Conjugated Natural Convection Heat Transfer Inside a Heated Domed Porous Cavity Containing a Solid Cylinder

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Abstract

The current effort is devoted to visualizing the natural convection flow behavior inside a dome shape trapezoidal porous media containing a heat-conducting solid cylinder to improve flow structure and natural convection heat transfer. Laminar, steady, incompressible flow behavior has been analyzed keeping the lower wall at high temperature, side walls at low temperature, and upper dome insulated. The porous media consists of fluid with higher thermal conductivity and also highly thermally conducting cylinder material to visualize thermal contour inside the cylindrical region. The finite element method has been applied for solving the non-dimensional form of continuity, Navier-Stokes and energy equation of fluid, and the energy equation of the solid cylinder. Computations have been conducted for a wide range of Grashof numbers (*Gr*: 10^3, 10^4, 10^5, 10^6) with a fixed Prandtl number (0.025) since the fluid is considered as mercury. Different materials for the solid cylinder are analyzed for heat transfer of varying thermal conductivity ($7.86 \leq Ks \leq 35.3$). The current study reveals that the Nusselt number can be increased by 4% and 11% respectively in contrast to trapezoidal and rectangular-shaped cavities without a dome. The result also shows that the trapezoidal dome shape facilitates the smoother circulation of fluid throughout the cavity which in turn establishes better natural convection heat transfer.

Keywords
Laminar flow, Finite Element Analysis, Natural Convection, Corrugated Heat transfer, Porous Medium.

1. Introduction
Natural convection with conduction heat transfer is known as conjugate natural convection has become a very popular topic for researchers and engineers because of its diverse fields of application in the various area including flow in a heat exchanger and boiler tubes, heat transfer in rooms and buildings, nuclear reactor, semiconductors, thermo-mechanical coupling, welding, thermoelastic damping, drying technology, food processing industries, cooling of electronics and so on. Heat transfer inside closed geometries is a prominent scientific area. But on the other side, the study of curvilinear geometries, including trapezoidal, and triangular-shaped, ellipsoidal, hexagonal, has become more difficult due to its meshing's inconsistency. Potential applications of trapezoidal cavities are heat controlling inside buildings, designing a solar collector, and so on. In recent years, several authors have been worked on heat transfer in trapezoidal shape cavities. Natural convection in the porous media becomes one of the most sustained areas of research.

2. Literature Review
In the last few decades, researchers have been investigated the conjugate natural convection of different shapes and media. Investigation of natural convection in the square cavity is very popular and has been lots of works in this field. C. Shu and H. Xue (1998) investigated the natural convection in a square cavity for comprising two approaches. C. Shu and K.H.A. Wee (2002) used the simple generalized differential quadrature method to study the natural convection in a square cavity. E.J. Braga and M.J.S. de Lemos (2005) studied the natural convection in a square cavity with circular and square rods and showed that the heat transfer with square rods is slightly high than circular rods.
A lot of studies of curvilinear geometries including, triangular, hexagonal, trapezoidal, and so on have already been done. E.H. Ridouane et al (2006) worked with a triangular enclosure and showed that heat transfer is high at the lower corners of the enclosure and is performed as conduction but far from the corner, heat is transferred by natural

convection. Then the researchers showed more interest to see the effect of the porous cavity from the non-porous cavity. Y. Varol et al (2009) studied the natural convection in a right-angle trapezoidal enclosure with porous media and he compared the numerical result with a differentially heated square porous cavity. H. Saleh et al (2011) also worked on natural convection in a trapezoidal enclosure with an inclined magnetic field and he showed heat transfer performance is reducing with reducing the angle of an inclined wall.

