# Algorithm for Integrated Problem of Workforce Allocation and Parallel Machines Scheduling with Sequence-dependent Setup and Machine Eligibility Restrictions

**Arajabhorn Chantavali and Anan Mungwattana**
Department of Industrial Engineering
Kasetsart University
Bangkok, Thailand
arajabhorn@hotmail.com, fenganm@ku.ac.th

## Abstract

Machine scheduling is a common problem found in many manufacturing and service industry. In this study, the stated problem is imitated from real-world beverage industry. We consider integrated problem of workforce allocation and parallel machine scheduling with sequence-dependent setup time and machine eligibility restrictions. Due to complexity of the problem, a rule-based heuristic is occurred to arrange production schedules in practice. Though this heuristic could make planner arrange the schedule effortlessly, much time is wasted on unnecessary setup which causes the completion time of all job fall behind what it is supposed to be. To overcome this circumstance, the problem is formulated into mathematical model with the aim to minimize makespan. Based on the result of the experiment, it can be concluded that the proposed model could obtain more effective result than those of the existing rule-based heuristic and LPT significantly.

## Keywords
Parallel Machines Scheduling, Sequence-dependent Setup Time, Machine Eligibility Restrictions, Workforce Allocation

## 1. Introduction

Industries are seeking for high utilization of their resources, particularly in the manufacturing and production industries. One of the most relevant is efficient machine scheduling. In previous studies of machine scheduling problem, they are frequently assumed that the setup time could be ignored or considered as a part of the processing time. On the other hand, these assumptions might be inappropriate for some manufacturing industries, which setup time is essential for cleaning and changing fixtures before processing, such as those in chemical processing, metal processing, and beverage industries. In addition, setup times become more crucial when the amount of setup time required strongly depends on the sequence of jobs to be processed on the machine or well-known as sequence-dependent setup time. A typical example is the printing or painting industry, the setup time of changing from black job to pale color is greater than changing from pale job to black job because extensive cleaning is required. As sequence-dependent setup time is the most complicated case for machine scheduling if setup time has occurred, many researchers have developed algorithms to solve sequence-dependent setup problems on various criteria. Bowers *et al*. (1994) applied a mathematical technique, Cluster Analysis, to a single machine scheduling with a sequence-dependent setup. With this technique, a product grouping procedure helps aggregating products which have similar setup requirements. Resulted in significantly improved production schedules. Luo *et al*. (2006) introduced dominance rules for single machine scheduling with sequence-dependent setup and due date and applied the rules to the Branch and Bound Approach, aiming to minimize maximum tardiness of all jobs. The result proved that the optimal solution was obtained with less computational time. Yet, there was no study on accommodating proposed rules with other computational techniques. A two-stage Ant Colony Optimization algorithm for unrelated parallel machines with sequence-dependent setup time was proposed by Arnaout et al. (2009). The performance of the heuristic was compared with those of Tabu Search and Meta-Heuristic for Randomized Priority Search proposed by Helal *et al*. (2006) and Rabadi *et al*. (2006) respectively. Another algorithm for dynamic parallel scheduling with sequence-dependent setup was developed by Lee *et al.* (2010), a restricted simulated annealing algorithm that incorporates a restricted search strategy with the elimination of non-effect job moves to find the best neighborhood schedule was presented. The computational experiments proved that the stated algorithm has a higher performance as compared with the common simulated annealing and the best solution from the benchmark problem obtained by Ovacik and Uzsoy (1995).

Nonetheless, it is undeniable for the real-life problem in many industries that proper allocation of limited resources, e.g., tools and workers, needs to be involved with machine scheduling. These two problems are decided as two sequential decision making in usual practice. The resource allocation might be selected then the production schedule will be decided depends on the previous decision in some industries or some might invert. However, these approaches are inflexible and limit the possible alternatives that may provide a better solution. Moreover, it is often found in manufacturing systems that to process the task on a given machine requires the assistance of a human operator with a specific skill to process the task. Surprisingly, there are only a few studies that resource allocation and machine scheduling are considered as one. One of those is a Schedule Generation Schemes and Genetic Algorithm for the Scheduling Problem with Skilled Operators and Arbitrary Precedence Relations proposed by Mencia *et al*. (2015).

Thus, we proposed a mixed integer linear programming approach for the integrated problem of workforce allocation and parallel machine scheduling. The considered problem is imitated from real-life beverage industry with parallel machines production system, which each machine requires the assistance of a human operator with a specific skill to process the task. There are several products to be processed various on flavor and net content volume. If the net content volume of the immediate successor job is different from the previous one, the changeover of the machine part is required, otherwise, it only requires fewer hours for cleaning. Each job is allowed to split and processed on more than one machine. And machine eligibility restriction is regarded.

