Marangoni Natural Convection in a Discretely Heated Cubic Cavity under Isoflux Heating Condition

HM Toufik Ahmed Zisan, Tahmidul Haque Ruvo and Sumon Saha
Department of Mechanical Engineering
Bangladesh University of Engineering and Technology (BUET)
Dhaka, Bangladesh
toufik.zisan1602@gmail.com, th.ruvo@gmail.com, sumonsaha@me.buet.ac.bd

Abstract

Marangoni natural convection inside a three-dimensional enclosure having a pair of discrete heaters at the bottom wall has been numerically analyzed in this study. Isoflux heating condition of the heaters has been considered, and the remaining walls except the top wall are kept insulated. The cavity is either completely or partly filled with water whose top surface is exposed to air and the surface tension gradient due to temperature at this air-water interface is responsible for the Marangoni convection. Three-dimensional, steady state governing Navier-Stokes and energy equations have been solved by finite element method. Rayleigh number in the range of $10^4$-$10^7$ has been selected for performing parametric simulation. Visualization of velocity and isothermal surface plots gives a qualitative comparison, whereas distribution of average Nusselt number with Rayleigh number under different filling conditions shows a quantitative illustration. Heat transfer regime shows a strong dependence on different values of governing parameter as well as different filling conditions.

Keywords
Marangoni convection, Isoflux heating, Discrete heater, Finite element method, Nusselt number.

1. Introduction:

Natural convection has been one of the major branches of research for quite a long time. Here the flow is initiated by the density difference arising from temperature gradient within the flow domain. The buoyancy driven flow has a high reliability, trifling cost and a great margin of power savings. It has got a wide range of applications in reactor core cooling, solar energy storage technology, food processing plants etc. When it comes to electronic equipment cooling, the added consideration of discrete heaters is gaining a far more importance. Improved cooling can eventually increase the performance of any electrical appliance.

Liquid cooling of the electric packages has been a major trend of modern era. However, besides the usual natural convection, Marangoni convection also occurs at the liquid-gas interface due to the presence of surface tension gradient. Occurrence of the gradient due to variation in solute concentration is called soluto-capillary effect and the other one, due to the temperature variation is called thermocapillary effect. Consideration of Marangoni effect in the analysis of systems resembling the liquid cooling scheme is a growing field of study. Proper understanding of such systems helps us in better understanding of the practical cooling scenario. Moreover, the effect of Marangoni convection is also dominant in other practical engineering cases like crystal growth, welding etc. which also needs to be studied in details.

1.1 Literature Review:

A benchmark solution for naturally convective heat transfer in a differentially heated cavity was provided by Vahl Davis (1982). Hamady et al. (1989) showed both experimentally and numerically the effect of inclination angle on the heat transfer. Increase in the Nusselt number for an increase of Rayleigh number was observed. Khanafer et al. (2003) analyzed heat transfer improvement of buoyancy driven flow in a 2D cavity for a range of Grashof number and nanofluid volume fractions. Later, using experimental and numerical analysis Mamun et al. (2003) obtained a new diamond orientation for a cubic cavity filled with air.
Analysis of Marangoni convection has been getting attention for past decades. Zebib et al. (1985) studied high Marangoni number thermocapillary convection for enclosures heated from side. The study also showed the effects of different Prandtl number in this type of convection. Bergman and Ramadhyani (1986) found that, presence of thermocapillary effect significantly alters the pattern of buoyancy driven flow inside a cavity. Investigation performed by Behnia et al. (1995) shows that for a cubic enclosure the domain of negative Marangoni number flow is stratified and it also induces some unusual critical zones. Oztop et al. (2013) also numerically showed the effect of combined natural convective flow and thermocapillary effect for a range of various positive and negative Marangoni numbers.