After then, researchers showed interest in conjugate natural convection. Conjugate natural convection is a combination of conduction (solid) and convection (fluid), so here the properties of solid material mainly thermal conductivity gives a vital role to improve the performance of the heat transfer. M. Lindstedt and R. Karvinen (2013) worked with the conjugate heat transfer and observed that to get better results to illustrate, lower thermal conductivity is useful for turbulent flow and also natural convection. J. Serrano-Arellano and M. Gijón-Rivera (2014) observed that convection heat transfer decreases as a result of radiative heat transfer but total heat transfer increases with high Rayleigh numbers by investigating conjugate natural convection inside a square cavity. Moreover, he used the finite volume method and also a k–ε turbulence model for high Rayleigh numbers. D.-D. Zhang et al (2016) investigated the factors that affect the inverse solution accuracy of inverse conjugate natural convection problem and thermal conductivity of solid to liquid is one of them. Thermal conductivity increases with increasing Rayleigh number was shown by M. Khatamifar et al (2017) in a differentially heated square cavity. M. Chakkingal et al (2019) studied different thermal conductivity ratio solid to liquid and Rayleigh numbers for certain fluid Prandtl numbers and also showed the heat transfer performance with thermal conductivity, and Rayleigh number.

Recently researchers performed conjugate natural convection in curvilinear geometries with porous media. Z.H. Khan et al (2021) investigated natural convection in a trapezoidal porous cavity and showed a relation among heat transfer performance, Rayleigh number, and domain shape. Nowadays, the regular shape is not popular in many practical applications. That's why researchers try to work with an irregular shape to get an idea about the heat transfer mechanism for regular and irregular shapes.

2.1 Objectives

Many studies have already been done for irregular enclosure in conjugate natural convection. But no studies have yet been done with the dome shape in porous media containing a solid cylinder for conjugating natural convection as per the authors' knowledge. Therefore, it is being a challenge to work with this problem and reach a conclusion. Effect of heat transfer performance has been shown for regular (rectangular), trapezoidal and domed shapes by filling the cavity with mercury taking as a working fluid.

3. Computational Model Description

The finite element method has been used to solve the non-dimensional equations of this present study. The domain consists of a trapezoidal domed cavity having a solid cylinder inside it and finer mesh is applied near the boundaries.

For this research, a domed-shaped porous cavity, with a solid cylinder inside the cavity is considered, as shown in figure 1. The geometry has a trapezoid of length L, and a height of H, where H=4L/5. On top of the trapezoid is a dome semi-circle whose diameter is D, where D = 0.70L. The trapezoid is symmetrical about its central vertical axis. There is a solid cylinder which diameter is d, where d = 0.2L inside the trapezoidal. The bottom wall, inclined sidewalls of the trapezoidal and upper dome are kept at hot temperature T_h, cold temperature T_c and adiabatic respectively. Here the heating is uniform.

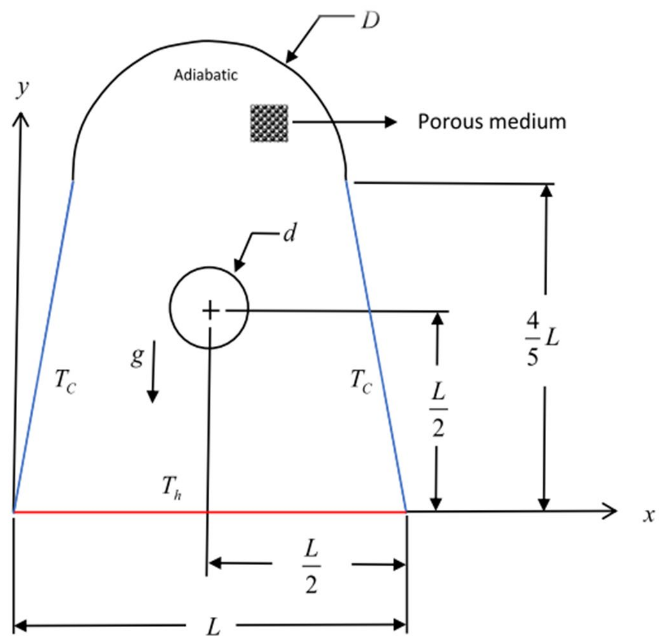


Figure 1. Schematic diagram of the dome porous cavity

Inside the cavity, the fluid is considered a Newtonian and incompressible fluid. Throughout the domain, laminar flow is considered. The Boussinesq approximation is considered to account for the variations of temperature as a function of density. The viscous effect is neglected and no heat generation is considered. As a result, the following governing equations can be written for the conservation of mass, momentum, and energy in the dimensionless form:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$\frac{1}{\varepsilon^2}\left[U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}\right]=-\frac{\partial P}{\partial X}+\frac{1}{\varepsilon}\left[\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right]-\frac{U}{Da}-\frac{F}{\sqrt{Da}}U\sqrt{U^2+V^2} \tag{2}$$