This paper consists of five sections. The following section describes statement of problem and mathematical model formulation. Illustrated examples could be found in section 3. In section 4, the computational experiments and results are discussed. And the conclusion and future research direction are given in the last section.

## 2. Mathematical Model Formulation

In this study, the stated problem from the previous section is formulated into a mathematical model with the objective to minimize total completion time. The proposed model has the following assumptions:

1. In the considered time period, resources including machine capacity and required raw materials are oversupplied.
2. Working hours are considered as a continuous time period.
3. Jobs could be split and processed on more than one machine.
4. Each operator can be assigned to assist only one machine at a time.

The notations used in formulation of model are as follows:

*Indexes*

| | |
|---|---|
| $n$ | the number of jobs. |
| $m$ | the number of machines. |
| $W$ | the number of operators. |
| $i, j$ | the index of jobs, $i, j = 0,1,2,\dots,n$+1; given job 0 is dummy job represents start job and job $n$+1 is dummy job for finish job of each machine. |
| $k$ | the index of machines, $k = 1,2,3,\dots,m$. |
| $w$ | the index of operators, $w = 1,2,3,\dots,W$. |

*Set*

| | |
|---|---|
| $L_w$ | Set of machines in the responsibility of worker. (To illustrate, if operator 1 responsible for machine 1 and 3 then $L_1 = \{1,3\}$). |

*Parameters*

| | |
|---|---|
| $P_i$ | total required processing time of job $i, j = 1,2,3,\dots,n$. |
| $S_{ijk}$ | the sequence-dependent setup times if job $j$ is the immediate successor of job $i$ on the machine $k$. |
| $T$ | maximum working hours of each operator in determined period. |
| $D_k$ | the total completion time of machine $k$. |
| $D_w$ | total working hours of operator $w$. |

*Decision Variables*

$Q_{ik}$ the proportion of job $i$ to be processed on machine $m$, $0 \leq Q_{ik} \leq 1$.
$Y_{ik}$ A binary variable equal to 1, if there is any of job $i$ processed on machine $m$, 0 otherwise.
$X_{ijk}$ A binary variable equal to 1, if job j is the immediate successor of job $i$ on machime $m$, 0 otherwise.
$U_{ik}$ A dummy variable for sub tour elimination.
$Cmax$ the maximum completion time of all jobs.

After the declarations of indexes, parameters, and decision variables, the objective function and constraints are described as below:

$$Minimize\ \ Cmax \tag{1}$$

Subject to:

$$\sum_{k=1}^{m} Q_{ik} = 1\,; i = \{1, 2, 3, \dots, n\}, \forall k \tag{2}$$

$$D_k = \sum_{i=1}^{n} Q_{ik}\, P_i + \sum_{i=1}^{n}\sum_{j=1}^{n} S_{ijk}\, X_{ijk}\ ;\ \forall k \tag{3}$$

