Conjugate Mixed Convection in a Differentially Heated Square Cavity with a Rotationally Oscillating Solid Cylinder

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Abstract

Heat transfer in a differentially heated square cavity containing a rotationally oscillating and heat conducting circular cylinder is studied numerically in this paper using a two-dimensional Galerkin finite element method. The problem involves unsteady laminar conjugate mixed convection heat transfer. The differential heating condition ensures heat dissipates from the high temperature wall to the system. Pressure-velocity forms of the Navier-Stokes and energy equations are used to represent the mass, momentum, and energy conservation of the fluid medium in the cavity. The heat conduction in the solid is represented by the conduction equation. These equations are solved numerically to generate the velocity as well as the temperature profiles. Computations of average Nusselt number are carried out by varying Reynolds numbers at unity Richardson number, and oscillation frequency ($1 \leq f \leq 10$), while the Prandtl number of the working fluid is kept fixed at 0.7 (air). These parameters are found to affect the heat transfer in the system to various extents. The effect of these parameters on heat transfer are analyzed in this study.

Keywords
Rotating Oscillating Cylinder, Nusselt Number, Frequency, Amplitude, Reynolds number.

1. Introduction

Study of heat transfer of a cylinder has been performed extensively as its area of application is plentiful. During these investigations, cylinders were given different types of boundary conditions. At some research, the cylinder was kept stationary, while at other works, cylinders were given oscillations - translational and rotational. Numerical and experimental research has been conducted to study heat transfer of a cylinder in a flow. Especially, forced convection works have been conducted considerably regarding this type of problem. This type of problem is often found in many engineering and industry-based fields such as heat exchangers in power plants and cooling of electronics etc.

In this paper, we are concerned with a differentially heated square cavity where one of the vertical sides are kept at a higher temperature while the other side is kept at a lower temperature. Also, the other 2 sides are kept insulated. At the center of the cavity, a solid cylinder of stainless steel is placed. This cylinder is our oscillating component of our problem. The cavity is filled with air where the flow is of laminar characteristics.

1.1 Literature Review

Different studies have been conducted to study the effect of rotational oscillating cylinder in fluid flow. Mahfouz *et al.* (2000) studied the effects of forced convection of a cylinder in a rotational oscillating motion placed in a uniform stream. Here the cylinder with a constant temperature was placed in a uniform stream and results included the effects of Nusselt Number with different frequencies, angular displacements, time etc. At low Reynolds numbers, Choi *et al.* (2002) explored the flow properties of a rotationally oscillating cylinder. For incompressible viscous flow past a rotating and translating cylinder, Ray (2011) devised a transformation-free HOC system. Beshok *et al.* (2012) tried to enhance heat transfer in a straight channel by a rotationally oscillating adiabatic cylinder. They placed an adiabatic cylinder in a parabolic flow which was passing through a channel where the walls were given the condition of constant heat flux. Kumar *et al.* (2013) conducted an experiment where they studied the flow past a cylinder. Their tests were conducted at a Reynolds number of 185. Sellapan and Pottbaum (2014) conducted an experiment to investigate the vortex shedding and heat transfer in rotationally oscillating cylinder. They found that heat transfer rate was greatly