$$\frac{1}{\varepsilon^2}\left[U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}\right]=-\frac{\partial P}{\partial Y}+\frac{1}{\varepsilon}\left[\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right]-\frac{V}{Da}-\frac{F}{\sqrt{Da}}V\sqrt{U^2+V^2}+Gr\theta \tag{3}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\Pr}\left[\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right] \tag{4}$$

For a solid circular cylinder, the non-dimensional energy equation is as follows,

$$\frac{K}{\Pr}\left[\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right]=0 \tag{5}$$

The non-dimensional parameters that appear in the above equations are the Grashof number $Gr=\frac{g\beta\Delta\theta L^3}{\nu^2}$, Forchheimer coefficient $F=\frac{1.75}{\sqrt{150\varepsilon^3}}$, Prandtl number $\Pr=\frac{\nu}{\alpha}$, and Darcy number $Da=\frac{k}{L^2}$. The variables used to nondimensional the governing equations are as follows,

$$X=\frac{x}{L},Y=\frac{y}{L},U=\frac{uL}{\nu},V=\frac{vL}{\nu},\theta=\frac{T-Tc}{Th-Tc},P=\frac{pL^2}{\rho\nu^2} \tag{6}$$

Nusselt number is defined as

$$Nu=-\int_0^1 \left.\frac{\partial\theta}{\partial X}\right|_{Y=0} dY \tag{7}$$

Here, x and y are the cartesian coordinate system, X and Y are the non-dimensional coordinate system, u and v are the velocity components along with x and y directions respectively, U and V are the dimensionless velocity components in X- and Y-directions respectively, p is the pressure, P is the dimensionless pressure, T is the temperature, Θ is the dimensionless temperature, ε is the porosity, β is the coefficient of thermal expansion of mercury, ν is the kinematic viscosity of mercury, α is the thermal diffusivity of mercury, and g is the gravitational acceleration.

The non-dimensional boundary conditions are used to investigate this problem are shown in table 1.

Table 1: Non-dimensional boundary conditions for the present problem

Boundary	Thermal condition	Velocity condition
Bottom wall	$\theta=1$	$U=V=0$ (All walls)
Left and right walls	$\theta=0$	
Dome wall	$\frac{\partial\theta}{\partial n}=0$	
Circular cylinder wall	$K\left(\frac{\partial\theta}{\partial n}\right)_s=\left(\frac{\partial\theta}{\partial n}\right)_f$	

4. Data Collection

Governing and geometrical parameters are used to solve this problem are shown in table 2.

Table 2: Selection of governing and geometrical parameters for the present simulation

Parameter	Value of interest
Prandtl number (*Pr*)	0.0251
Darcy number (*Da*)	10^{-5}
Porosity (ε)	0.2
Grashof number (*Gr*)	10^3

Thermal conductivities from solid to liquid are used to solve the present simulation are given in table 3.

Table 3: Selection of material for the present simulation

Material	*Ks*	*Kf*	*K*
Bismuth	7.86	8.515	0.92
Nichrome	12	8.515	1.41
SS-AISI 302	15.1	8.515	1.77
Boron	27	8.515	3.17
Lead	35.3	8.515	4.15

5. Results and Discussion

5.1 Graphical Results

The present study reveals that a dome-shaped trapezoidal surface is more efficient in heat transfer than trapezoidal, and rectangular shapes without a dome. These results validate our choice of choosing a particular shape. From the bar chart below, it is visible that the Dome shape has the highest Nusselt number than trapezoidal, and rectangular shapes with the variation of Grashof number. The maximum Nusselt number indicates higher heat transfer.

