

Exponentially Weighted Moving Average Chart Employing Curtailed Inspection for Monitoring Attributes

Salah Haridy ^{a,b}

^a Department of Industrial Engineering and Engineering Management,
University of Sharjah, Sharjah 27272, United Arab Emirates.

^b Benha Faculty of Engineering, Benha University, Benha, Egypt.
sharidy@sharjah.ac.ae

Mohammad Shamsuzzaman and Imad Alsyouf

Department of Industrial Engineering and Engineering Management,
University of Sharjah, Sharjah 27272, United Arab Emirates.

mshamsuzzaman@sharjah.ac.ae, ialsyouf@sharjah.ac.ae

Ahmed Maged

Benha Faculty of Engineering, Benha University, Benha, Egypt.

ahmed.maged@bhit.bu.edu.eg

Abstract

Exponentially weighted moving average (EWMA) chart is one of the most powerful techniques for detecting small and moderate shifts. The curtailed inspection has been broadly used in acceptance sampling plans to reduce the average sample number substantially. This research presents an EWMA chart employing curtailed inspection (curtailed EWMA chart) to monitor the fraction nonconforming p of a process with attribute characteristics. A design algorithm is developed to optimize the charting parameters of the curtailed EWMA chart. The proposed curtailed EWMA chart is compared with the traditional EWMA without curtailed inspection (abbreviated as EWMA chart in this research) using the same false alarm rate to ensure a fair comparison. The overall detection ability of the charts is measured in terms of the expected average run length over a wide range of shifts, and the p shift is assumed to follow a uniform distribution. The findings of this research reveal that the curtailed EWMA chart has a superior overall performance than the EWMA chart without curtailed inspection. On average, the former is more effective than the latter by 35%, considering different circumstances. The high overall effectiveness of the curtailed EWMA chart is mainly attributable to curtailed inspection.

Keywords

Control chart, Curtailed inspection, Average run length, EWMA chart.

1. Introduction

A control chart is one of the most useful tools in Statistical Process Control (SPC). It is commonly used as an on-line surveillance technique in the manufacturing industries and service sectors. Recently, many control charts and SPC surveillance techniques have been developed (Shamsuzzaman *et al.* 2014, Chiu and Lu 2015, Khoo *et al.* 2015, Haridy *et al.* 2017a). The binomial Exponentially Weighted Moving Average (EWMA) chart is widely used for monitoring the fraction nonconforming p of a process. The extensive application of the binomial EWMA chart and attribute charts, in general, is attributable to many factors. These factors are not just related to the simplicity of handling attribute quality characteristics, and the capability of checking multiple quality requirements, but also to the ease by which they allow people at different levels of an

organization to communicate the information, and the prevalence of count data in various industries and non-manufacturing sectors.

When dealing with small and moderate process shifts, the EWMA chart is usually found to be a very effective tool in detecting these type of shifts. To detect an upward p shifts, a statistic C_t is updated and plotted for the t th sample in an EWMA chart

$$\begin{aligned} C_0 &= 0 \\ C_t &= \lambda(d_t - d_0) + (1 - \lambda)C_{t-1} \end{aligned} \tag{1}$$

where d_t is the number of nonconforming units found in the t th sample, d_0 is the in-control value of $d_t = n \times p_0$ (the product of the sample size n and the in-control fraction nonconforming p_0), and λ ($0 < \lambda \leq 1$) is a weighting parameter. When an increasing p shift occurs, C_t tends to become larger and larger. Eventually, an out-of-control signal is produced when C_t to exceeds the control limit H of the EWMA chart.

The speed of signaling an out-of-control condition is usually measured by the average run length (ARL), that is, the average number of samples required to signal an out-of-control condition after its occurrence. Charts are more effective when the out-of-control ARL is smaller as the problem can be signaled earlier. The smaller the ARL is, the easier it is to determine when the p shift occurs (Pignatiello and Samuel 2001). Thus minimization of ARL improves the capability of the control chart to diagnose the cause.

It is quite common to design an attribute control chart on 100% inspection of all the output of a process (Montgomery 2013), which is mainly due to the simplicity of the inspection of the attributes. This article mainly refers to a 100% inspection where the implementation of rational subgrouping may not be enforced (Reynolds and Stoumbos 1999, Montgomery 2013). However, there should be an optimal value of n that minimizes ARL (i.e., maximizes the detection effectiveness) of the control chart must exist.