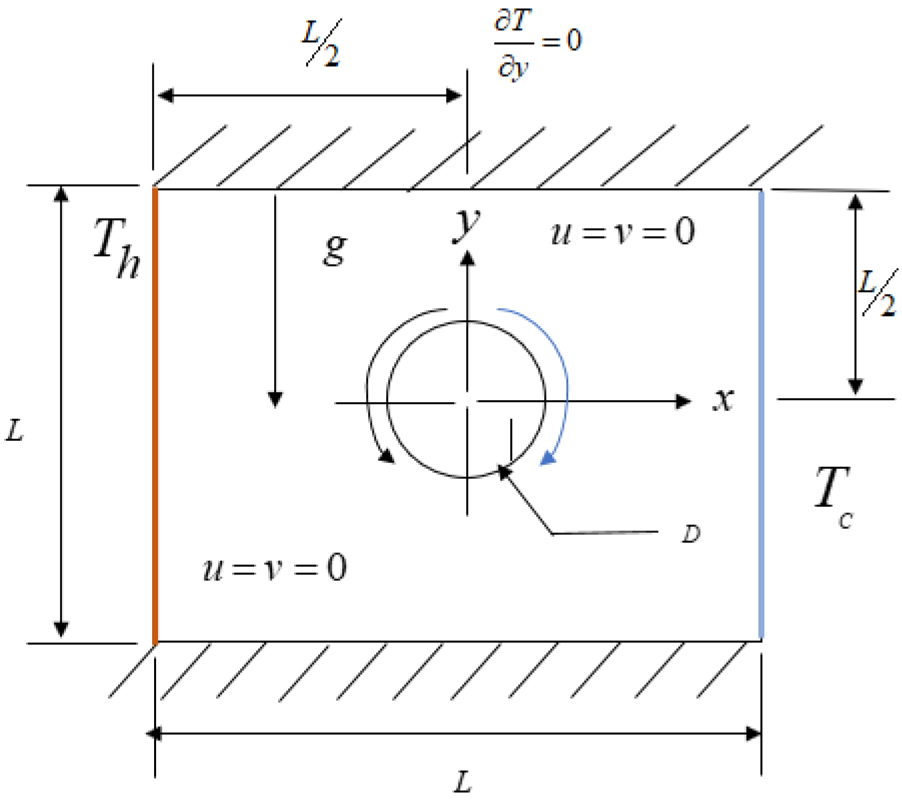


FIGURE 1. Schematic diagram of the differential square cavity with an oscillating solid cylinder in Cartesian coordinate system

increased when forcing frequency of the cylinder was close to Strouhal frequency. Mittal *et al.* (2017) used a transformation free HOC approach to simulate beginning flow past an impulsively initiated rotationally oscillating circular cylinder. Hadžiabdić *et al.* (2019) investigated heat transfer in a flow around a rotationally oscillating cylinder at a high subcritical Reynolds number ($Re = 1.4 \times 10^5$). Yawar *et al.* (2019) numerically investigated heat transfer over a rotationally oscillating cylinder which is subjected to gust impulse. During their study, they kept Reynolds number constant at 110 and used different forcing Strouhal frequency. They found that Nusselt number is increased by 7% in the gust impulse case in comparison to streamlined flow. Ozalp *et al.* (2021) did an experimental investigation where they studied the flow patterns of a rotationally oscillating cylinder in the wake region. They used dye visualization and Particle Image Velocimetry (PIV). The rotationally oscillating cylinder affects the wake flow architecture and vortex shedding patterns.

1.2 Objectives

Objective of this research is to study the effects of mixed convection in a cavity filled with air where a solid cylinder is oscillating rotationally. This work investigates the effects of convection process due to the variation of Reynolds number and changing of oscillating frequency. While many works have been conducted by researchers to study rotational oscillating motion for forced convection, very few works have been done to study this type of problem for conjugate mixed convection.

2. Computational Methods

To solve the problem in hand, we consider the flow to be incompressible, two dimensional. The fluid is a Newtonian and the flow is of laminar characteristics. As both natural and forced convection is prominent in this problem, we use Boussinesq approximation to calculate density variation. Moreover, we assume that the thermophysical properties such as thermal conductivity, viscosity, specific heat capacity is constant with temperature variation. Also, it is assumed that the thermal conductivity of the solid cylinder is constant with temperature variation.

By maintaining these assumptions, the continuity equation, the Navier Stokes equation, the energy equation can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\frac{\partial v}{\partial t}+\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

$$\rho C_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{5}$$

We will solve this problem by using Non dimensional equations. To convert the equation (1) – (5) into non-dimensional form, use the non-dimensional scales below.

$$X=\frac{x}{L},\ Y=\frac{y}{L},\ U=\frac{uL}{\alpha},\ V=\frac{vL}{\alpha},\ \theta=\frac{T-T_c}{T_h-T_c},\ Gr=\frac{g\beta\Delta TL^3}{\nu^2},\ \Pr=\frac{\nu}{\alpha},$$
$$\mathrm{Re}=\frac{u_m L}{\nu},\ Ri=\frac{Gr}{\mathrm{Re}^2},\ \theta_s=\frac{T_s-T_c}{T_h-T_c} \tag{6}$$