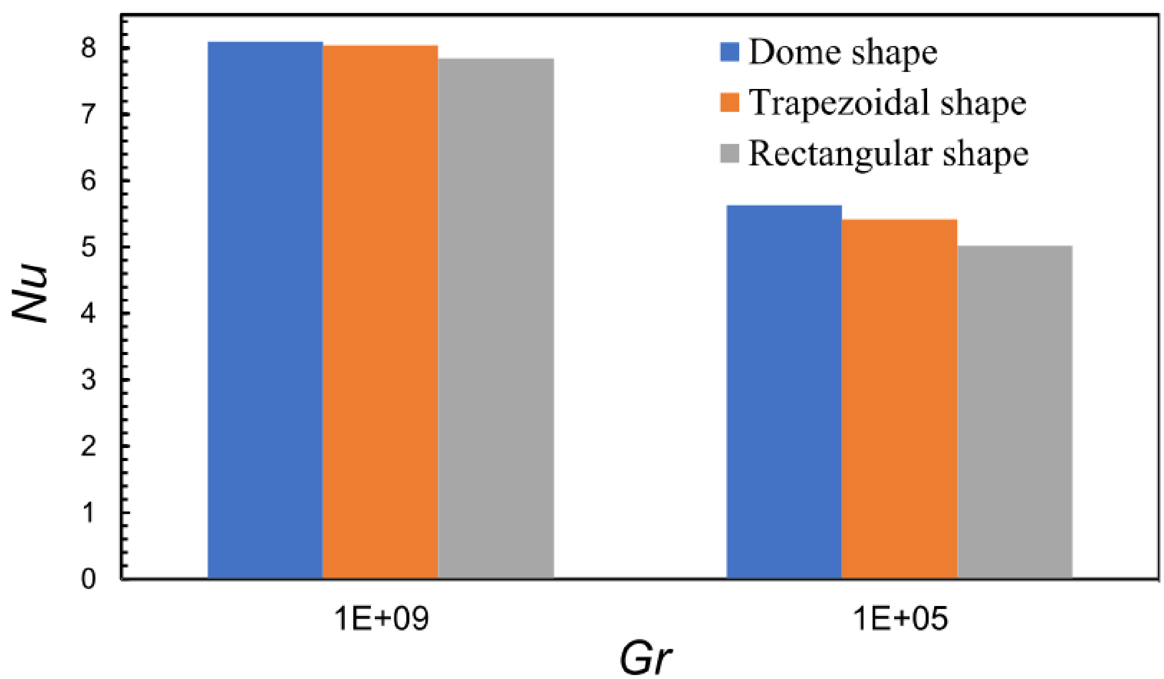


Figure 2. Variation of Nusselt number with Grashof number for different shapes (Dome, Trapezoidal, and Rectangular)

Streamlines and isothermal contours are given in figure 3 to show a comparison among the dome, trapezoidal, and rectangular shapes.

