Problems of Demand and Effects in Portfolio Based Selection of Utility Functions

Riaman, Sukono, Eman Lesmana, Agus Supriatna
Department of Mathematics, Faculty of Mathematics and Natural Sciences,
Universitas Padjadjaran, Indonesia
riaman@unpad.ac.id, sukono@unpad.ac.id, man.msie@gmail.com, asupriatna55@ymail.com,

Abdul Talib Bon
[3]Department of Production and Operations,
University Tun Hussein Onn Malaysia, Malaysia
talibon@gmail.com

Abstract

Stochastic rules and their inequality are very useful tools in various economic fields and financial problems. The purpose of this paper is to determine the main results obtained from the use of stochastic rules on financial optimization. Emphasis is placed on demand problems and the effects of changes in portfolio selection problems. Some other examples are not directly related to the optimization problem, this is given to show a broad view on the application of stochastic rules in financial matters. Stochastic orders and their inequalities have been used in various fields on problems related to opportunities (probability) and statistics in general. For example, a queuing system with a single service with Increasing Failure Rate (IRF) service time and service discipline that prioritizes job savings. Suppose T_i states of the ergodic arrival time of a customer when service discipline is used. In this paper it is assumed that the market guarantee is comprehensive and mutually independent.

Keywords:
Portfolio selection, stochastic financial, utility functions

1. Introduction

Stochastic orders and their inequalities have been used in various fields on problems related to opportunities (probability) and statistics in general. For example, a queuing system with a single service with Increasing Failure Rate (IRF) service time and service discipline that prioritizes job savings. Suppose T_i states the ergodic arrival time of a customer when service discipline is used.

In the financial sector, the future price (or rate of return) of financial assets is shown by random variables that are uncertain. Therefore, a tool is needed to solve the optimization problem which cannot be an exception for the field, where many of them use stochastic rules. Likewise in many portfolio selection issues many use stochastic rules.

Thus, stochastic rules are very useful tools in the problem of portfolio selection. Investors are considered as a utility function for u, that want to invest wealth into one or two assets or even more. In the case of two assets, call A and B, take X and Y as random variables that show the results of assets A and B.

Furthermore, according to the principle of utility expectations, these investors overestimate asset A than B, if and only if (Kahneman, Daniel and Amos Tversky. 1979)

$$E[u(X)] \geq E[u(Y)] \quad (1)$$

The above inequality shows, that for all investors, the utility functions are included in several function classes, which are part of stochastic rules for X and Y.

The purpose of this paper is to introduce stochastic rules on economic problems, namely more specifically on the problem of demand and the problem of the effects of changes in portfolio selection. The next section discusses the problem of formal portfolio selection, demand problems, and their comparative applications regarding market equilibrium price systems. Chew Hong Soo, Itzhak Zilcha (1990).

2. Methodology

Suppose an investor will invest initial capital of 1 ($W = 1$). Suppose there are also opportunities to invest, namely X_1, X_2, ... , X_n as random variables. the problem of a single period portfolio that allocates wealth to n investments is to maximize the utility function of the final value of wealth, carried out by selection on the portfolio. The problem of portfolio selection is done by maximizing the mean of the utility, i.e. Levy, Haim. (1973)

$$\max_{a\in A}\left[u\left(\sum_{i=1}^{n} a_i X_i\right)\right] \tag{2}$$

where u represents a utility function. This expectation value is taken from n random variables with A as the boundary in R_n. The boundary function A, expressed by

$$A = \{a = (a_1, a_2, a_3, \dots, a_n); \sum_{i=1}^{n} a_i = 1,\ a_i > 0\} \tag{3}$$

or A = Rn if there is no specified boundary. If the utility function is u, in case where u '> 0 and u "≤ 0, it is assumed that the investor will refuse to take risks, except on the contrary statement.
A portfolio is a shaped random variable

$$P = \sum_{i=1}^{n} a_i X_i, (a_1, a_2, a_3, \dots, a_n) \in A \tag{4}$$

which is a solution to the problem of equation 2.
If X_i is normally distributed with an average $\mu_i = E[X_i]$ and variance $\sigma_i^2 = V[X_i]$, this problem can be reduced to the Markowitz (1995) model. In this case the portfolio $P = \sum_{i=1}^{n} a_i X_i$ is also normally distributed with the average $E(P) = \sum_{i=1}^{n} a_i \mu_i$, and its variance $V(P) = \sum_{i=1}^{n}\sum_{i=1}^{n} a_i \sigma_{ij} a_j$, and $\sigma_{ij} = cov(X_i X_j)$; $i, j = 1, 2, 3, \dots, n$, represents covariance between X_i and X_j, and $\sigma_{ij} = V(X_i)$. The characteristic probability of P is determined by the average $E(P)$ and variance $V(P)$, the utility expectation $E[u(P)]$ is also a function of $E(P)$ and $V(P)$. The problem is formulated as follows:

