths of the delivery time windows were 500. The capacity of the vehicle also used the same benchmark problem value.

In the experiments, the distance between sites was calculated as the Euclidean distance. We assumed that the vehicle velocity is one distance per time.

Table 2 shows the details of the instances. Cases 1-1 to 1-4 are the same problem and are solved using different clustering criteria. Cases 2-1 to 2-4 are the same problem and are solved using different clustering criteria.

Table 1. Conditions of computational experiments

Object	Item	Value
Potential depot	Number of potential depots	5/10
	Location	Prins et. al. (2007)
	Fixed costs	Prins et. al. (2007)
Customer	Number of customers	20/50/100
	Location/Delivery amount	Prins et. al. (2007)
	Delivery time windows	U[0-500]+250
Vehicle	Capacity	70
	Working time	1000
	Fixed costs/Variable costs	1000/1
Outsourced delivery	Unit delivery cost	30

Table 2. Detail of instances

Case	Num. of facilities	Number of customers	Ratio of num. of clustered customers
1-1	5	20	0.2
1-2	5	20	0.4
1-3	5	20	0.6
1-4	5	20	0.8
2-1	5	50	0.2
2-2	5	50	0.4
2-3	5	50	0.6
2-4	5	50	0.8
3	5	100	0.8
4	10	100	0.8

4.2 Results and discussions

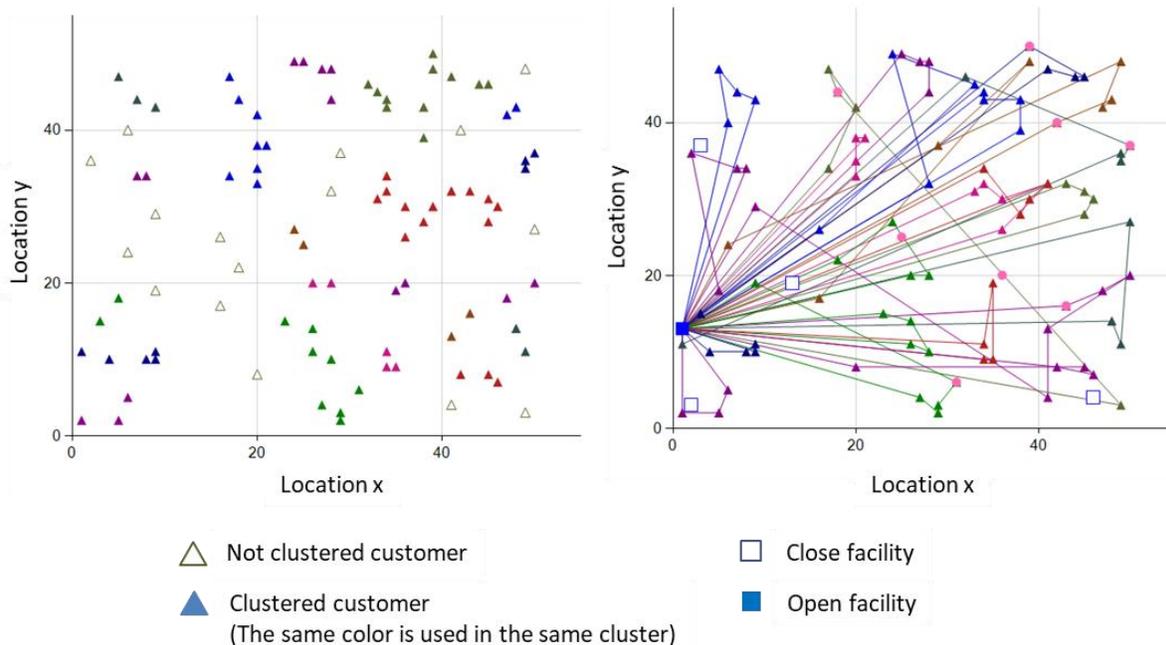
Table 3 shows the results of the computational experiments. The six data were calculated by two methods: MIP and the proposed algorithm. The MIP solver was stopped after one hour.

Cases 1-1 to 1-4 and 2-1 to 2-4 show that if the clustering criteria are selected properly, better solutions can be obtained with a short run time. When the number of clustered customers is small or most customers belong to the same cluster, a long calculation time is necessary because the clustering is not valid. The results show that better solutions are obtained when 80 to 90 % of all customers are divided into clusters. Therefore, the clustering criteria of cases 3 and 4 are selected so that 80% or more of the customers were divided into clusters.

Figure 3 shows the clustering results and vehicle routes. There are 19 clusters, and each cluster has less than 20 customers. Customers who do not belong to a cluster were assigned to vehicles to reduce costs in terms of global minimization.

Table 3. Results of computational experiments

Case	Num of clusters	Num. of clustered customers	Total costs	Total costs by MIP (at 3,600 sec.)	Run time (sec.)
1-1	4	8	11,463	11,462	300
1-2	4	8	11,463	11,462	300
1-3	5	12	11,460	11,462	197
1-4	5	13	11,460	11,462	69
2-1	11	24	24,452	24,492	302
2-2	11	25	24,452	24,492	302
2-3	14	33	21,803	24,492	300
2-4	12	46	18,372	24,492	28
3	19	84	69,318	88,101	666
4	22	80	78,848	87,015	606



(a) Clusters (b) Vehicle routes and outsourced deliveries
Figure 3. Results of case 3

5. Conclusion

To improve delivery efficiency, we proposed a new location routing problem (LRP) model with outsourced delivery. The model includes not only selection of depot location and vehicle routes, but also selection of delivery mode, that is selection of non-outsourced delivery or outsourced delivery. And a clustering-based heuristic is proposed to solve LRP with outsourced delivery. The algorithm divides the original LRP into smaller problems to make it easier to solve, without reducing the optimization potential as much as possible. The idea of clustering is not to divide all customers into clusters, but to cluster only close customers. In the next step, non-clustered customers are assigned to vehicles by MIP. The computer experiments on small-scale problems showed that the case in which about 80% of the customers were grouped into a cluster could get better solutions. Test instances based on LRP benchmark instances were solved

by using the algorithm. The results of the computational experiments show that the proposed algorithm can find a 20% better solution than MIP within an hour for problems with a hundred customers.

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