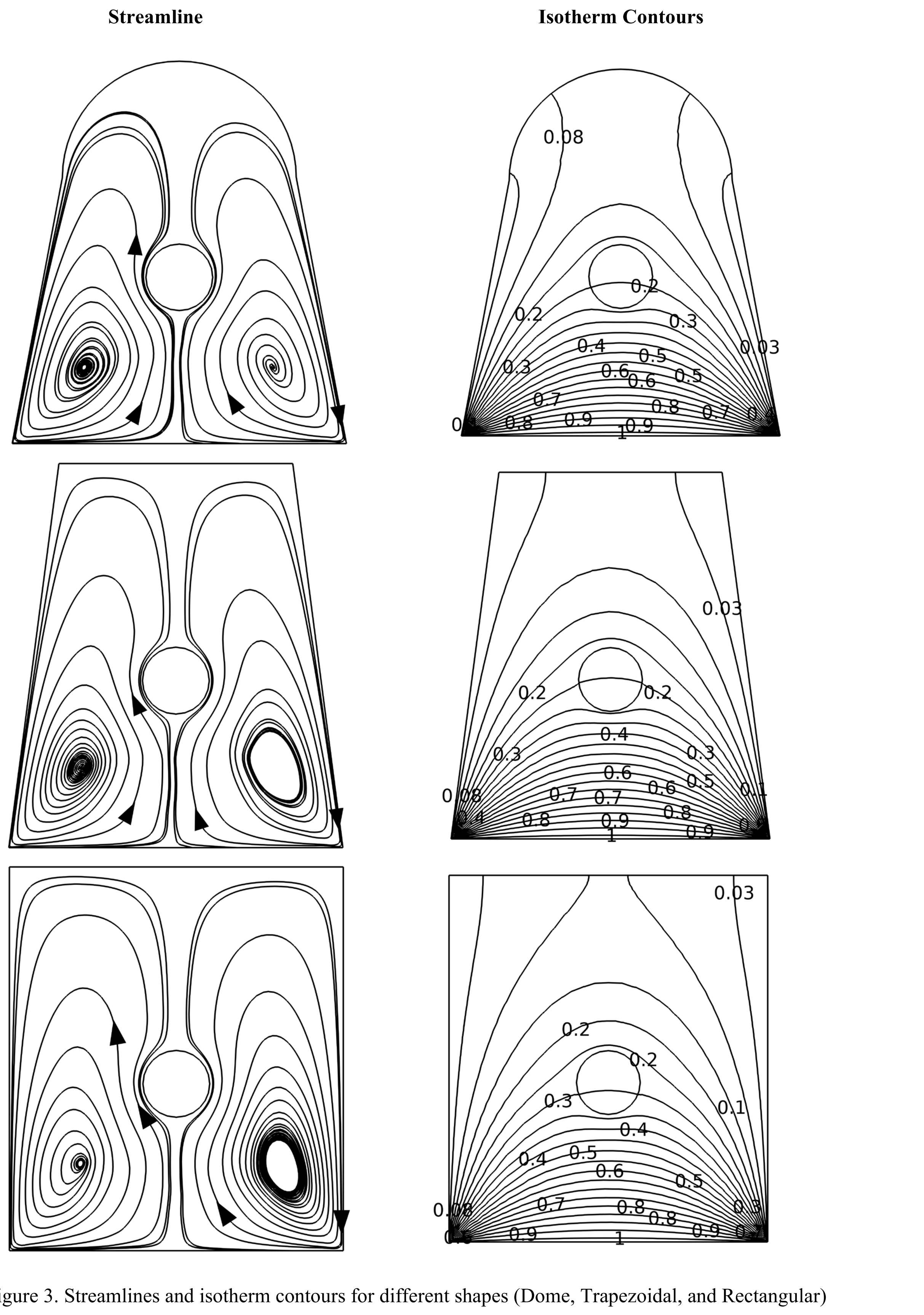