In acceptance sampling plans, the curtailed inspection means that the inspection of a sample is terminated and the associated lot is rejected when the number of observed nonconforming units exceeds an acceptance number (Montgomery 2013). This curtailment can result in a significant reduction in the average sample number. The curtailment technique was employed by Wu *et al.* (2006) and Haridy *et al.* (2013) to develop an np chart and cumulative sum (CUSUM) chart, respectively. The np and CUSUM charts with curtailment were found to significantly outperform the conventional np and CUSUM charts without using curtailment. Haridy *et al.* (2017b) developed an EWMA chart with curtailment for monitoring p . Haridy *et al.* (2017b) assumed that the p shift follows a Rayleigh distribution and evaluated the overall performance of the proposed chart in terms of the average number of defectives. It was found that the curtailment technique can improve the overall detection speed of the EWMA chart while maintaining the false alarm rate at a specified level.

In this article, a binomial EWMA chart with curtailed inspection (curtailed EWMA chart) is proposed to monitor the fraction nonconforming p . This research assumes the random number d (the number of nonconforming units found in a sample) follows a binomial distribution with a known in-control fraction nonconforming p_0 . The p shift is assumed to follow a uniform distribution. The performance of the curtailed EWMA chart and the traditional EWMA chart without the curtailed inspection (shortened as EWMA in this paper) are compared in terms of the expected average run length ($EARL$) which is a popular measure of the overall performance of the control charts. Control charts for attributes are most often used to detect either an increase in fraction nonconforming or a deterioration in quality (Lucas 1985, Reynolds and Stoumbos 1999). Therefore, the focus of this research is to detect increasing p shifts.

The paper is organized as follows. In section 2, the implementation of the curtailed EWMA chart is explained. In section 3, the objective function *EARL* is introduced, and the optimal design of the curtailed EWMA chart is then described in detail. In section 4, a comparative study is presented, and finally, the conclusion is drawn in the last section.

2. Implementation of the curtailed EWMA chart

Unlike the EWMA chart, a curtailed EWMA chart can detect an out-of-control status before having inspected all the n units in a sample. The statistic C_{t-1} for the $(t-1)$ th sample of a curtailed EWMA chart can be found by Equation (1). Let g_t be the count of defective units found in the t th sample. g_t will increase one by one, during inspecting the t th sample. Using Equation (1), the process will be out of control when:

$$\lambda(g_t - d_0) + (1 - \lambda)C_{t-1} > H \quad (2)$$

$$\text{or } g_t > G_t \quad (3)$$

$$\text{where } G_t = \frac{H - (1 - \lambda)C_{t-1} + d_0}{\lambda} \quad (4)$$

G_t is defined as the upper bound of curtailed inspection for the t th sample. If $g_t > G_t$, the process is considered to be out-of-control and there is no need to inspect the remaining items in the t th sample. This is the salient feature of the curtailed inspection. A curtailed EWMA chart can be implemented as:

- (1) Initialize the EWMA statistic C_0 as zero and set $t = 1$.
- (2) Determine the upper bound G_t by Equation (4).
- (3) Set the g_t of the nonconforming units at zero in the beginning of the t th sample.
- (4) Increase g_t by one when a nonconforming unit is observed. If $g_t > G_t$, the inspection is stopped and go to step (8).
- (5) Otherwise (i.e., $g_t \leq G_t$), the process is deemed to be in control and make $d_t = g_t$.
- (6) Update C_t by Equation (1)

$$C_t = \lambda(d_t - d_0) + (1 - \lambda)C_{t-1} \quad (5)$$

- (7) Increase t by one. Then, go back to step (2), and take the next sample.
 - (8) The process is considered to be out of control and should be terminated for investigation.
- Clearly, the curtailed inspection may detect the out-of-control signal in step (4) before inspecting all items in the sample, and thus the detection effectiveness is enhanced. This curtailed inspection is the distinctive feature of the curtailed EWMA chart when compared to the EWMA chart.

3. Design of the curtailed EWMA chart

Four specifications are needed to carry out the design of a curtailed EWMA chart:

- (1) The allowable minimum value τ of the in-control average run length (ARL_0).
- (2) The in-control fraction nonconforming p_0 .
- (3) The maximum shift δ_{max} in fraction nonconforming p .
- (4) The minimum allowable value (n_{min}) of the sample size n .

