Combined Effect of Reynolds and Grashof Numbers on Mixed Convection Heat Transfer in Turbulent Pipe Flow

Kazi Afif Salam and Sumon Saha
Department of Mechanical Engineering
Bangladesh University of Engineering and Technology (BUET)
Dhaka, Bangladesh
kaziafifsalam@gmail.com, sumonsaha@me.buet.ac.bd

Abstract. This paper represents a 3D numerical study of turbulent mixed convection of air in horizontal smooth pipe with the variation of Reynolds number (*Re*=74100, 33130, 23423, 10480,7410) and Grashof number while keeping the Richardson number constant (*Ri*=1). The mathematical model used here consists of Reynolds averaged Navier-Strokes and energy equation with *REAL-k-ε* turbulence model which is solved using finite volume method. Validation is done with the previous works in terms of axial velocity profile, dimensionless mean axial velocity (u^+), and mean temperature (T^+) profiles in the inner and outer region of the pipe. Later, the effect of variation of Reynolds number and Grashof number in the fully developed turbulent mixed convection regime is observed in terms of axial velocity and isothermal contours, secondary velocity vector with streamlines, u^+ and T^+ profiles. From the result, it is seen that axial velocity profiles are asymmetric for all Reynolds numbers with slight variations and maximum velocity is displaced towards the bottom of the pipe. In the inner region, as the Reynolds number and Grashof number decrease, the non-dimensional velocity u^+ and temperature T^+ increase but an opposite trend is seen for the outer region of the pipe.

Introduction

When air in turbulent forced convection flow is heated, a free convection effect is superimposed on the main flow. Under certain conditions this effect can become significant and can lead to marked distortion of the flow field. In this type of flow where momentum & heat are transferred due to combined effects of shear and buoyancy is known as mixed convection flow. This type of phenomenon represents an important mechanism of heat transfer which can be found in several applications such as nuclear reactors, heat exchangers, cooling systems for electronics, VHCS (ventilated hollow core slab), solar collectors, various manufacturing processes etc. In mixed convection, hydrodynamics & thermal fields are interdependent. The buoyant forces modify the velocity and temperature profiles of pure forced convection and transition to turbulence is occurred at a Re number much lower than conventional value of 2300 in case of pipe (Metais 1963). In a horizontal pipe, the forces of free and forced convection are reciprocally orthogonal. Temperature variations in the fluid may lead not only to velocity profiles modifications, but also to a secondary flow, which can significantly increase the heat transfer performance, with respect to the case of pure forced convection. (Kakac¸et al. 1987). The variation of Re number as well as Gr number with constant Ri number (pure mixed convection) on hydrodynamic & thermal fields has not received the attention it deserves however the effects are still significant and much remains to be learned. So, this is the main scope of this paper.

Literature Review

Very few investigations have considered buoyancy effect on flow field & heat transfer due to turbulent mixed convection in horizontal pipe. Petukhov and Polyakov (1988) observed experimentally the secondary flow pattern of two counter rotating vortices and the evolution of velocity and temperature structures. Carr et al. (1973) presented the velocity & temperature profiles of air in a vertical pipe with constant heat flux at Reynolds numbers of 5000 to 14000. Grassi and Testi (2007) found out the best velocity, temperature, and turbulence measurements in air for pipe flow with combined free and forced convection. Faheem et al. (2016) worked with air with constant Re, Ri no of 23000 & 1.04 respectively to evaluate the accuracy of different turbulence models. Mohammed et al. (2018) numerically simulate the effect of variation of Re & Ri with constant Gr number on u^+ and T^+ profiles with k-ε turbulence model. M. Piller investigate the interactive shear and buoyancy effects on the turbulent momentum and heat transport in fully developed mixed convection in horizontal pipe with friction Reynolds number of 360, Prandtl number of 0.71, Richardson number of 0, 0.1, 0.3, 0.7,1. In mixed convection buoyancy forces generate a secondary flow which is symmetric with respect to the vertical midplane with the warmer fluid moving upward close to the pipe wall and, by continuity, flowing downward through the core region (Li et al. 1998). In fully developed turbulent mixed convection with increasing heat flux, the skin friction first decreases and then increases in upward heated flow, while it changes little in downward heated flow (You 2003). Santis et al. (2018) worked with turbulent mixed convection in a channel flow with differentially heated walls for a fixed Richardson number (0.5) and three different Prandtl numbers (1, 0.1, 0.01). Lastly, Guo (2021) simulate turbulent mixed convection heat transfer with fixed Ri of 0.25 and four different Prandtl number (0.025, 0.05, 0.1, 0.71).

