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Scheduling of M jobs on N machines using a new proposed sequencing method

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Abstract

This paper focuses on the problems faced during scheduling of large job shops (manufacturing units) to sequence their jobs on machines available in the such a way that results in all processes completed in minimum time. There are already different methods available to do sequencing of this kind of problems. Branch and Bound method (BB), Timed Petri net (TPN) models for sequencing are two methods which are considered as standard methods to solve any scheduling problems and to get an optimum answer. In above-mentioned methods, there are some limitations and due to these limitations these methods sometimes do not lead to optimum make span or it takes large computational time or sometimes it becomes difficult to do scheduling manually for a relatively low number of jobs and machines and also difficult to do scheduling of relatively large amount of numbers on a computer. In this paper, we have proposed a new method which can easily solve problems for same flow for the path of operation and also removes limitations of Branch and Bound method and takes lesser time than Timed Petri net models for sequencing of jobs. We have done many computation experiments which show that the proposed method successfully solves the limitations of Branch and Bound method with almost no difficulty level.

Keywords

Scheduling of large job-shops, Branch and Bound method(B&B), Timed Petri Net model(TPN)

1. Introduction

The purpose of scheduling is to deal with how to assign the available jobs or resources to machines in the way that make span of all products gets the minimum. This paper proposes the 2 different types of problems which can be faced during sequencing of any large job shop and can be solved by our method. These problems can be defined as follows:

A. Sequencing of M jobs on N machines while all jobs have to go to every machine in the same sequence.
B. Sequencing of M jobs on N machines but not all jobs go to all machines and they have the same path of operations.

For above-mentioned problems, sequencing can be done by following methods and they give an optimum solution. In (A) types of problems, B&B method can be applied to get an optimum solution but sometimes two branches will have same lower bound values and this kind of problems can be solved by checking both branches, which is considered to be the best way to get an optimum solution when this kind of conflicts happen.

But if we calculate all conflicting branches then it can be a huge trouble (more time) for larger values of M and N (8-9). With just 3 to 4 conflicts will increase the calculation by 300 to 400 percent. Some reference problems are taken from given references (Pachpor et al. 2017) (Kumar 1995):

In table 1& 2, we have shown a simple sample problem for the B&B algorithm taken from reference (Kumar 1995). While solving this problem a conflict occurs during calculating 1st iteration. This problem has just two branches conflicting to each other due to which remaining problem increases its size to twice compared to problem without any conflict. We can also use TPN model to solve this problem but it is not preferred because it checks every possible branch while B&B removes the branches with each iteration and chooses the best branch among available branches.

Table 1: Type A Problem(Q1)

J/M	Processing time on machine		
	I	II	III
1	6	8	20
2	4	30	6
3	30	4	5
4	2	5	1
Total	41	47	32

Table 2: Type A Problem(Q2) (Kumar 1995)

J/M	I	II	III
1	4	3	2
2	4	1	4
3	3	2	3
4	1	3	3
Total	12	9	12

In (B) type of problems, where jobs have their processing time but they do not have to go to all machine. This kind of problem is taken from reference (Pachpor et al. 2017) and shown in table 3. To solve this kind of problems, we can use B&B method (Ashour et al. 2007) but the same problem can occur which occurs in type (A) problem, which leads us to use another and a better method than B&B. Timed Petri net model for scheduling is used to solve type (B) problems. TPN model uses reachability tree to check every possible transition firing sequence and give a sequence which takes minimum time as output. TPN model does not have any conflicting problem as B&B because it checks all possible firing sequence without cutting out any branches which takes a quite significant amount of time for larger problems and this same also become a limitation which does not allow it to take large sequencing problems.

Table 3: Type B Problem (Pachpor et al. 2017)

J/M	Operation sequence						Time for operation					
	A	B	C	D	E	F	A	B	C	D	E	F
1	0	1	1	0	1	1	0	5	4	0	6	7
2	1	0	0	1	0	1	7	0	0	3	0	2
3	0	1	1	0	0	0	0	3	5	0	0	0
4	1	0	0	1	0	0	2	0	0	5	0	0
5	1	1	0	0	0	0	6	4	0	0	0	0
6	1	0	0	1	0	1	4	0	0	6	0	3
7	0	0	1	0	1	1	0	0	9	0	2	3
8	1	0	0	1	1	1	1	0	0	4	5	3

In above-shown table 3, a sample problem for problem type B is given. As mentioned earlier this type of problem contains different operation sequences with a different number of operations for different jobs. In table 3, operation sequence is given, if job 1 has an operation on given machine than it is shown as 1 otherwise it is shown as 0. We have also considered that jobs go to machines in A to F order. To solve this type of problems, we can use B&B and TPN both methods but as we can see the size of problem is also larger than previous problems and difficulty level of problem also increases, using B&B to solve this problem is not advisable because there are different number of operations and also different paths and B&B method does not have any specific properties to deal with them. TPN model can be used to solve this kind of problem but still, thesame problem occurs as it checks all possible transition route and gives the route with minimum time (optimum schedule of jobs). TPN method leads to optimum answer but

it takes considerably more time to solve the problem. We have proposed a new method which can easily deal with this kind of problems with reducing complexity in finding the optimum schedule for Job shop problems.