Hence the non-dimensional equations take the following form:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{7}$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{8}$$

$$\frac{\partial V}{\partial t}+U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{\mathrm{Re}}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ri\theta \tag{9}$$

$$\frac{\partial \theta}{\partial t}+U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{\mathrm{Re}\Pr}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{10}$$

$$\frac{\partial^2 \theta_s}{\partial X^2}+\frac{\partial^2 \theta_s}{\partial Y^2}=\frac{\partial \theta_s}{\partial t} \tag{11}$$

Here, L is the length of the sides of the square cavity. The non-dimensional coordinate system's axes are X and Y. τ is the non-dimensional time. θ and θ_s are the non-dimensional temperature of the fluid and the solid respectively. U and V is the non-dimensional velocity components. P is the non-dimensional pressure.

The boundary conditions for the equation (6) – (10) are as follows:

$$\begin{gathered} U(0,Y)=U(1,Y)=U(X,0)=U(X,1)=0 \\ V(0,Y)=V(1,Y)=V(X,0)=V(X,1)=0 \\ \frac{\partial\theta}{\partial Y}(X,0)=\frac{\partial\theta}{\partial Y}(X,1)=0 \\ \theta(0,Y)=1, \theta(1,Y)=0 \end{gathered} \tag{12}$$

For the solid oscillating cylinder, the wall movement (Beskok *et al.*) imparted is-

$$\phi=\phi_m * \pi f_e * D * sin(2\pi f t) \tag{13}$$

Here, ϕ is the angular motion of the cylinder. ϕ_m is the maximum angular displacement. f is the oscillating frequency and diameter of the cylinder is D.

Heat would be conducted between the fluid and the cylinder. It followed this condition: $\frac{\partial\theta}{\partial n}=k\frac{\partial\theta_s}{\partial n}$ (14)

K is the ratio of the thermal conductivity between the solid and the fluid.

Nusselt number is determined as:

$$Nu\ (t)=-\int_0^1 \frac{\partial\theta}{\partial X}\, dy \tag{15}$$

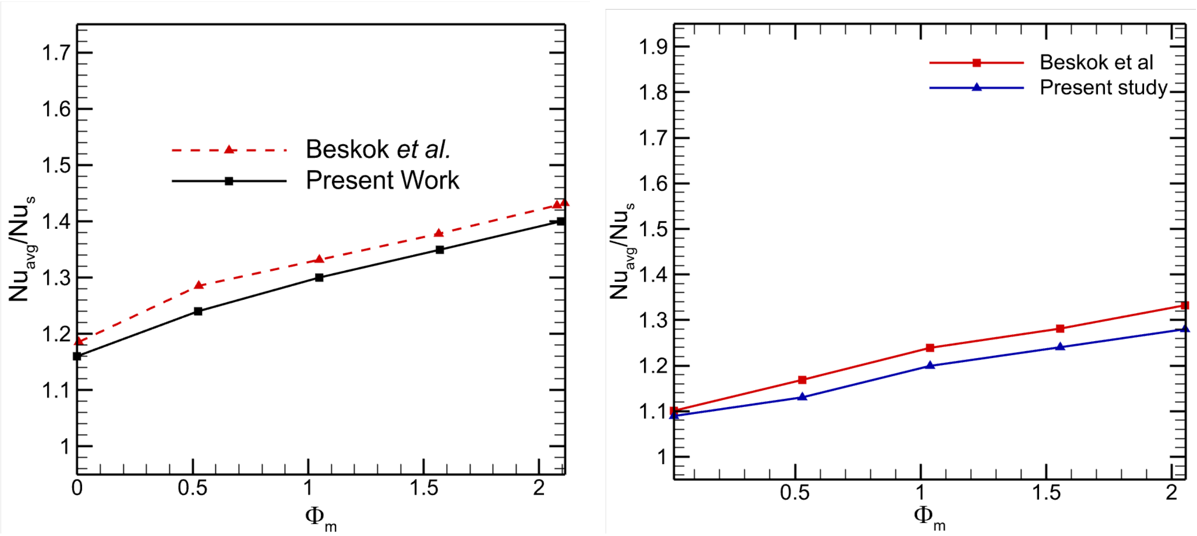
Time average Nusselt number is obtained as, $\overline{Nu}=\frac{1}{t}\int_0^t Nu(t)\, dt$ (16)

3. Simulation Procedure:

The non-dimensional equations from (7)-(11) are solved using Finite element method. Both non-uniform triangular and quadrilateral elements are used to discretize the domain. At the edges, a finer mesh is used. Not very fine mesh in general is used in the solution as it would take a lot of time for computation.