Considering the above papers, this study focuses on the influence of varying Re & Gr number with constant Ri in case of horizontal turbulent mixed convection smooth pipe flow in terms of hydrodynamic and thermal fields which will definitely provide new scopes for further research in the application of heat exchanger, electronics cooling and other pipe flow applications.

Numerical Model & Procedures

Geometry, Boundary Conditions & Flow Parameters

In this present study, a horizontal smooth pipe of length 12m with diameter of 0.18m is taken with fixed inlet air temperature of 293 K. The variation of inlet velocity & wall heat flux is shown in Table 1. The turbulence intensity of the incoming air flow at the inlet is set to 5%. The outlet is set to the outflow condition.

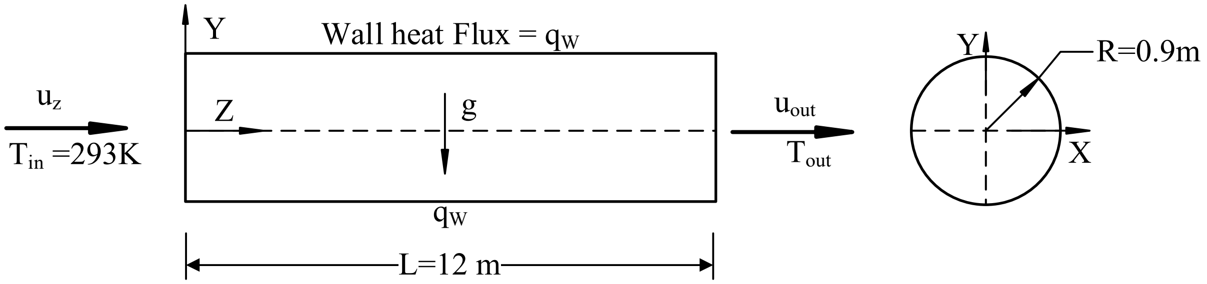


Figure 1: Geometry & Boundary Conditions of Horizontal Smooth Pipe

Table 1: List of Boundary Conditions & Flow Parameters

No	Inlet Velocity (u_z) (m/s)	Inlet Temperature (T_{in}) (K)	Wall Heat Flux (q_w) (W/m^2)	Reynolds number ($Re = \frac{u_z d}{v}$)	Grashof number ($Gr = \frac{g\beta q_w d^4}{\lambda v^2}$)	Richardson number ($Ri = \frac{Gr}{Re^2}$)
1	6.38	293	997.75	74100	5.49×10^9	1
2	2.85		199.1	33130	1.91×10^9	
3	2.017		99.72	23423.23	5.49×10^8	
4	0.902		19.94	10480	1.91×10^8	
5	0.638		9.98	7410	5.49×10^7	

The properties of air at 293 K is taken as follows: Heat capacitance, C_p=1005 J/kg K; Density, ρ=1.18 kg/m^3; Thermal conductivity of fluid, λ= 0.0261 W/m.K; Thermal expansion coefficient, β=0.00335 1/K; Kinematic viscosity, v=1.55×10^{-5}

Governing Equations

In this study, Reynolds-Averaged Navier-Stokes (RANS) approximation is used to model the Navier-Strokes equations. The mass, momentum and energy conservation with usual Boussinesq approximation are written in cartesian tensor form as follows:

$$\frac{\partial(\rho u_j)}{\partial x_j}=0 \tag{1}$$

$$\frac{\partial(\rho u_i u_j)}{\partial x_j}=-\frac{\partial p}{\partial x_i}+\frac{\partial}{\partial x_j}\left[\mu\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)-\rho\overline{u_i'u_j'}\right]-\rho_o\beta(T-T_o)g_i \tag{2}$$

$$\frac{\partial(\rho u_j T)}{\partial x_j}=\frac{\partial}{\partial x_j}\left[\frac{\mu}{\Pr}\frac{\partial T}{\partial x_j}-\rho\overline{u_j'T'}\right] \tag{3}$$

The components of acceleration due to gravity g_i are (0, g, 0) when i=1, 2, 3. The terms $\rho\overline{u_i'u_j'}$, $\rho\overline{u_j'T'}$ are Reynolds stresses and turbulent heat fluxes, respectively and are modeled numerically to close the RANS equations.