Consideration of open-ended cavities is also important for simulating any standard cooling systems. Chan and Tien (1985) presented numerical study for two dimensional square open cavities. It was found that, at higher $Ra$ recirculation zone becomes visible near the bottom zone. Mohamad (1995) showed work on a cavity having open side wall. The flow pattern and the direction of incoming air and outgoing air seemed to change at higher $Ra$. With the consideration of combined conduction, convection and radiation in an open top cavity, Dehghan and Behnia (1996) showed that, consideration of the radiation does not actually improve the heat transfer coefficient and also, omitting of radiation effect adds very little inaccuracy in the analysis of such systems. Hinojosa et al. (2005) showed that, for heat transfer in an inclined open top cavity, convective Nusselt number changes significantly with the change in $Ra$ and inclination angle.

Study of cavities with discrete heat sources has received a significant importance due to various practical applications. Combined experimental and numerical analysis done by Calcagni et al. (2005) for discrete heater at the bottom of cubic cavity shows that increase in heat source size causes an intended increase in heat transfer. Aminossadati and Ghasemi (2009) studied cooling of localized heater inside separate enclosures filled with different nanofluids. Mahmoudi et al. (2010) showed that both height and length of discrete horizontal heaters mounted on the sidewalls immensely effect the heat transfer. In the comparative study performed by Aminossadati and Ghasemi (2011) it was found that in between two separate configurations for a pair of heat source and sink, the asymmetric configuration gives the optimum heat transfer. Gibanov and Sheremet (2018) studied transient case inside a cubic cavity for 5 different heat source shapes. Trapezoidal shaped heater seemed to give the best performance. Numerical investigation performed by Moutaouakil et al. (2021) showed that for discrete heaters inside a cubic cavity the optimum positioning and size of the heaters significantly affect the rate of heat transfer.

1.2 Objective:
From the literature survey stated, it is clear that separate studies have been performed on natural convection, discrete heaters and Marangoni convection. However, none of the cases mentioned above combines all the effects together, which is very much essential in proper understanding and analysis of any practical liquid cooling systems. That’s why this paper proposes to investigate the cooling performance inside a cubic cavity with open top side and two discrete heaters at the bottom together with the consideration of both buoyancy driven and surface tension gradient driven flow.

2. Model Description

2.1 Physical Modelling
The cubic cavity considered for this study is shown in figure 1 which has two heating elements at the bottom wall while remaining portion of the wall is kept adiabatic. Left and right wall of the cavity is taken as cold wall and rest of the walls are in insulated condition. Heating elements are maintained at isoflux condition throughout the study.

Working fluid is water for the fully filled cavity and for partially filled cavities a portion of the cavity contains water and the rest is filled with air. A water-air interface is found at the top wall for first configuration and inside the cavity for the other cases. This interface allows Marangoni convection due to the temperature dependent surface tension gradient. At the air water interface, surface tension varies linearly with relation $\sigma_0(1-\theta(T-T_o))$. Here $\sigma_0$, $T_o$ denotes reference surface tension and reference temperature respectively. $\theta$ is the temperature coefficient of surface tension defined as $\theta=(-1/\sigma_0) (\partial \sigma/\partial T)$. The side length of the cavity is $L=1m$ and the heating elements width is $e=0.2L$ which are positioned at the left and right side of the bottom wall. For partially filled cases, height of water is considered as $0.5L$ and $0.3L$. 
2.2 Mathematical Modelling

Air and water both are Newtonian fluid. For first configuration, there is only one domain of water and other configurations contain two separate domains of water and air. Governing equation is taken as continuity, momentum and energy equations separately for air and water to solve the problem numerically. For this study, steady state, incompressible and laminar flow conditions are considered to observe the flow behavior along with thermal field. Governing Equation for water domain:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]  

\[
\rho \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial x} + \mu_\nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)
\]
\[
\rho_w \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial y} + \mu_w \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \tag{3}
\]

\[
\rho_w \left( u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu_w \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) + \rho_w \beta_w (T - T_c) \tag{4}
\]

\[
\rho_a C_{p,a} \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) = k_a \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \tag{5}
\]

Governing Equation for air domain:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{6}
\]

\[
\rho_a \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial x} + \mu_a \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) \tag{7}
\]

\[
\rho_a \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial y} + \mu_a \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) \tag{8}
\]

\[
\rho_a \left( u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu_a \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) + \rho_a \beta_a (T - T_c) \tag{9}
\]

\[
\rho_a C_{p,a} \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) = k_a \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \tag{10}
\]