$$\min_{a\in A} \sum_{i=1}^{n} \sum_{j=1}^{n} a_i \sigma_{ij} a_j$$

Where the boundary condition is

$$\sum_{i=1}^{n} a_i \mu_i = \mu_1 \tag{5}$$

for several μ. Equation (5) above is quadratic programming with linear constraints, where the problem is not easily solved for n large enough.
When X_i is normally distributed, it is assumed that each other is free from each other. For a random variable X and marginal function utility u', it is defined as generalized harmonic mean (GHM) as

$$GH(X; u') = \frac{E(Xu'(X))}{E(u'(X))} \tag{6}$$

Note that GHM at 6 is well defined, if $u' > 0$. Take S which states the choice of sets of assets and assume that the assets are mutually independent of each other. For a risk averse utility function, among others, shows that none of the assets $X \notin S$, will be included in the optimal portfolio of the set of choices $S \cup \{X\}$ if GH $GH(P_S^*; u') \geq E[X]$ and the expected value of some of the assets included in the optimal portfolio exceeds the expected value of some assets that are not selected for the optimal portfolio. The optimal portfolio of the selected set S is represented as P_S^*. So, the structure of the optimal portfolio consists of a number of positive selected assets.
For the two selected assets X and Y, the problem of portfolio selection is given by:

$$\max_{a\in[0,1]}[\phi(a)];\ \phi(a) = E[u(aX + (1-a)Y)] \tag{7}$$

For an investor who is reluctant to take risks. Lower (7) towards a, obtained

$$\phi'(a) = E[(X - Y)u'(aX + (1-a)Y) \tag{8}$$
$$\phi''(a) = E[(X - Y)^2 u''(aX + (1-a)Y)]$$

The utility function u is concave for $u''(x) < 0$, by following (7) where $\phi(a)$ is strictly concave in a. Henceforth for $X = 0$ obtained $\phi'(a) = E[(X - Y)u'(Y) \leq 0$. Or equivalent to (Meyer, Jack and Michael B. Ormiston. (1989).)

$$E(X) \geq \frac{E\big(Xu'(Y)\big)}{E\big(u'(Y)\big)} = GH(Y;u') \tag{9}$$

3. Illustration Analysis

Let F be a function class in R. The relation of the stochastic rule $X \phi Y$ is generated from F, if

$$E[f(X)] \geq E[f(Y)] \text{ untuk semua } f \in F \tag{10}$$

The rules used to describe equation 10 in a perspective economy, suppose there is an investor with a utility function u. Furthermore, a rule or stochastic relation $X \geq_F Y$, if and only if, all investors whose utility functions are in class F, which shows relation X to Y. Suppose the first candidate for the class of function is $F_{FSD} = \{f: f(x)$ is increasing at $x\}$. (Tversky, Amos and Daniel Kahneman. (1992))

The stochastic rule is obtained from F_{FSD} which is called the First Dominant Dominance rule (*First Order Stochastic Dominance*) and is expressed as $X \geq_{FSD} Y$, which is also, in general, an application of the usual stochastic rules. Furthermore $X \geq_{FSD} Y$ applies, if and only if, $F_X(x) \leq F_Y(y)$ for all $x \in R$ with $F_X(x)$ and $F_Y(y)$ respectively express the distribution function of X and Y. (Levy, Haim, (1992))

Then, investors are considered to have risk aversion, which is described by its utility function, which is conclusive. Therefore, the function is described as $F_{MPC} = \{f: f(x)$ is concave in $x\}$. The stochastic rule is generated from F_{MPC} which is called the Mean-Preserving Contraction (MPC) and is expressed as $X \geq_{MPC} Y$, where the Mean-Preserving Contraction is described as an increased concave rules.

because the number of investors can increase or decrease, then the third candidate from his function class is $F_{SSD} = \{f: f(x)$ is concave in $x\}$. The stochastic rule is generated from F_{SSD} called Second Order Stochastic Dominance (SSD) and is represented by $X \geq_{SSD} Y$. The Second Order Stochastic Dominance (SSD) is described as an increasing concave rule. Thus $X \geq_{SSD} Y$ applies, if and only if (Safra, Zvi and Itzhak Zilcha. (1988).)

$$\int_{-\infty}^{x} F_X(u)du \leq \int_{-\infty}^{y} F_Y(u)du, x \in R,$$

so that it is obtained

$$X \geq_{MPC} Y \Leftrightarrow E(X) = E(Y) \text{ dan } X \geq_{SSD} Y \tag{11}$$

a. Demand problem
   Suppose the portfolio problem (7) is simplified to be

$$\max_{a \in A}[\phi(a)];\ \phi(a) = E[u(aX + (1 - a)Y)] \tag{12}$$

where $A \in R$ is a closing interval and assume that $½ \in A$. If no short-term sales are allowed, then $A = [0, 1]$ or $A = R$ if the limit is not specified. However in this case A is not limited to $½ \in A$.