2. Current Methodology and Its Limitations

As this paper focused on B&B and TPN methods' limits, they are explained below:

2.1 Branch And Bound Method

Branch and Bound method use the terminology that it would be not a good way to evaluate all the permutations for choosing the optimum one (Brucker et al.1992).It is a type of search by which certain classes of permutations not leading to an optimum solution, will be discarded in thevery early stage. But when two classes of permutations have same answers(lower bound), which one to discard and which one to choose to calculate further (Clausens 1999). We have solved type (A) problems below and showed how it increases calculations when two classes have asame optimum answer.

- The solution for type (A) problem using thetypical branch and bound method(table 1 problem):

Table 4: 1st Iteration For Lb For Q1

J/M	I	II	III	L_1	L_2	L_3	Max(L_1,L_2,L_3)
1	6	8	20	48	54	46	54
2	4	30	8	48	52	66	66
3	30	4	5	48	78	66	78
4	2	5	1	51	54	39	54

In table 4, it is seen that job 1 and job 4 have lowest and same lower bound(LB) due to which job to select as priority job becomes difficult. Solving this problem by selecting Job 4 as apriority here will lead us to optimum answer but Job 1 will not. Hence, the calculation of this simple problem is increased by 2 times which is not desired by any means. In table 5, Q2 is solved using B&B method. After 1st iteration, job 4 is successfully selected but during the second iteration, it is not possible to prioritize any job over another. Q2 is also taken to show the conflict in proposed method and what this conflict actually represents.

Table 5: 2nd Iteration For Lb For Q2

J/M	I	II	III	L_1	L_2	L_3	Max(L_1,L_2,L_3)
4	1	3	3	17	12	16	17
4-1	5	8	10	17	14	17	17
4-2	5	6	11	17	13	15	17
4-3	4	6	10	17	12	15	17

2.2 Timed Petri Net Model

Petri nets(PN) are graphical tools which can be used to model and simulate FMS (Murata 1989). It was introduced by C.A. Petri in 1962. Timed Petri nets can be used to do scheduling of FMS and its mathematical representation is shown below:

TPN is a 2-tuple R = (C,d)

Where, C is Petri net C=(P,T,F,M_0,W) and dis atime associated with transitions. Here, P = set of places of PN, T = set of transitions, F = set of arcs, M_0 = initial marking and W = weights of arcs.

As mentioned TPN model can be used to solve all types(A,B) of problems. PN models for problems (B) are shown:

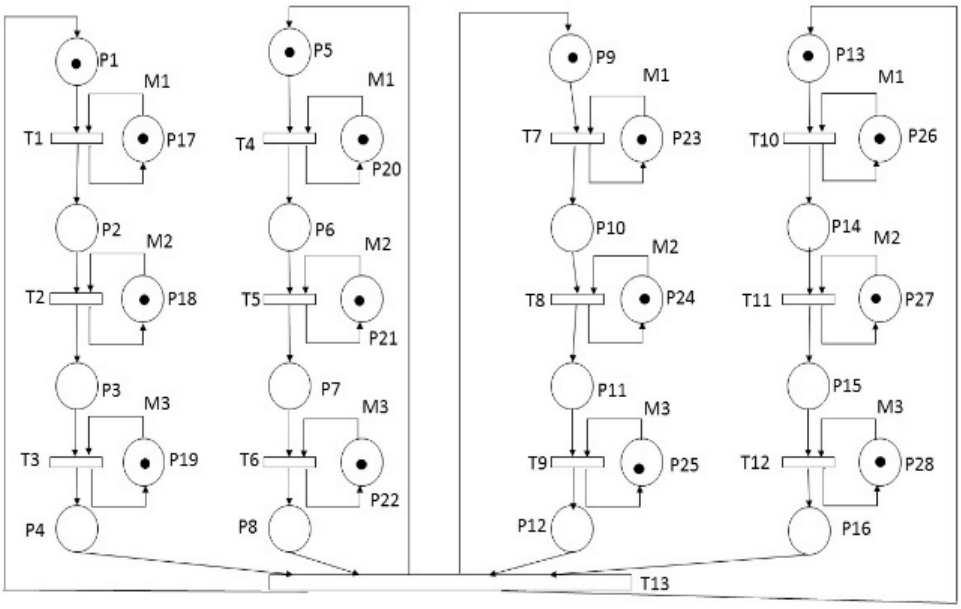

Figure 1: PN model for problem Q1 and Q2

In figure 1, PN model for theproblem (A) is given and which consists of 28 places and 13 transitions. In order to do scheduling using TPN model, a reachability tree is formed which gives all possible markings and all possible state a PN can achieve. By analyzing the reachability tree for any PN, all possible routes to get solution can be found and optimum one is selected. TPN gives makespan of 56 seconds with transition firing order (T1-T4-T7-T10-T2-T5-T8-T11-T3-T6-T9-T12) for Q1 and makespan of 17 seconds with transition firing order (T10-T7-T11-T4-T8-T12-T1-T5-T9-T2-T6-T3). There are 5 more firing sequences available which give the same answer. For problem (B) Comparing this TPN methodology for scheduling. It can be easily seen that TPN checks every possible sequence for theproblem and gives the best one as ananswer but B&B will be comparing different branches or route and cut downs branches which will not lead to anoptimum answer. But as it was shown above there is a huge possibility that many conflicts can occur during calculations. Here B&B will take considerably small time while dealing with a large number of jobs and machines compare to TPN, which will check every possible answer for anoptimum answer. A TPN model for theproblem (B) is shown in figure 2. It consists of 54 places and 24 transitions. TPN gives makespan of 28 seconds with transition firing order (T20-T17-T1-T14-T21-T8-T5-T12-T10-T22-T23-T18-T2-T15-T13-T6-T9-T11-T3-T16-T19-T7-T4).

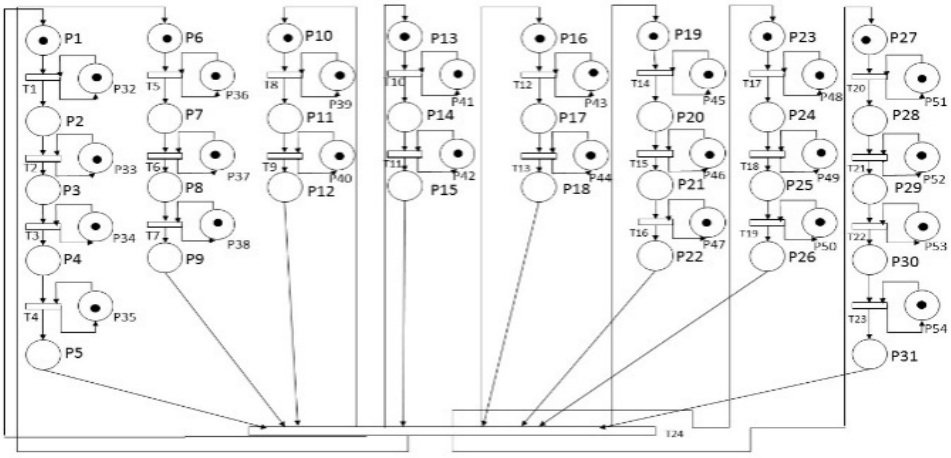

Figure 2: PN model for problem (B)

3. Proposed Methodology

3.1 Introduction of proposed method

The proposed method uses the concept of comparing a sequence of two jobs for machining time with given same sequence. It is a simple mathematical method which compares completion time of 2-job sequences. This method overcomes Branch and Bound in following manners: (i) Less complexity, (ii) Fewer calculations, (iii) Less time.

3.2 Algorithm

Step 1: First divide all M jobs into aset of thesame number of operations. After dividing these sets into pairs of two jobs. While every job must have a set which pairs it with every other given job. It will create a 2M number of sets.

Step 2: With different sequences, different completion time can be observed & for two specific jobs there can be two different sequences.

Step 3: Start calculating completion time for thelast job in each pair of the sequence. From these completion times of pairs, priority for each job can be defined.

Step 4: Sometimes 2 jobs having same completion time will be dealt using Gantt charts & jobs which don't have amachine in common for any given operation can be sequenced using Gantt chart.

Step 5: If jobs are divided into different sets based on their number of operations then ajob with more number of operations will be given priority.

3.3 Mathematical Explanantion

A mathematicalexplanation of this given algorithm is given by solving stated examples.