Here, \( x, y \) and \( z \) axis construct 3D cartesian coordinate system, \( u, v, w \) are the dimensional velocity components in the \( x, y \) and \( z \) direction respectively, \( \rho \) is dimensional pressure and \( T \) is dimensional temperature, \( \rho, C_p, \mu, \beta, k, \nu, \alpha \) are density, specific heat capacity, dynamic viscosity, thermal expansion coefficient, thermal conductivity, kinematic viscosity and thermal diffusivity respectively. Subscripts \( w \) and \( a \) are for water and air respectively. Non-dimensional governing parameter is defined as \( Ra = \rho \beta \Delta T / (\nu \alpha) \) to compare the non-dimensional performance parameter Nusselt number.

**Table 1.** Thermo-physical properties at 20°C

<table>
<thead>
<tr>
<th>Properties</th>
<th>Water</th>
<th>Air</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu ) [Pa.s]</td>
<td>1.01E-03</td>
<td>1.81E-05</td>
</tr>
<tr>
<td>( C_p ) [J/(kg.K)]</td>
<td>4.19E+03</td>
<td>1.01E+03</td>
</tr>
<tr>
<td>( \rho ) [kg/m³]</td>
<td>9.98E+02</td>
<td>1.20E+00</td>
</tr>
<tr>
<td>( k ) [W/(m.K)]</td>
<td>5.94E-01</td>
<td>2.58E-02</td>
</tr>
<tr>
<td>( \beta ) [1/K]</td>
<td>2.20E-04</td>
<td>3.41E-03</td>
</tr>
</tbody>
</table>

Dimensional boundary conditions for the current work are given in Table 2.

**Table 2.** Dimensional boundary conditions
Boundary | Velocity Condition | Thermal Condition | Surface Tension
--- | --- | --- | ---
Left and Right wall | \( u = 0, \ v = 0, \ w = 0 \) | \( T = T_c \) | \\
Top Wall | \( w = 0 \) (configuration -1) \( \frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = \frac{\partial w}{\partial z} = 0 \) (configuration -2, 3) | \( \frac{\partial T}{\partial z} = 0 \) (configuration -1) \( \left( \frac{\partial T}{\partial z} \right)_{out} = 0 \) \( T_{in} = T_c \) (configuration -2,3) | \( \frac{\partial u}{\partial z} = -\frac{\partial \sigma}{\partial T} \frac{\partial T}{\partial x} \), \( \frac{\partial u}{\partial z} = -\frac{\partial \sigma}{\partial T} \frac{\partial T}{\partial y} \) (configuration -1)
Front and Rear wall | \( u = 0, \ v = 0, \ w = 0 \) | \( \frac{\partial T}{\partial y} = 0 \) | \\
Bottom wall (isoflux heating elements) | \( u = 0, \ v = 0, \ w = 0 \) | | \( q'' \) \\
Bottom wall (adiabatic) | \( u = 0, \ v = 0, \ w = 0 \) | \( \frac{\partial T}{\partial z} = 0 \) | \\
Water-Air interface (Configuration-2, 3) | \( u_a = u_w, \ v_a = v_w, \ w_a = 0 \) | \( T_a = T_w, k_a \frac{\partial T}{\partial z} = k_w \frac{\partial T}{\partial z} \) | \( \mu_a \frac{\partial u_a}{\partial z} - \mu_w \frac{\partial u_w}{\partial z} = -\frac{\partial \sigma}{\partial T} \frac{\partial T}{\partial x} \), \( \mu_w \frac{\partial u_w}{\partial z} = -\frac{\partial \sigma}{\partial T} \frac{\partial T}{\partial y} \)

Average Nusselt number is calculated along the heating elements using following expression:

\[
\frac{1}{L} \int_{0}^{L} \frac{\partial T}{\partial z} dA
\]

\[
\frac{\int_{A} (T - T_c) dA}{Nu} = \int_{A} \frac{\partial T}{\partial z} dA
\]

(11)

### 3. Numerical Procedure

Dimensional governing equations are solved using finite element method based “COMSOL Multiphysics 5.6”. The domain is at first halved in the XZ plane in order to reduce the computational cost. That is followed by discretization of the domain into tetrahedral elements. Boundary layers are treated with finer mesh. Using mesh independence test the optimum number of selected elements is 23050.