4. Model Validation:

The current method of solution is validated against the work of Beskok *et al.* (2012). The work of Beskok *et al.* is based on an adiabatic cylinder oscillating in rotational motion in a straight channel. Here, Reynolds number is kept at 100. 2 cases are shown in fig 2. In the first case, unit Prandtl number is used, while F is 1. One the second case, Prandtl number is 0.1 and the F is 0.8. In both cases, oscillation is occurring sinusoidally.



(a) (b)

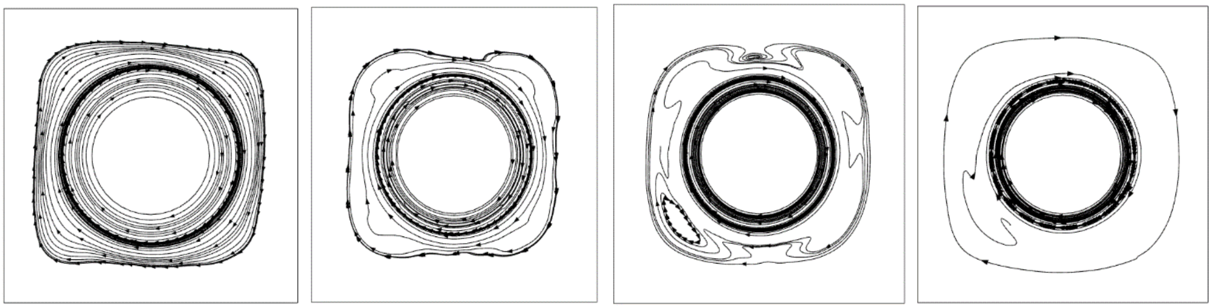
Figure 2. Average Nusselt number ratio of a channel containing rotational oscillating cylinder and straight channel with maximum angular displacement (a) F=1, Pr = 1 (b)F=0.8, Pr =0.1

5. Results and Discussion

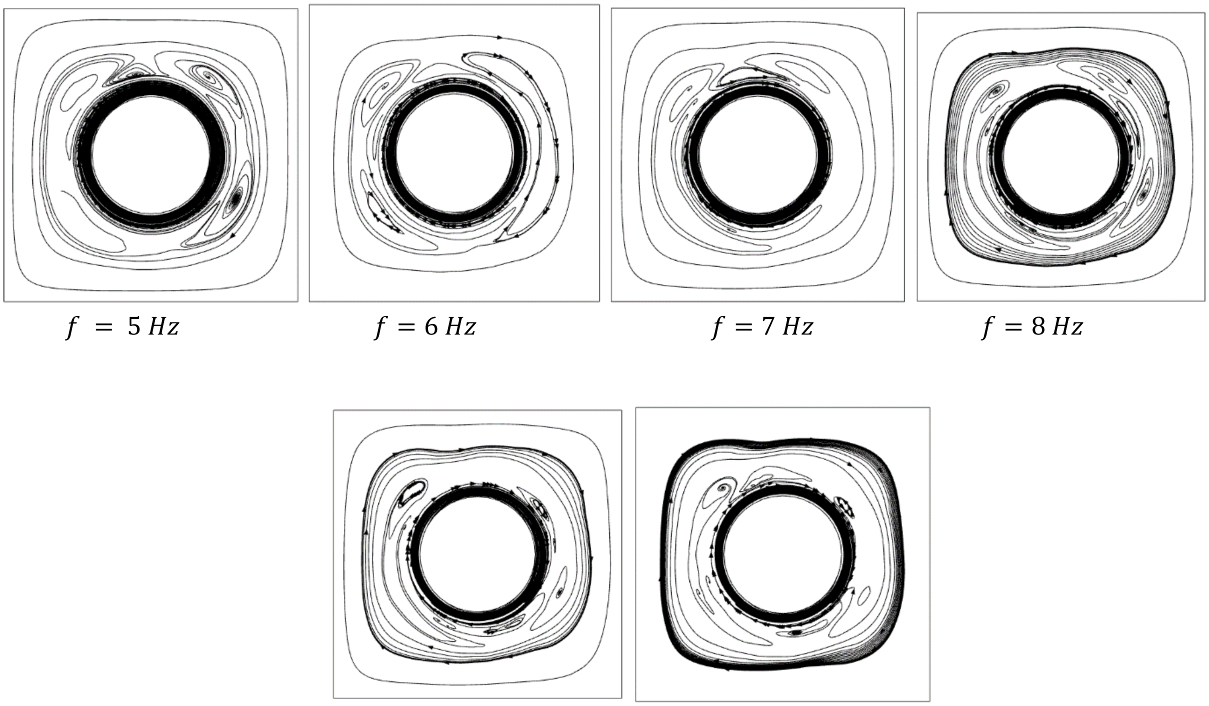
Simulations are run for a variety of frequencies ($0 < f < 10$ Hz) and various Reynolds number ($100 < Re < 1000$). Figure 3 showcases the streamline plots for different frequencies. Figure 4 manifests the isotherm plots for the same conditions.