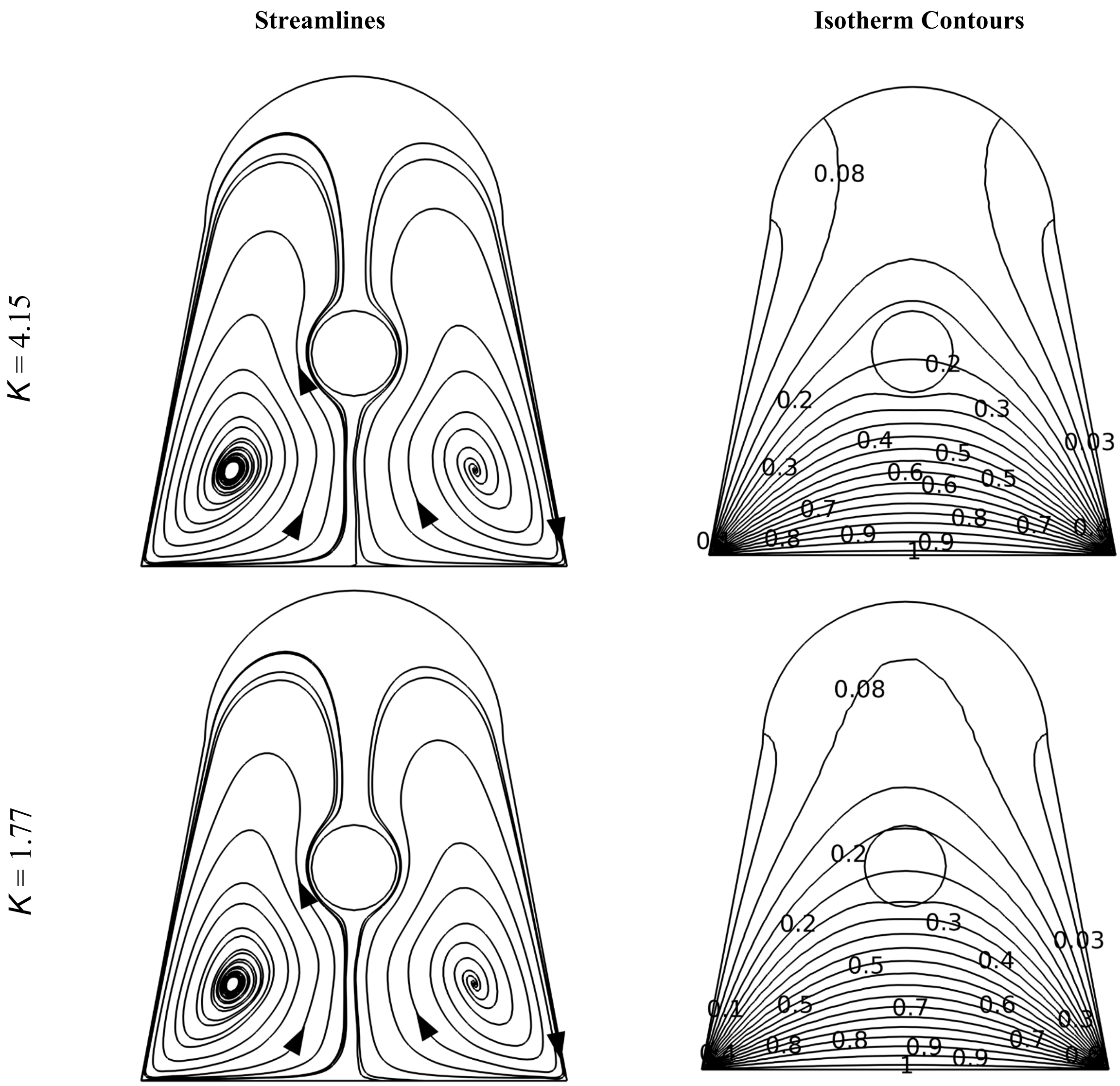