The value of τ is decided with regard to the false alarm. The value of p_0 is usually estimated from the historical data observed when the process is in control. The value of δ_{max} may be chosen based on the knowledge about a process (e.g., the maximum p shift in a process) or taken as the shift range the users are interested in. When a process shift occurs, the fraction nonconforming p will change to:

$$p = \delta \times p_0 \quad (6)$$

where δ ($1 \leq \delta \leq \delta_{max}$) is the increasing p shift in terms of p_0 . Finally, the value of n_{min} is determined in order to accommodate some operational and managerial considerations. It is worth mentioning that in a 100% inspection, varying the sample size n simply means adjusting the grouping of the inspected items. In this article, the *EARL* is used to measure the overall performance of a control chart (Ryu *et al.* 2010).

$$EARL = \int_1^{\delta_{max}} ARL(\delta) \times f_{\delta}(\delta) \, d\delta \quad (7)$$

where $ARL(\delta)$ is the out-of-control *ARL* produced by a control chart at δ . The integration in Equation (7) can be evaluated by Legendre–Gauss Quadrature method. The random shift δ in p is assumed to follow a uniform distribution (Castagliola *et al.* 2011, Haridy *et al.* 2013) with a probability density function $f_{\delta}(\delta)$ of:

$$f_{\delta}(\delta) = \frac{1}{\delta_{max} - 1} \quad (8)$$

In this article, *EARL* is used as the objective function to be minimized. The design of the curtailed EWMA chart is carried out using the following optimization model:

$$\begin{array}{ll} \text{Objective:} & \text{Minimize } EARL \\ \text{Constraint:} & ARL_0 \approx \tau \\ \text{Design variables:} & n, \lambda, H. \end{array} \quad (9)$$

The optimization design aims at identifying the optimal values of n , λ and H that minimize *EARL* over a shift range of ($1 < \delta \leq \delta_{max}$) and meanwhile ensure that $ARL_0 \approx \tau$. The minimization of *EARL* in turn leads to a smaller out-of-control *ARL* over the entire range of p shifts. The optimization design is detailed as follows:

- (1) Identify the specifications τ , p_0 , δ_{max} and n_{min} .
- (2) Initialize $EARL_{min}$ as a very large number, say 10^7 ($EARL_{min}$ is used as the minimum value of *EARL*).
- (3) Explore the optimal value of n by increasing it from n_{min} at the first level. The search at this top level is stopped when *EARL* can be no longer decreased.
- (4) At the second level with the n determined at the first level, search the optimal value of λ . For a given n and λ ,
 - (4.1) Find the H that satisfies $ARL_0 \approx \tau$.
 - (4.2) When the values of all three charting parameters, n , λ and H , are preliminarily determined, the *EARL* is calculated by Equation (7).
 - (4.3) If the calculated *EARL* is smaller than the current $EARL_{min}$, set $EARL_{min} = EARL$ and the current values of n , λ and H are stored as a temporary optimal solution.
- (5) Once the two-level search is completed, the optimal curtailed EWMA chart that produces the minimum *EARL* and satisfies $ARL_0 \approx \tau$ is identified. The corresponding optimal n , λ , and H are also identified.

4. Comparative Studies

In this section, the detection effectiveness of the curtailed EWMA and EWMA charts is investigated and compared. The design of both charts is to find the best combination of the control limit H and the weighting parameter λ so that the chart produces the minimum *EARL* (Equation (7)) and meanwhile has an ARL_0 very close to τ .

The two charts are first studied under one case in which p_0 is estimated as 0.005. Based on the experience of the quality engineer in the process, the maximum shift δ_{max} is set to 10. The allowable

minimum τ is set as 500. Based on some operational considerations, a minimum value of 50 is used as a lower bound for the sample size n . The design specifications are summarized as follows:
 $\tau = 500, p_0 = 0.01, \delta_{max} = 10$ and $n_{min} = 50$ (10)

The two charts are designed, and the results are shown below:

EWMA chart: $n = 50, \lambda = 0.175, H = 0.1010, EARL = 90.61.$

Curtailed EWMA chart: $n = 95, \lambda = 0.125, H = 0.0108, EARL = 59.67.$

The values of the in-control ARL_0 (where $\delta = 1$) and out-of-control ARL (where $1 < \delta \leq 10$) of the two charts are calculated within the process shift range, and the results are displayed in Table 1. The curves of the normalized ARL (i.e., $ARL/ARL_{Curtailed EWMA}$) of the two charts are shown in Figure 1. Table 1 and Figure 1 indicate the following:

- (1) Each of the two charts generates an ARL_0 value very close to τ (constraint (9)) when the process is in control. This suggests that both charts have nearly identical false alarm rates.
- (2) The curtailed EWMA chart is always more effective (has smaller ARL values) than the EWMA chart over the entire p shift range. It is apparent that the curtailed inspection makes the curtailed EWMA chart very effective from an overall standpoint.
- (3) The superiority of the curtailed EWMA chart over the EWMA chart increases with increasing δ . For example, when $\delta = 10$, the ARL value of the curtailed EWMA chart is larger than that of the EWMA chart by 158%.

Table 1: Comparison of the EWMA and curtailed EWMA charts

δ	ARL	
	EWMA chart	Curtailed EWMA chart
1	505.235	501.380
2	150.895	110.768
3	98.015	67.649
4	77.340	51.125
5	66.855	40.066
6	60.400	32.901
7	56.765	28.667
8	54.250	25.174
9	52.535	22.146
10	51.750	20.082

$ARL_{EWMA} / ARL_{Curtailed EWMA}$

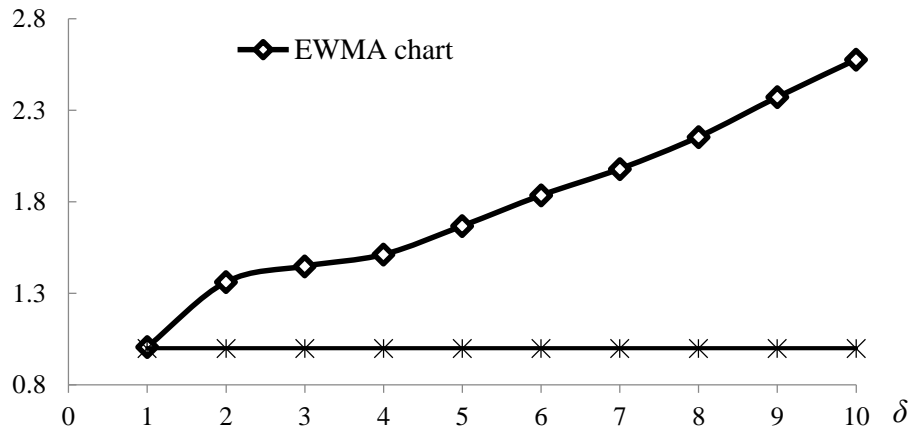


Figure 1: Normalized ARL of the EWMA and curtailed EWMA charts

The $EARL$ values (Equation (7)) of the two charts are also calculated. The ratio of $(EARL_{EWMA}/EARL_{Curtailed\ EWMA}) = 90.61/59.67 = 1.52$. This value indicates that, for this case, the overall detection effectiveness of the curtailed EWMA chart over the entire range of shifts ($1 < \delta \leq 10$) is more than that of the EWMA chart by 52%.

The two charts are further studied under four more cases with different settings of the design specifications as shown below:

τ :	300,	800.
p_0 :	0.005,	0.02.
δ_{max} :	5,	10.

The levels of the factors are determined with reference to those commonly used by many authors (Haridy *et al.* 2014, Shamsuzzaman *et al.* 2016). The fourth specification n_{min} is fixed at 50 for all cases.

For each case, the EWMA and curtailed EWMA charts are designed, and each of them produces an ARL_0 which is approximately equal to τ . The charting parameters of the two charts and the overall performance, as reflected by $EARL$, are listed in Table 2 for the four cases. It can be seen from Table 2 that the value of $EARL_{EWMA}/EARL_{Curtailed\ EWMA}$ is always larger than one. This indicates that the curtailed EWMA chart is always superior to the EWMA chart for detecting different sizes of p shifts. In these four cases, the relative detection effectiveness of the charts is similar to that revealed in Table 1. Namely, the curtailed EWMA chart always produces smaller out-of-control ARL values than the EWMA chart over the entire range of p shifts.