In figure 3, it can be seen that with the increase of frequency, number of small circulation of fluids also increases. The isotherm plots (figure 4) show that there is not really much difference with the variation of frequency.

Figure 5 and figure 6 shows the stream line plot and isotherm plot for different Reynolds number. From figure 5, it can be deducted that with the increase of Reynolds number, the streamline around the cylinder increases in density. The isotherm plots (figure 6) show that in lower Reynolds number, the isotherm lines are more evenly distributed. If we look at the figures for higher Reynolds number, it is seen that the isotherm lines are more densely packed at the heated wall.



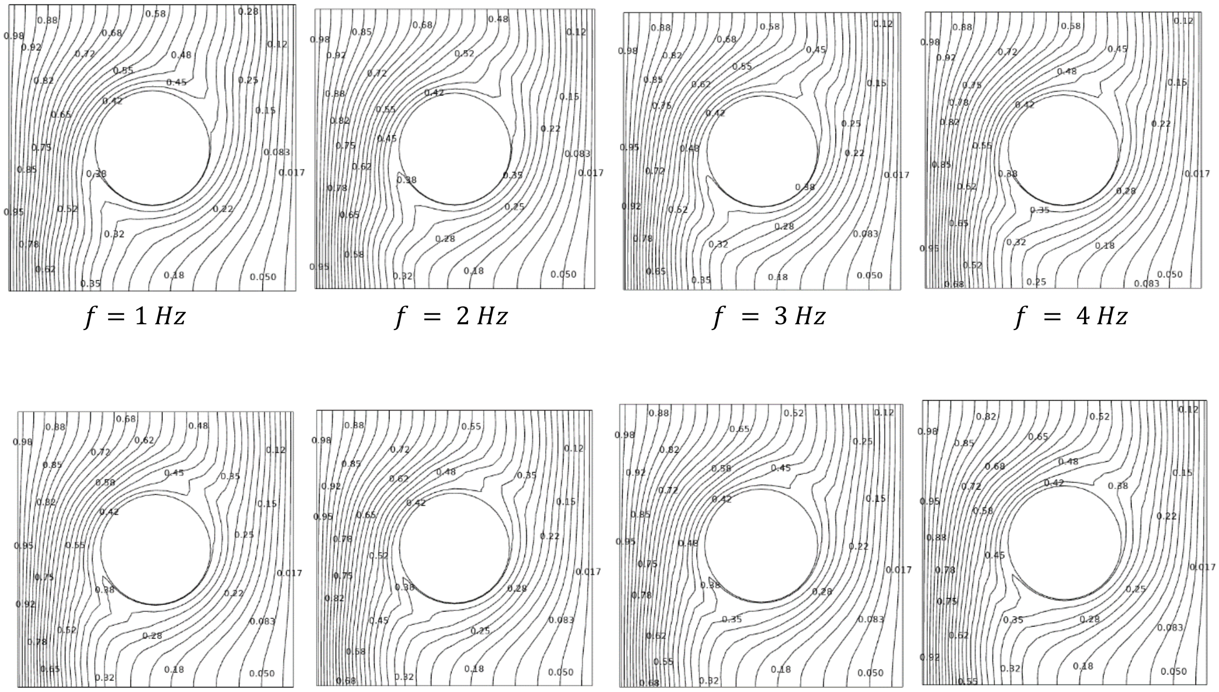
$f = 1\,Hz$ $f = 2\,Hz$ $f = 3\,Hz$ $f = 4\,Hz$