Numerical Method and Mesh Generation

Ansys fluent 20 is used to solve the problem numerically using finite volume method. A suitable grid is identified by considering three grid arrangements which consists of a combination of structured and unstructured regions. the grid contains only hexahedral elements (prisms with rectangular base). The suitability of the proposed grids for use in the numerical simulations is evaluated based on the near wall y^+ value.

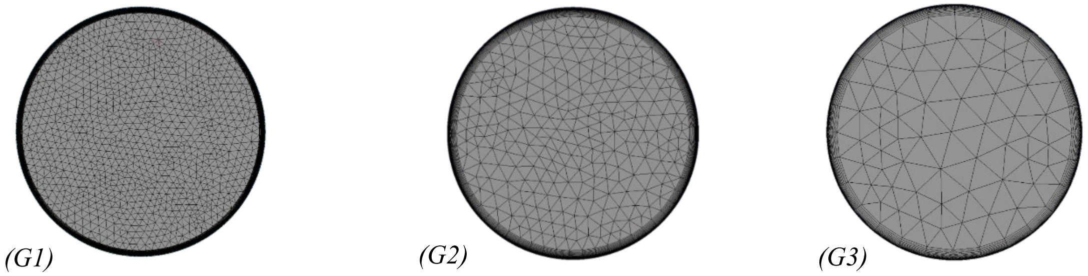

(G1) *(G2)* *(G3)*

Figure 2: Cross-section view of grid arrangement of the computational domain

Table 2: Characteristics of the different grid arrangements considered in the present study

Grid	Growth Ratio	No of Inflation layers	First layer thickness (mm)	Max face size (mm)	Total No of cells	Near wall y^+ values
G1	1.05	35	0.05	5	8764611	0.47
G2	1.1	25	0.06	10	2473859	0.74
G3	1.2	15	0.1	20	369748	3.67

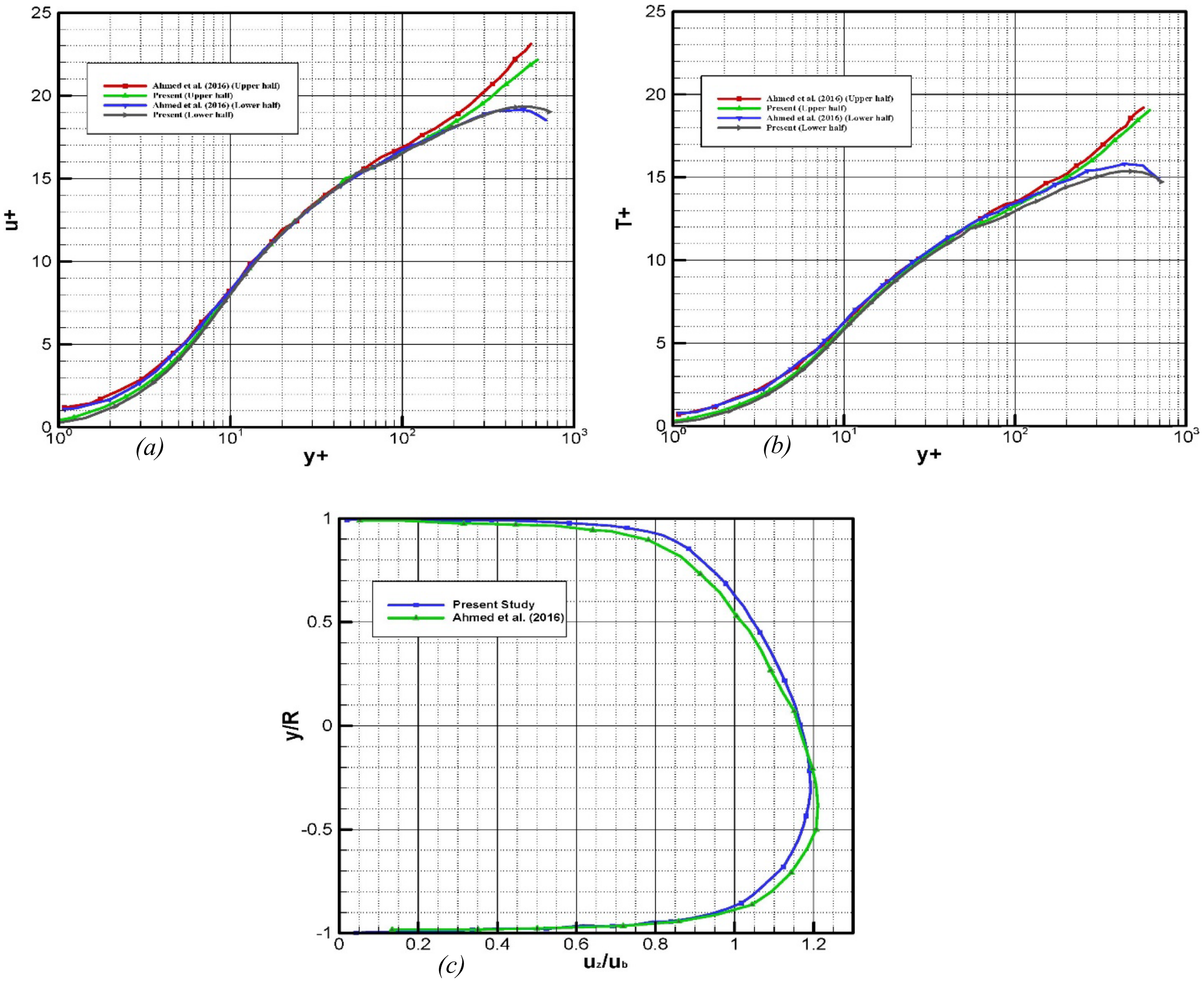
Based on the near wall y^+ comparisons, G2 is adopted for the numerical simulations performed in the following, because it is computationally more efficient. This simulation is performed in steady state. The upwind scheme to discretize the governing equation is of second order. SIMPLE method is carried to couple the velocity and pressure fields. A convergence criterion for all governing equations has been set to 10^{-6}.