Figure 3. Streamlines and isotherm contours for different shapes (Dome, Trapezoidal, and Rectangular)

To understand the thermal structure, three different thermal conductivities of 4.15, 1.77, and 0.92 are chosen keeping the thermal conductivity of the fluid (Hg) the same but choosing different heat-conducting solids. The simulation results are shown in figure 4. with streamlines and isotherm contours. Streamlines show very little variation with the change of thermal conductivity ratio's but there are noticeable changes in isothermal contours. With the decrease of thermal conductivity ratio, the isothermal contours get flatter at the position of heat-conducting solid.



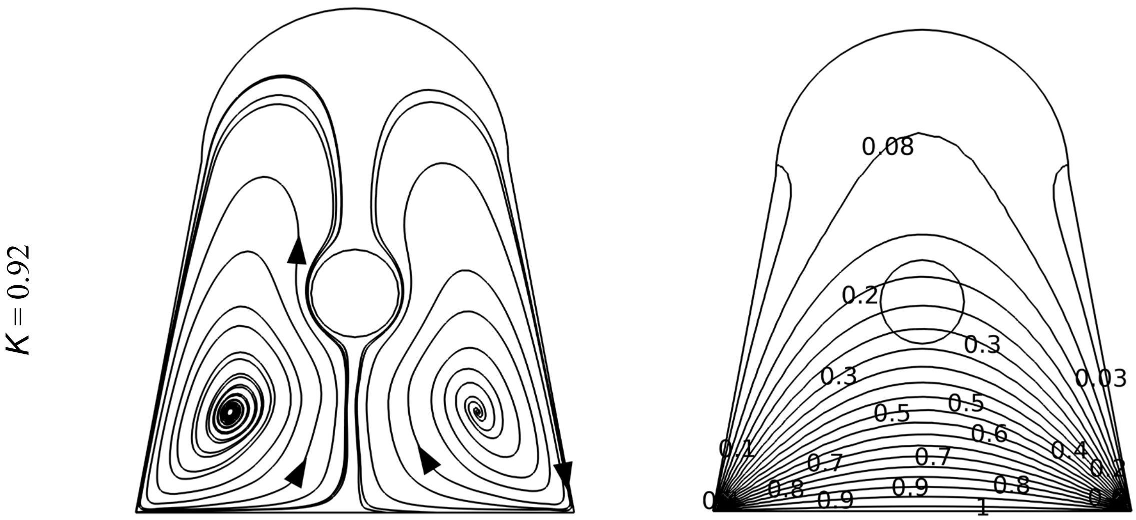


Figure 4. Streamlines and isotherm contours for different thermal conductivity ratios K = 4.85, 1.77 and 0.92

In figure 5 Nusselt number is plotted against different Grashof numbers for various thermal conductivity ratios. For a particular value of thermal conductivity ratio, Nusselt number is the same for a wide range of Grashof numbers (10^3 to 10^5) and then there is a gradual rise in Nusselt number. It is also noticeable that the value of the Nusselt number increases with the increase of the thermal conductivity ratio.

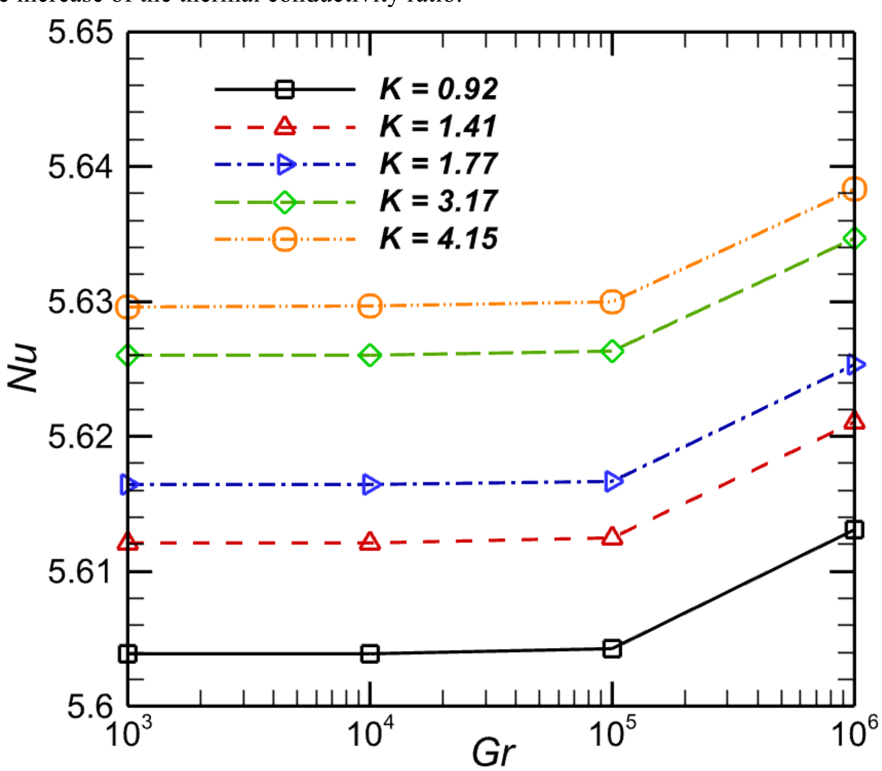


Figure 5. Nusselt number for different Grashof numbers, the thermal conductivity of solids

5.2 Validation

For any numerical simulation, the model has been validated with the previously published results. The present simulation model is validated with the work of P. Biswal et al (2016) shown in figure 6.