$f = 5\,Hz$ $f = 6\,Hz$ $f = 7\,Hz$ $f = 8\,Hz$

$f = 9\,Hz$ $f = 10\,Hz$

Figure 3. Stream line plots for different frequencies at $\phi_m = \pi$ and $Re = 100$ at unity Richardson number



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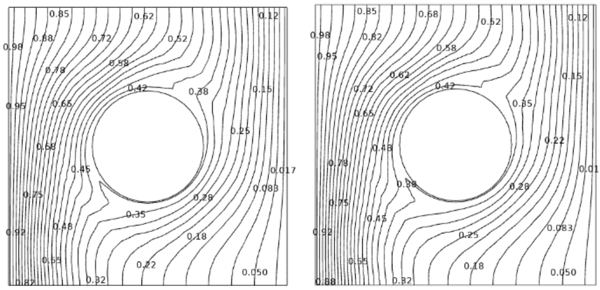
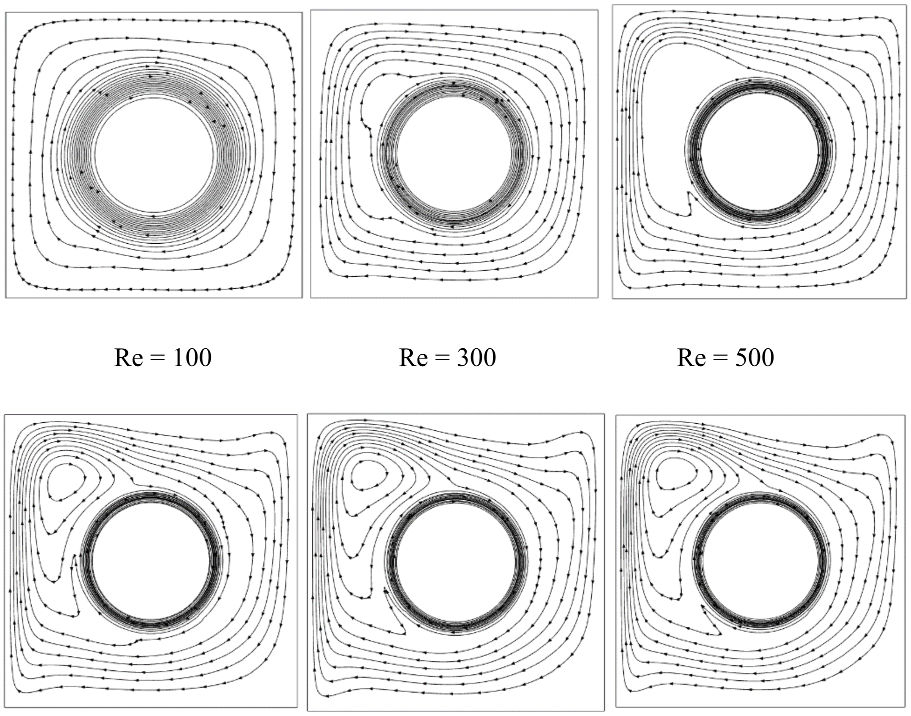