Code validation for discrete heating system is performed against the work of Gibanov and Sheremet (2018). Here the heat transfer and flow effect of different three-dimensional heat source configuration is analyzed. The quantitative plot of Nusselt number in figure 2(a) shows considerable similarity. Another validation is performed for combined Marangoni convection and natural convection in open ended cavity done by Bergman and Ramadhyani (1986). For \( Ma=1000 \), current code seems to be in well agreement with the published result.
From both of these validations, it can be stated that current model can accurately predict the heat transfer and flow domain of discrete heating system and Marangoni convection.

Figure 2. (a) Comparison with the work of Gibanov and Sheremet (2018) for $\alpha=56^\circ$, (b) Comparison with the work of Bergman and Ramadhyani (1986) for $Ma=1000$

4. Result and Discussion
A comparative study is shown for three different arrangements. $Ra$ was varied within the range of $1 \times 10^4$-$1 \times 10^7$. Fluids used are air ($Pr=0.71$) and water ($Pr=7$) whose temperature dependent properties are stated in the Table 1.

4.1 Visualization of Thermal Field
Isosurface maps of temperature are shown in figure 3. For configuration 1, the isothermal surface is shown only within the water domain. At lower $Ra$ numbers isothermal surfaces shows a slower development. As the $Ra$ number increases, a notable development is observed through the middle of the domain representing a significant increase in heat transfer. For configuration 2 and 3, separate water and air domains are present within the enclosure. In both of the cases, a hindered growth of isotherm is observed within the water domain. At higher $Ra$, this is accompanied by an irregular and chaotic surface formation near the water-air interface due to the effect of surface tension gradient. These layers ultimately flatten near the lateral cold walls giving up heat. For all of the cases, a small amount of heat transfer occurs to the air domain at the interface. This causes almost gentle development of isosurfaces within the air domain for all of the cases. With the increase of $Ra$, this isosurfaces formation becomes steeper. It is also to be noted that, for configuration 3, a small portion of water is available to transfer the heat to cold side wall. That’s why in this case a relatively greater share of heat is transferred to air domain than in the case of configuration 2. This can be observed by the faster development of air domain temperature isosurfaces in configuration 3 accompanied by a slight distortion at $Ra 1.5 \times 10^7$.

4.2 Visualization of Flow Field
Velocity isosurfaces plotted for different $Ra$ number in the range $7.5 \times 10^4$-$1.5 \times 10^7$ are provided in figure 4, to visualize the flow behavior of the selected domain. In configuration 1 where working fluid is only water, isosurfaces show a different growing behavior with $Ra$ number than the configurations having both water and air. Here, at low $Ra$ number water shows a smoother flow with a lower heat transfer rate. As $Ra$ number increases flow starts developing at a moderate rate and a strong circulation is observed at $Ra$ number $1.5 \times 10^6$-$1.5 \times 10^7$ which ultimately increases the heat transfer rate. In configuration 2 and 3, circulation occurs in both water and air where air shows stronger circulation. This is due to the fact that, density change of air is more than that of water. Here air domain takes the top surface of water as heat source and construct the flow towards the cold walls. And, with the increment of $Ra$ number, flow field
becomes stronger in both water and air. Also, for these two cases, the considered open boundary condition allows a

![Temperature isosurfaces inside the cavity for different configurations and different Ra](image)

**Figure 3:** Temperature isosurfaces inside the cavity for different configurations and different Ra
Figure 4: Velocity isosurfaces maps inside the cavity for different configurations and different $Ra$

free flow of air in and out of the cavity. That is why three different open-ended vortices are formed – two adjacent to the cold walls and one in the center of the air flow.
For all the cases, the water-air interface including the temperature dependent surface tension allows shear stress and makes additional vortex in the flow field. This additional vortex has effects on the temperature distribution and an overall effect on heat transfer.