Table 2: Comparison of the EWMA and curtailed EWMA charts under different settings

Case	τ	p_0	δ_{max}	Chart	n	λ	H	$EARL$	$\frac{EARL_{EWMA}}{EARL_{Curtailed\ EWMA}}$
1	300	0.005	10	EWMA	50	0.200	0.0198	84.83	1.53
				Curtailed EWMA	63	0.175	0.0057	55.43	1.00
2	300	0.02	5	EWMA	50	0.500	0.4335	84.26	1.42
				Curtailed EWMA	64	0.410	0.2868	59.36	1.00
3	800	0.005	10	EWMA	50	0.440	0.3569	150.79	1.23
				Curtailed EWMA	110	0.440	0.3011	122.39	1.00
4	800	0.02	5	EWMA	55	0.410	0.6835	141.91	1.22
				Curtailed EWMA	67	0.440	0.7246	116.73	1.00

Finally, a grand average $\frac{EARL_{EWMA}}{EARL_{curtailed\ EWMA}}$ is calculated indicating the average of the $EARL_{EWMA}/EARL_{Curtailed\ EWMA}$ values encompassing all the four cases in Table 2. The result is $\frac{EARL_{EWMA}}{EARL_{curtailed\ EWMA}} = 1.35$. This shows that from the most comprehensive viewpoint (covering all different values of τ , p_0 , and δ_{max}), the curtailed EWMA chart is more effective than

the EWMA chart by 35%. This reflects the contribution of the curtailed inspection to improve the overall detection effectiveness of the EWMA control chart. The curtailed inspection allows the curtailed EWMA chart to use a reasonably large sample size so that it is sensitive to small p shifts. On the other hand, when a large p shift takes place, the curtailed inspection enables the curtailed EWMA chart to signal the large p shift before all of the n units in a sample are inspected. This ensures that the curtailed inspection mechanism can improve the performance of the control charts for detecting both small and large p shifts.

5. Conclusion

This research proposes a binomial EWMA chart employing curtailed inspection for detecting increasing shifts in fraction nonconforming p . The curtailed inspection was able to considerably improve the overall detection effectiveness of the EWMA chart compared with that of the traditional EWMA without curtailed inspection. The developed curtailed EWMA chart is superior to the conventional EWMA chart under different settings. On average, the former is more effective than the latter by 35% in terms of expected average run length (*EARL*).

The curtailed EWMA chart can be implemented using a well-developed procedure. Once the curtailed EWMA chart is designed, it can be used continuously as long as there is no change in the process and the improvement in detection speed can be gained. The observed ARL_0 of the curtailed EWMA chart is usually very close to the desired false alarm rate, despite the discrete nature of attribute characteristics.

In this article, the curtailed EWMA chart is implemented with a 100% inspection. It can also be used to detect increasing p shifts when sampling inspection is employed. Another prospective future work is to use the curtailed inspection to enhance the power of other control charts such as the time-between-events and synthetic control charts.

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Biographies

Salah Haridy is an assistant professor in the Department of Industrial Engineering and Engineering Management at the University of Sharjah, UAE. He received his M.Sc. and Ph.D. degrees from Benha University, Egypt and Nanyang Technological University, Singapore in 2008 and 2014, respectively. He is the recipient of the 2013 Mary G. and Joseph Natrella Scholarship awarded by the American Statistical Association (ASA) and the 2014 Richard A. Freund International Scholarship awarded by the American Society for Quality (ASQ). His research interests cover quality engineering, statistical process control and design of experiments.

Mohammad Shamsuzzaman is currently an associate professor in the Department of Industrial Engineering and Engineering Management at the University of Sharjah, UAE. He obtained his Ph.D. in Systems and Engineering Management in 2005 from Nanyang Technological University, Singapore. His current research focuses on quality control and improvement, reliability, simulation, and multi-criteria decision-making. He is a member of the American Society for Quality.

Imad Alsyouf is an associate professor of Industrial Engineering, employed by the University of Sharjah, UAE. He is the founder and coordinator of the Sustainable Engineering Asset Management (SEAM) Research Group. He has produced more than 30 conference and journal papers. He has about 27 years of industrial and academic experience in various positions in Jordan, Sweden, and UAE. His research interests include reliability, quality, maintenance, and optimization. He has developed and taught more than 25 post and undergrad courses. He delivered training courses in Kaizen, TQM, and organizational excellence.

Ahmed Maged is a teaching assistant in the Department of Mechanical Engineering at Benha University, Egypt. He received his M.Sc. degree in Industrial Engineering from the same institution. His research interests are focused on machine learning, quality engineering, statistical process control and design of experiments.