$f = 9\ Hz$ $f = 10\ Hz$

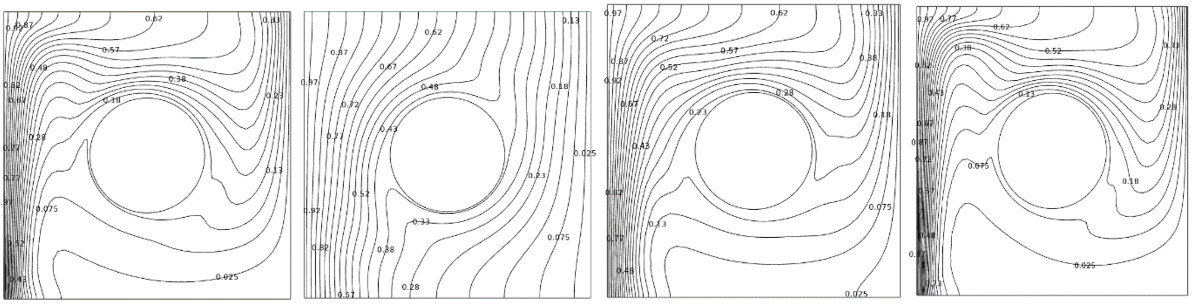
Figure 4. Isotherm contours for different frequencies at $\phi_m = \pi$ and $Re\ =\ 100$ at unity Richardson number



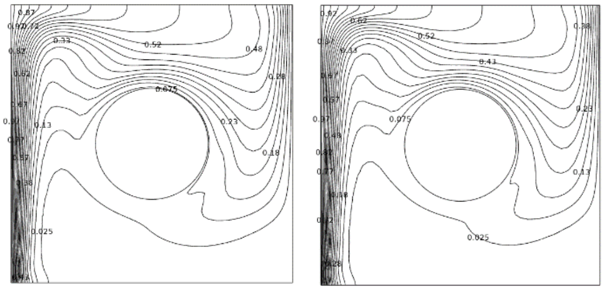
Re = 100 Re = 300 Re = 500

Re = 700 Re= 900 Re = 1000

Figure 5. Streamlines for different Re at $\phi_m = \pi$ and $f\ = 1\ Hz$ at unity Richardson number



Re = 100 Re = 300 Re = 500 Re=700



Re=900 Re = 1000

Figure 6. Isotherm contours for different Re at $\phi_m = \pi$ and $f = 1\ Hz$ at unity Richardson number

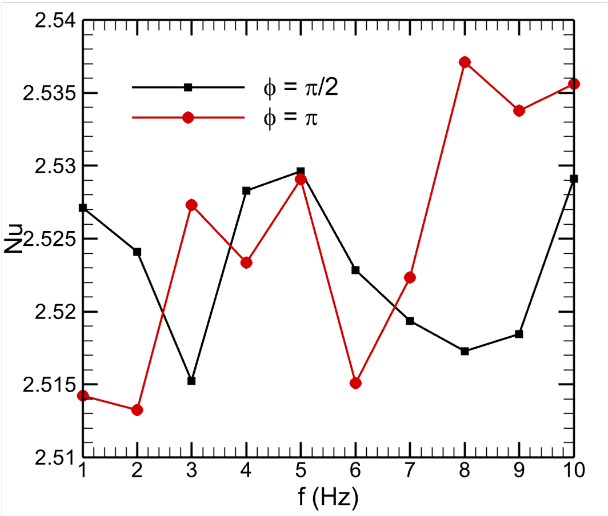


Figure 7. Nusselt Number Variation with Frequency

Figure 7 plots Nusselt number with respect to different frequencies. Here, 2 cases are considered, where the maximum angular displacement are $\phi_m = \pi$ and $\phi = \pi/2$. For both of the cases the Reynolds number is kept at 100 and the Richardson number is kept at unity. By looking at both cases, it can be said that the plots generally do not follow anu particular trend lines. If the above mentioned 2 cases are considered, it is evident that the case with $\phi_m = \pi$ has higher Nusselt number in most cases. Ergo, it can be said that larger angular displacement produces more convective performance in most cases

Figure 8 is the plot of Nusselt number which is fluctuating in relation to Reynolds number. Again, 2 cases are considered. For both of the cases, Richardson number is kept at unity. The maximum angular displacement is kept at π. For our first case, the oscillating frequency of the cylinder is kept at 1 Hz. The Nusselt number rises in lockstep with the Reynolds number. Same argument can be used when oscillating frequency is 5 Hz. In fact, both of the results are so similar that the plot lines coincide with each other. Hence, frequencies do not play a significant rule with the variation of Nusselt number.