### 4.3 Quantitative Comparison of Thermal Performance

Considering all the effects of Marangoni convection within open cavity, buoyancy driven convection and discrete heating system, the average $Nu$ at the hot surfaces is plotted in the figure 5. It shows that, heat transfer increases in all three configurations with the increase of $Ra$ as expectedly with the presence of severe vorticity formation. The trend of heat transfer increment is almost identical for each of these cases.

On observing the figure, it becomes clear that there is only a negligible difference in the $Nu$ number in between configuration 1 and 2. In configuration 2, despite the decrease in the length of cold sidewall available to water domain, the heat transfer remains same owing to the increased circulation of air towards the cold walls and through the open boundary.

While in configuration 3, the average $Nu$ decreases than the other two cases. Here, the amount of water is not sufficient enough to reject the same amount of heat as in other scenarios. Air circulation cannot compensate for the decrease in heat transfer to the cold walls through water medium. This decreases overall heat transfer in configuration 3.

![Figure 5: Variation of $Nu$ at different $Ra$ for different configurations](image)

5. Conclusion

In this study, natural convection in conjunction the Marangoni effect in the water-air interface inside a cubic cavity along with an opening at top end was analyzed. Three different configurations were numerically investigated across a range of $Ra$ ($7.5 \times 10^4$-$1.5 \times 10^7$). Following conclusions can be drawn:

- In configuration 1, flow fields and thermal fields are continuous while in configuration 2 and 3, isosurfaces of two domains expand in different manner having a discontinuity in between.
- At the open end of cavity, three different vortices are formed due to the circulation of air through the boundary in case of configuration 2 and 3, which increases the outgoing convective heat flux.
- For Marangoni effect, shear stress builds up due to the surface tension gradient and causes to initiate an additional vortex.
- For all configurations, average Nusselt number increases with Rayleigh Number in a similar pattern.
In this type of heating arrangements, half-filled cavity is enough for adequate heat transfer. But when water fills up the cavity by less than half of the cavity volume, decrease in the heat transfer to cold side wall through water domain outweighs the increase in outgoing convective heat flux through air domain resulting in a reduced overall heat transfer.

Further analysis can be conducted with other liquids to better understand and compare the flow behavior and overall heat transfer in such configurations along with an added consideration of economic aspect.

Acknowledgements
The authors would express their earnest gratitude to the Computational Fluid Dynamics and Heat Transfer (CFDHT) Research Group of Department of Mechanical Engineering, Bangladesh University of Engineering and Technology (BUET) for providing continuous support throughout the research works.

References:


**Biographies**

**HM Toufik Ahmed Zisan** is pursuing B. Sc. in Mechanical Engineering from Bangladesh University of Engineering and Technology (BUET). His research interest is focused to computational heat transfer, CFD, nanofluids, molecular dynamics. He already presented one research paper in International Conference.

**Tahmidul Haque Ruvo** is pursuing B. Sc. in Mechanical Engineering from Bangladesh University of Engineering and Technology (BUET). His research interest involves heat transfer, CFD, nanofluids, manufacturing process, nanomaterials. He already presented one research paper in International Conference.

**Dr. Sumon Saha** received his PhD in Engineering from the University of Melbourne, Victoria, Australia in 2014. He completed his B.Sc. and M.Sc. in Mechanical Engineering from Bangladesh University of Engineering and Technology (BUET), Dhaka, Bangladesh on 2004 and 2007, respectively. His major field of study is numerical analysis on problems of thermo-fluid. He is now working as a Professor in the Department of Mechanical Engineering of Bangladesh University of Engineering and Technology (BUET). He already published more than 140 research papers in International Journals and Conference Proceedings and coauthor of two books in engineering field. His fields of interests are turbulent flows, computational fluid mechanics, computational heat transfer and thermal postbuckling analysis. Dr. Saha is the editor of one international journal and reviewers of several international conference proceedings and international journals. He is currently senior member of International Association of Computer Science and Information Technology (IACSIT), Singapore. Moreover, he is a life member of Bangladesh Solar Energy Society. He has received many professional awards like International Postgraduate Research Scholarship by the Australian federal government; Melbourne International Research Scholarship by the University of Melbourne; RHD Studentship by University of Melbourne, and so on.