$$D_w = \sum_{k=1}^{m} D_k\ ; m \in\ L_w\ , \forall w \tag{4}$$

$$C_{max}\ \geq \max_{1 \leq w \leq W} D_w \tag{5}$$

$$Q_{ik} \leq Y_{ik}; \forall i, \forall k \tag{6}$$

$$\sum_{j=1}^{n+1} X_{ijk} = Y_{ik}\ ;\ \forall k \tag{7}$$

$$\sum_{i=1}^{n} X_{ijk} = Y_{ik}\ ;\ \forall k \tag{8}$$

$$\sum_{j=1}^{n} X_{ojk} = 1\ ;\ \forall k \tag{9}$$

$$\sum_{i=1}^{n} X_{o(n+1)k} = 1\ ;\ \forall k \tag{10}$$

$$X_{iik} = 0\ ;\ \forall i \tag{11}$$

$$U_{ik} - U_{jk} + nX_{ijk}\ \leq n - 1; \forall i, \forall j \tag{12}$$

$$D_w \leq T\ ; \forall w \tag{13}$$

$$X_{ijk}, Y_{ik} = binary \tag{14}$$

$$Q_{ik}, U_{ik}, Cmax \geq 0 \tag{15}$$

The proposed mathematical model pursues the minimum value of $Cmax$, which is representing the total completion time (makespan) of all jobs, as shown in equation (1). Equation (2) guarantees that total amount of job $i$ is assigned to machine(s). Equation (3) explains how to calculate the total completion time of machine $k$, consisting of processing time and setup time of all jobs assigned to machine $k$. Equation (4) determines the total working hours of worker $w$, which equals to the sum of completion time of machine(s) that worker $w$ is responsible to. Although $\max_{1\le w\le W} D_w$ could represent the maximum completion time of all jobs by itself, setting $\max_{1\le w\le W} D_w$ as an objective function would make the model nonlinear. Thus, $C_{max}$ is introduced as shown in equation (5) to provide linearity for the model. Equation (6) ensures that part of job $i$ could be processed on machine $k$ only when job $i$ is selected to be processed on machine $k$. Equation (7) forces that every job $j$ processed on machine $k$ could only have one predecessor job $i$, which also has to be processed on machine $k$. Likewise, every job $i$ processed on machine $k$ could only have one successor job $j$ processed on machine $k$ as shown in equation (8). For each machine, there must be only one first and last job as formulated in equation (9) and (10) respectively. Equation (11) forces that the identical jobs $i$ could not be assigned contiguously to same machine $k$. Equation (12) takes care of sub-tour elimination from the model. Equation (13) is to ensure that total working hours of each worker is not exceeding the maximum working hours. Equation (14) and (15) are the basic restrictions on the decision variables.

## 3. Illustrated Example

As a proposed model was developed to overcome unnecessary setup times, which are the result of a rule-based heuristic occurred to arrange production schedules in practice. Hence, an experiment was conducted to test the efficiency of the proposed mathematical model by comparing the result, total completion time, from the model to those from a rule-based heuristic and a common dispatching rule. To schedule with the existing rule-based heuristic, the shortest processing time job will be processed on any eligible machine as soon as it is available. Yet, if none of the operators who are able to assist the machine available, the jobs could not be processed until the operator is free. The selected dispatching rule in this paper is using the Longest Processing Time (LPT) dispatching rule to arrange the schedules. In order to demonstrate the experiment, we randomly selected 4 examples with 10 different jobs ($n = 10$) consisting of 3 various net contain volume and 5 various flavors (represented by A, B, C, D, E), 5 machines ($m = 5$), and 4 operators ($W = 4$). Setup times and other properties of each machine are stated in Table 1. The processing times of each jobs and jobs descriptions are given by Table 2.

Table 1. Properties of Machines

| Machine $k$ | Eligible Product Flavor | Setup time Required (Hrs.) | | Assist Operator |
|---|---|---|---|---|
| | | Same Net Contain Volume | Different Net Contain Volume | |
| 1 | A, C, D, E | 4 | 6 | Operator 1 |
| 2 | B | 5 | 5 | Operator 1 |
| 3 | A, C, D, E | 4 | 6 | Operator 2 |
| 4 | A, C, D, E | 6 | 6 | Operator 3 |
| 5 | A, C, D, E | 5 | 7 | Operator 4 |

Table 2. Job Descriptions

| | Job *i* | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Test example 1 | Processing Time (Hrs.) | 35 | 32 | 35 | 48 | 49 | 39 | 42 | 43 | 37 | 40 |
| | Flavor | A | C | E | C | B | A | A | C | E | D |
| | Net Contain Volume (mL) | 350 | 380 | 500 | 350 | 380 | 500 | 380 | 500 | 380 | 500 |
| Test example 2 | Processing Time (Hrs.) | 35 | 50 | 46 | 50 | 45 | 34 | 34 | 40 | 28 | 27 |
| | Flavor | C | C | B | A | D | A | E | B | A | D |
| | Net Contain Volume (mL) | 350 | 380 | 500 | 350 | 380 | 500 | 380 | 500 | 380 | 500 |
| Test example 3 | Processing Time (Hrs.) | 34 | 32 | 48 | 49 | 40 | 44 | 29 | 43 | 36 | 44 |
| | Flavor | B | A | C | D | C | A | E | D | D | E |
| | Net Contain Volume (mL) | 350 | 380 | 500 | 350 | 380 | 500 | 380 | 500 | 380 | 500 |
| Test example 4 | Processing Time (Hrs.) | 35 | 29 | 38 | 44 | 47 | 33 | 44 | 48 | 28 | 37 |
| | Flavor | C | A | E | D | D | A | E | B | E | C |
| | Net Contain Volume (mL) | 500 | 380 | 500 | 380 | 500 | 350 | 380 | 500 | 350 | 380 |

## 4. Computational Results

The presented mathematical model in section was implemented in Microsoft Excel 2016, and Microsoft Excel Add-in OpenSolver was used to solve the model, and run on a personal computer with Intel® Core™ i5-7200U CPU, 2.5 GHz processor, and 32 GB RAM. The result from calculation of each illustrated example, including proposed mathematical model, existing rule-based heuristic, and LPT dispatching rule mentioned in section 3, are given in Table 3.