In this section, it is assumed that the problem of the demand is the condition, ie the demand at X is greater than the demand at Y, for example a $a^* \geq ½$ is true.

For the problem defined as

$$g(x, y) = xu' \frac{x+y}{2}, x, y \in R \tag{13}$$

So, based on (12) obtained

$$\phi'(\frac{1}{2}) = E[(X - Y) - E[\Delta g(x, y)] \quad (14)$$

where $\Delta g(x, y) = g(x, y) - g(y, x)$
If $\phi'(\frac{1}{2}) \geq 0$, then the optimal fraction of a^* cannot be less than ½ because $u'' < 0$. This follows the bivariate characteristic of the stochastic rule that is useful for the demand problem.

Take *G* as the bivariate function class in R^2. For two random variables *X* and *Y*, the relation of the stochastic rule is $X \phi Y$ which is generated from *G*, if;

$$E[(X*, Y*)] \geq E[g(Y*, X*)], \text{for all } g \in G \quad (15)$$

Where X^* and Y^* are mutually independent random variables such that $X* \overset{d}{=} X*$ and $Y* \overset{d}{=} Y*$. In this case, it is stated that $X \geq_G Y$, with a note, that the stochastic rule relation is only dependent on marginal distributions.

b. Aplication

In this sub-section, it will be explained about comparative stochastic about the system of equilibrium in the whole market. Also introduced is what is called the Arrow-Debreu guarantee and the definition of state-price, so that the monotonous properties of the price guarantee are obtained. For example, regarding the effects of changes on investors that are uncertain and unclear. Suppose that statements in end-period economies are classified into one from *n* state $\Omega = \{\omega_1, \omega_2, \dots, \omega_n\}$, where *n* denotes guarantees traded in that market. This guarantee is characteristic of vector $x_i = (x_{i1}, x_{i2}, \dots, x_{in})$ where x_{ij} is pay off or dividend with guarantee payment *i* in state state ω_j at the end of the period. Hirayama, T and Kajima, M. (1989)
In this paper, it is assumed that the market guarantee is comprehensive, i.e. x_i, $i = 1, 2, 3, \dots, n$, which are mutually independent. Without eliminating the general nature, it is assumed that the Arrow-Debreu δ_i guarantee, i = 1,2,3, ..., n, is traded on the market, where δ_i pays one unit of the account in the ω_j state, and does not pay at another place.
Suppose that there are *m* investors taking part in the market, and for each investor defined by the utility function and an initial value. Suppose that every investor tries to build a portfolio that maximizes utility expectations at the end of the usage period with the budget as a limiter. Next take $\pi = (\pi_1, \pi_2, \pi_3, \dots, \pi_n)$ as the usual probabilistic trusted by investors, where $\pi_i > 0$ is a probabilistic state estimate ω_i that appears at the end of the period. Suppose also $p = (p_1, p_2, p_3, \dots, p_n)$ as a system of prices, where p_i is positive and is the price of guarantee δ_i in the position of competitive balance. Furthermore the market price *q* on the guarantee $x = (x_1, x_2, \dots, x_n)$ of a portfolio on δ_i is $q = \sum_{i=1}^{n} x_i p_i$. In this case $\sum_{i=1}^{n} p_i$ is the market price in a portfolio that is not at risk, that is, securities by paying one unit of a certain account. Because the system state price is shown only in a multiplication constant, without omitting the general property taken $\sum_{i=1}^{n} e_i p_{i=1}$ where e_i is the aggregate consumption level in state ω_i.
According to (Kajima, M and Ohnishi M. Ohnisi (1999)) the price system can be characterized as

$$p_i = \frac{\pi_i u'(e_i)}{\sum_{k=1}^{n} \pi_i \{e_k u'(e_k)\}}, i = 1, 2, 3, 4, \dots, n \quad (16)$$

Where *u* is the utility function of a pool of investors in the market guarantee with $u' > 0$ and $u'' \leq 0$. The return rate is *R*,from assets that are not at risk, namely

$$1 + R = \frac{\sum_{k=1}^{n} \pi_i \{e_k u'(e_k)\} \pi_i}{u'(e_i)} \quad (17)$$

Normalize the state price p_i with $\alpha_i = \frac{p_i}{\sum_{i=1}^{n} p_j} = (1 + R)p_i$, so that the probability vector is obtained, namely $\alpha = (\alpha_1, \alpha_2, \alpha_3, \dots, \alpha_n)$, take

$$q = \frac{1}{1+R}\sum_{i=1}^{n} x_i \alpha_i \tag{18}$$

a guarantee of market prices $x = (x_1, \dots, x_n)$. Vector probability α is a probability at risk of being neutral.