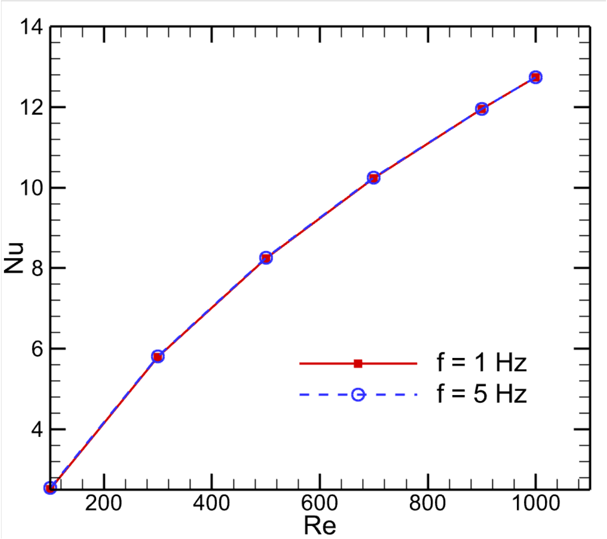


Figure 8. Nusselt Number Variation with Reynolds Number

6. Conclusion:

The present study analyzes the conjugated mixed convection in a differentially heated square cavity where a rotationally oscillating cylinder is placed. The data show that the Nusselt number rises in lockstep with the increase in Reynolds number. Larger angular displacement also improves convective performance. Frequency of the oscillating cylinder do not play a crucial role.

References:

Mahfouz, F.M. and Badr, H.M., 2000. Forced convection from a rotationally oscillating cylinder placed in a uniform stream. *International journal of heat and mass transfer*, *43*(17), pp.3093-3104.

Choi, S., Choi, H. and Kang, S., 2002. Characteristics of flow over a rotationally oscillating cylinder at low Reynolds number. *Physics of Fluids*, *14*(8), pp.2767-2777.

Ray, R.K. (2011) A Transformation-free HOC Scheme for Incompressible Viscous Flow Past a Rotating and Translating Circular Cylinder. *J Sci Comput* **46,** 265–293

Beskok, A., Raisee, M., Celik, B., Yagiz, B. and Cheraghi, M., 2012. Heat transfer enhancement in a straight channel via a rotationally oscillating adiabatic cylinder. *International journal of thermal sciences*, *58*, pp.61-69.

Kumar, S., Lopez, C., Probst, O., Francisco, G., Askari, D. and Yang, Y., 2013. Flow past a rotationally oscillating cylinder. *Journal of Fluid Mechanics*, *735*, pp.307-346.

Sellappan, P. and Pottebaum, T., 2014. Vortex shedding and heat transfer in rotationally oscillating cylinders. *Journal of Fluid Mechanics*, *748*, pp.549-579.

Mittal, H.V.R., Ray, R.K. and Al-Mdallal, Q.M., 2017. A numerical study of initial flow past an impulsively started rotationally oscillating circular cylinder using a transformation-free HOC scheme. *Physics of Fluids*, *29*(9), p.093603

Hadžiabdić, M., Palkin, E., Mullyadzhanov, R. and Hanjalić, K., 2019. Heat transfer in flow around a rotary oscillating cylinder at a high subcritical Reynolds number: A computational study. *International Journal of Heat and Fluid Flow*, *79*, p.108441.

Yawar, A., Ebrahem, M., Manzoor, S., Sheikh, N.A. and Ali, M., 2019. Transient cross flow and heat transfer over a rotationally oscillating cylinder subjected to gust impulse. *International Journal of Heat and Mass Transfer*, *137*, pp.108-123.

Ozalp, C., Soyler, M., Polat, C., Saydam, D.B. and Yaniktepe, B., 2021. An experimental investigation of a rotationally oscillating cylinder. *Journal of Wind Engineering and Industrial Aerodynamics*, *214*, p.104679.

Biography

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