A solution to problem type A:

Table 6: Completion Time For Different Available Sequences(Q1)

J/M	I	II	III
1	6	8	20
2	4	30	6
3	30	4	5
4	2	5	1
1-2	10	44	50
2-1	10	42	62
2-3	34	38	45
3-2	34	64	70
3-4	32	39	40
4-3	32	36	41
1-4	8	19	35
4-1	8	16	36

In above table 6, completion of different sequences is shown. Comparing sequence 1-2 & 2-1, the completion time of sequence 1-2 is less which leads to the conclusion that if job 1 is sequenced ahead of job 2, it will take less time & sequence (1-2) is better than (2-1) & priority can be given as Job 1>Job2. By this method comparing all sequences, priority Job1>Job2>Job3>Job4. Which can be interpreted as ajob sequence (1-2-3-4) and 56 seconds of makespan. The problem which is solved above does not have any conflicting sequence where 2 sequences have same completion time. Q2 question is prepared based on that conflict and it is solved below to show the results, what happens when to completion time are same:

In table 7, the completion time of 6 sequences is the same. Here, these completion times are the same which can be taken as any of these jobs can be taken first but as it can be seen in Q1 that it is not mandatory that every time branch

and bound will give the optimum for all sequences but in this method the completion time of each operationis considered to do sequencing of problem. This simply means that if both sequences have same completion time then it won't matter which one goes first in thesequence.

Table 7: Completion Time For (Q2)

J/M	I	II	III
1	4	3	2
2	4	1	4
3	3	2	3
4	1	3	3
1-2	8	9	13
2-1	8	11	13
2-3	7	9	12
3-2	7	8	12
3-4	4	8	11
4-3	4	6	9
1-4	5	10	13
4-1	5	8	10
1-3	7	9	12
3-1	7	10	12

- The solution for problem type B:

Table 8: Completion Time For Problem B

J/M	A	B	C	D	E	F	Set
1	0	5	4	0	6	7	O_4
2	7	0	0	3	0	2	O_3
3	0	3	5	0	0	0	O_2
4	2	0	0	5	0	0	O_2
5	6	4	0	0	0	0	O_2
6	4	0	0	6	0	3	O_3
7	0	0	9	0	2	3	O_3
8	1	0	0	4	5	3	O_4
8-1	-	5	9	9	16	23	O_4
1-8	1	1	1	5	20	25	
2-6	11	11	11	17	17	20	O_3
6-2	11	11	11	14	14	16	
6-7	-	-	9	9	11	16	
7-6	4	4	4	10	10	17	
2-7	-	-	9	9	11	15	
7-2	7	7	7	10	10	16	
3-4	2	2	2	7	7	7	O_2
4-3	-	3	8	8	8	8	
4-5	8	12	12	12	12	12	
5-4	8	8	8	13	13	13	
3-5	6	10	10	10	10	10	
5-3	-	13	18	18	18	18	

In table 8, jobs are divided into sets using their number of operations and sequencing of these sets are done after diving them in pairs of two job sequences. For these kinds of problems, CT of 2nd job in all sequences are calculated considering on which machines they have their respective operations. Here, the critical situation arises when two jobs which completion time is being checked does not have any common machine for any of their operation. It can happen that job with less CT will be selected but it will be incorrect because if jobs don't have any common machine for any of their operation, actually these jobs don't need sequencing or can't be sequenced. From this table, (8-1-6-2-7-3-4-5) or (8-1-6-2-7-4-3-5) sequence can be given after considering all steps which give the makespan of 28 seconds.

4. Computational Results

When this method is compared with B&B and TPN for calculation time, it wins with a good amount of idle time after calculation while B&B and TPN are still calculating answers. For more when this comparison is performed on thecomputer, proposed method also wins when a large number of jobs and machines are given but for a small number of jobs and machine, there is nonoticeable difference in acurrent era of highly powered computers and laptops. We have done this comparison for B&B and proposed method on C++ and proposed method on C++ and TPN on PIPE, VOOPAD for a large number of jobs and machines. Here also proposed amethod with optimized logic on C++ wins. The reason behind this is simply better logic than current methods which results in less calculation.

5. Conclusion

In this paper, a new method for sequencing is proposed which removes the limitation of Branch and Bound method and reduces a large amount of calculation compared to Branch and Bound method. The proposed method is based on completion time for sequencing of only two jobs in two different orders eg 1-2 and 2-1 .The order of sequence having minimum competion time will lead question to optimum answer. For the same purpose Branch and Bound method will compare the lower bounds of all branches but theproposed method is more practical and provides a better logic which results in less calculation. The same logic can be applied to conflicting lower bounds in B and B technique.When this proposed method is compared with Timed Petri Net method for sequencing, proposed method takes the comparably low amount of time with anoptimum answer while TPN will go through every branch of reachability tree and gives minimum makespan. After all these reasons benefits of theproposed method over B&B, TPN is listed as below:

1. A simpler method than Branch and Bound method and Timed Petri Net model for scheduling.
2. Less calculation than both methods.
3. Better results than Branch and Bound method and also removes the limitations of Branch and Bound method.

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