4. Conclusions

Based on the analysis and discussion that has been done then it can be concluded that: Stochastic rules and inequality are very useful tools, especially in financial economic problems, especially on demand issues and the effects of changes in portfolio selection. Likewise, bivariate characteristics, risk aversion, average harmonic price generalization and equilibrium prices are a broad spectrum of applications for stochastic rules on financial problems.

Acknowledgments
Acknowledgments are conveyed to the Rector, Director of Directorate of Research, Community Involvement and Innovation, and the Dean of Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, with whomthe Internal Grant Program of UniversitasPadjadjaran was made possible to fund this research.The grant is a means of enhancing research and publication activities for researchers at Universitas Padjadjaran.

References

Chew Hong Soo, Itzhak Zilcha. (1990). .Invariance of the Efficient Sets when the Expected Utility Hypothesis is Relaxed., Journal of Economic Behavior and Organization 13, 125-131.
Kahneman, Daniel and Amos Tversky. (1979). .Prospect Theory of Decisions Under Risk., Econometrica 47(2), 263-291.
Hirayama, T and Kajima, M. (1989). An extremal property of FIFO discipline in G/1FR/I queues. Advance Mc Entire PL. 1984. Portfolio Theory for Independent Assets. Management Science.
Kajima, M and Ohnishi M. (1999) Portfolio Selection Problems via The bivariate Characterization of Stochastic Dominance Relations. Mathematical Finance.
Levy, Haim. (1973). .Stochastic Dominance Among Log-Normal Prospects., International Economic Review 14(3), 601-614.
Levy, Haim, (1992). .Stochastic Dominance and Expected Utility: Survey and Analysis., Management Science 38(4), 555-593.
Meyer, Jack and Michael B. Ormiston. (1989). .Deterministic Transformatons of Random Variables and the Comparative Statics of Risk., Journal of Risk and Uncertainty 2, 179-188.18
Safra, Zvi and Itzhak Zilcha. (1988). .Efficient Sets with and without the Expected Utility Hypothesis., Journal of Mathematical Economics 17, 369-384.
Tversky, Amos and Daniel Kahneman. (1992). .Advances in Prospect Theory.

Biographies

Riaman is a teaching staff in the mathematics study program at the Mathematics Department of the University of Padjadjaran since 1997. The mathematics degree was won from the Mathematics Department of the University of Padjadjaran in 1995, while the Masters of Actuarial Science was obtained from the Bandung Institute of Technology in 2000. The field of research currently being undertaken is the field of applied mathematics. More specifically are: fields of financial mathematics, actuarial mathematics, survival models, mathematical statistics, and reliability. Since 2001 until now he has joined as a member of IndoMS, and since 2017 joined IAENG.

Sukono is a lecturer in the Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran. Currently serves as Head of Master's Program in Mathematics, the field of applied mathematics, with a field of concentration of financial mathematics and actuarial sciences.

Eman Lesmana is a lecturer at the Department of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran. He was born in Ciamis on July 9, 1961. He received a degree in mathematics from the Mathematics Study Program, Faculty of Mathematics and Natural Sciences, University of Padjadjaran Bandung in

1986. He completed a master's program in Industrial Technology from the Bandung Institute of Technology in 1992. Since 1987 it has been lecturer at Padjadjaran University, Bandung, Indonesia in the field of Applied Mathematics. He teaches in Operational Research , Forecasting, Inventory Control, Linear Programming, Optimization Models and Decision Theory.

Agus Supriatna is a teaching staff in the mathematics study program at the Mathematics Department of the University of Padjadjaran since 1984. The mathematics degree was won from the Mathematics Department of the University of Padjadjaran in 1980, while the Masters of Actuarial Science was obtained from the Bandung Institute of Technology in 2001. The field of research currently being undertaken is the field of applied mathematics. More specifically are: forecasting, time series, multivariable calculus.

Abdul Talib Bon is a professor of Production and Operations Management in the Faculty of Technology Management and Business at the Universiti Tun Hussein Onn Malaysia since 1999. He has a PhD in Computer Science, which he obtained from the Universite de La Rochelle, France in the year 2008. His doctoral thesis was on topic Process Quality Improvement on Beltline Moulding Manufacturing. He studied Business Administration in the Universiti Kebangsaan Malaysia for which he was awarded the MBA in the year 1998. He's bachelor degree and diploma in Mechanical Engineering which his obtained from the Universiti Teknologi Malaysia. He received his postgraduate certificate in Mechatronics and Robotics from Carlisle, United Kingdom in 1997. He had published more 150 International Proceedings and International Journals and 8 books. He is a member of MSORSM, IIF, IEOM, IIE, INFORMS, TAM and MIM.