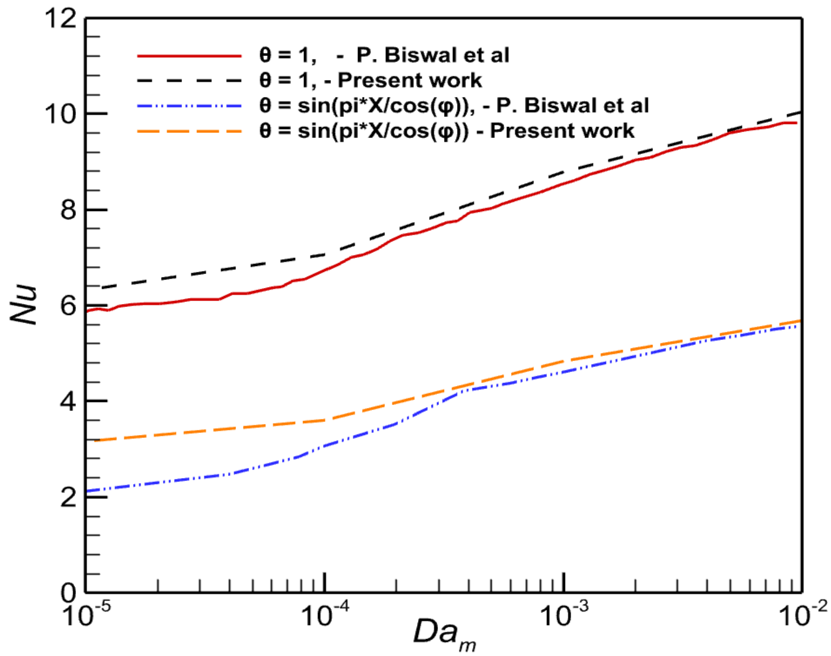


Figure 6. Comparison of Nusselt number with Darcy number between the present model and Biswal et al. (2016)

6. Conclusion

The present study represents a comprehensive analysis of natural convection heat transfer inside a heated domed porous cavity containing a solid cylinder. The main target was here to find an optimized configuration for maximum heat transfer. Based on the different analyses, the following conclusions can be drawn:

- Dome-shaped porous cavity demonstrates more heat transfer than trapezoidal or rectangular-shaped cavities.
- For a particular value of thermal conductivity ratio, the Nusselt number is the almost same for a wide range of Grashof numbers.
- Nusselt number increases gradually after a certain value of Grashof number.
- Nusselt number increases with the increase of thermal conductivity ratio.

Acknowledgments

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Biography

Sakib Javed is an undergraduate student of Mechanical Engineering (ME) at Bangladesh University of Engineering and Technology (BUET), Dhaka. He passed his HSC from Notre Dame College, Dhaka, and SSC from Akij Collegiate School, Jashore. His topics of interest include Numerical analysis, Machine Designing, System Dynamics, Thermodynamics, Heat Transfer, and Fluid Flow.

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Dr. Sumon Saha received his Ph.D. in Engineering from the University of Melbourne, Victoria, Australia in 2014. He completed his B.Sc. and M.Sc. in Mechanical Engineering from Bangladesh University of Engineering and Technology (BUET), Dhaka, Bangladesh in 2004 and 2007, respectively. His major field of study is numerical analysis on problems of thermo-fluid. He is now working as a Professor in the Department of Mechanical Engineering of Bangladesh University of Engineering and Technology (BUET). He already published more than 130 research papers in International Journals and Conference Proceedings and is co-author of two books in the engineering field. His fields of interest are turbulent flows, computational fluid mechanics, computational heat transfer, and thermal post-buckling analysis. Dr. Saha is the editor of one international journal and a reviewer of several international conference proceedings and international journals. He is currently the senior member of the International Association of Computer Science and Information Technology (IACSIT), Singapore. Moreover, he is a life member of the Bangladesh Solar Energy Society. He has received many professional awards like the International Postgraduate Research Scholarship by the Australian federal government; the Melbourne International Research Scholarship by the University of Melbourne; the RHD Studentship by the University of Melbourne, and so on