Table 3. Computational Results

| Maximum completion time of all jobs (Hrs.) | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Test example 1 | | | Test example 2 | | | Test example 3 | | | Test example 4 | | |
| Proposed Mathematical model | Rule-based Heuristic | LPT | Proposed Mathematical model | Rule-based Heuristic | LPT | Proposed Mathematical model | Rule-based Heuristic | LPT | Proposed Mathematical model | Rule-based Heuristic | LPT |
| 114 | 135 | 132 | 114 | 174 | 123 | 116 | 136 | 126 | 112 | 120 | 129 |

Based on the result of experiments, it is apparent that the existing rule-based heuristic is most likely to give the worst solution, in terms of maximum completion time of all jobs, as obtained in test example 1, 2 and 3 that completion time of all jobs is reduced from 135, 174, and 136 to 132, 123, and 126 respectively. Although handling these examples with LPT dispatching rule seems to acquire a better solution, sometimes it could be worse, for instance in test example 4 where completion time of all jobs from LPT is 9 hours grated than rule-based heuristic's result. For this reason, we employed the proposed mathematical model to solve these example problems and obtain an optimal solution. The experimental results substantiate that the proposed mathematical is effective. The minimum value of maximum completion time of all jobs gained from a rule-based heuristic and LPT for each test example whereas equals to 132, 123, 126, and 120, is decreased to 114, 114, 116, and 112 consecutively.

However, it is important to note that the proposed model could only be applied to the parallel machine scheduling production system with same speeds. For the future research, the production system of parallel machines with different speed could be included.

## 5. Conclusions

Many effective algorithms for parallel machine scheduling were developed in the past several decades. Yet, there are only few studies that consider machine scheduling together with workforce allocation. Thus, this paper considers the integrated problem of workforce allocation and parallel machine scheduling with sequence-dependent setup time and machine eligibility restrictions production system, which is imitated from is imitated from real-world beverage industry. The mathematical model is proposed to solve such problem. The experiment was set to test the efficiency of the proposed mathematical model by comparing the results of the introduced model with those of the existing rule-based heuristic in real practice and the common LPT dispatching rule. The experimental results establish that the suggested model could reduce value of maximum completion time of all jobs from the existing rule-based heuristic and LPT significantly.

## References

Arnaout, J.-P., G. Rabadi, and R. Musa, A Two-Stage Ant Colony Optimization Algorithm to Minimize the Makespan on Unrelated Parrallel Machines with Sequence-dependent Setup Times, *Journal of Intelligent Manufacturing*, vol. 21, pp. 693-701, 2010.

Helal, M., Rabadi, G., & Al-Salem, A. A Tabu Search Algorithm to Minimize The Makespan for Unrelated Parallel Machines Scheduling Problem with Setup Times. *International Journal of Operations Research*, vol. 3, pp. 182–192, 2006.

Lee, Z., Lin, S., and Ying, C., Scheduling jobs on dynamic parallel machines with sequence-dependent setup times, *The International Journal of Advanced Manufacturing Technology*, vol.47, pp. 731-781, 2010.

Luo, X., Liu, X., Lui, Z., Wang, C., Dominance Rules for Single Machine Schedule with Sequence Dependent Setup and Due Date, *Journal of Control Theory and Applications*, vol. 4, pp. 364 – 370, 2005.

Mencia, R., Sierra, M.R., Mencia, C., and Varela, R., Schedule Generation Schemes and Genetic Algorithm for the Scheduling Problem with Skilled Operators and Arbitrary Precedence Relations, *Proceedings of the Twenty-Fifth International Conference on Automated Planning and Scheduling*, Jerusalem, Israel, June 7–11, 2015.

Rabadi, G., Moraga, R., and Al-Salem, A., Heuristics for the Unrelated Parallel Machine Scheduling Problem with Setup Times. *Journal of Intelligent Manufacturing,* vol.17, pp.85–97, 2006.

Ovacik, I., Uzsoy, R., Rolling horizon procedures for dynamic parallel machine scheduling with sequence-dependent setup times. *International Journal of Production Research*, vol. 33, pp. 3173–3192, 1995.

## Biographies

**Arajabhorn Chantavali** holds a Bachelor of Engineering Degree in Industrial Engineering from Kasetsart University and currently pursues a master degree in Industrial Engineering at Kasetsart University. Her interests include manufacturing system management, computer integrated scheduling and planning, and optimization. Her undergraduate project involved increasing the productivity of a production line using simulation.

**Anan Mungwattana** has a doctoral degree in Industrial and Systems Engineering. Currently, he is an associate professor at the Department of Industrial Engineering, Kasetsart University. His research interests include facilities planning, supply chain and logistics management.