Model Validation

In this study, the model is validated with the data of Ahmed et al. (2016) using *REAL-k-ε* turbulence model in terms of axial velocity profile, dimensionless mean axial velocity (u^+), and mean temperature (T^+) profiles in the inner and outer region of the fully developed flow (z/d=61.1). Some of the expressions are as follows:

$$y^+ = \frac{yu_\tau}{v}, u^+ = \frac{u_z}{u_\tau}, T^+ = \frac{T_w - T}{T_\tau}, u_\tau = \sqrt{\frac{\tau_w}{\rho}}, T_\tau = \frac{q_w}{\rho C_p u_\tau}$$

Here, u_τ is the friction velocity, T_τ is the friction temperature, T_w is the wall temperature, and T is the fluid temperature. It is seen from the Fig.3 that the u^+, T^+ and axial velocity profiles of the present work and the work of Ahmed et al. (2016) are almost similar. The deviations may be attributed due to the major simplification of the computational model. Therefore, it can be demonstrated that, the present simulation model is reasonably accurate to analyze.



(a) (b) (c)

Figure 3: Comparison of the variation of (a) u^+ profiles, (b) T^+ profiles (c) axial velocity profiles in the fully developed region at z/d=61.1 with Ahmed et al. (2016)

Result & Discussion

Axial Velocity Profiles

The effect variation of Re & Gr number on the vertical axial velocity profile is shown in Fig. 4. Due to the effect of buoyancy, the maximum velocity is displaced towards the bottom of the pipe. As, temperature rises, the density of air becomes smaller, so, the hot air moves upward, and the cold air moves downward. This also creates an asymmetric profile. It is seen that for all Reynolds number, the velocity profiles are almost similar. This is because the Richardson number is fixed to 1. The velocity of air decreases with corresponding decreases in wall heat flux and to maintain the constant value of Richardson number, similar velocity profiles are obtained.

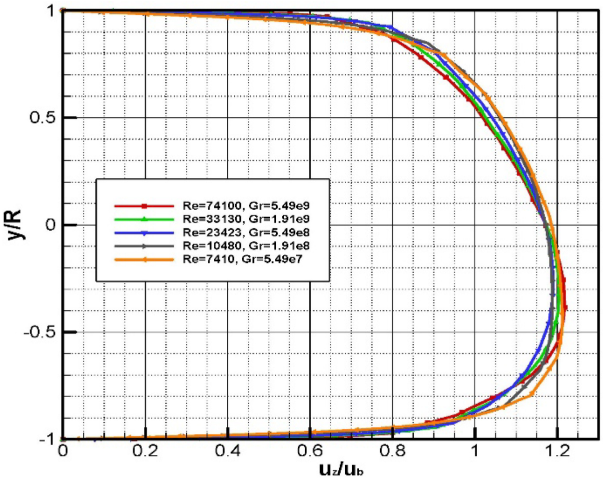
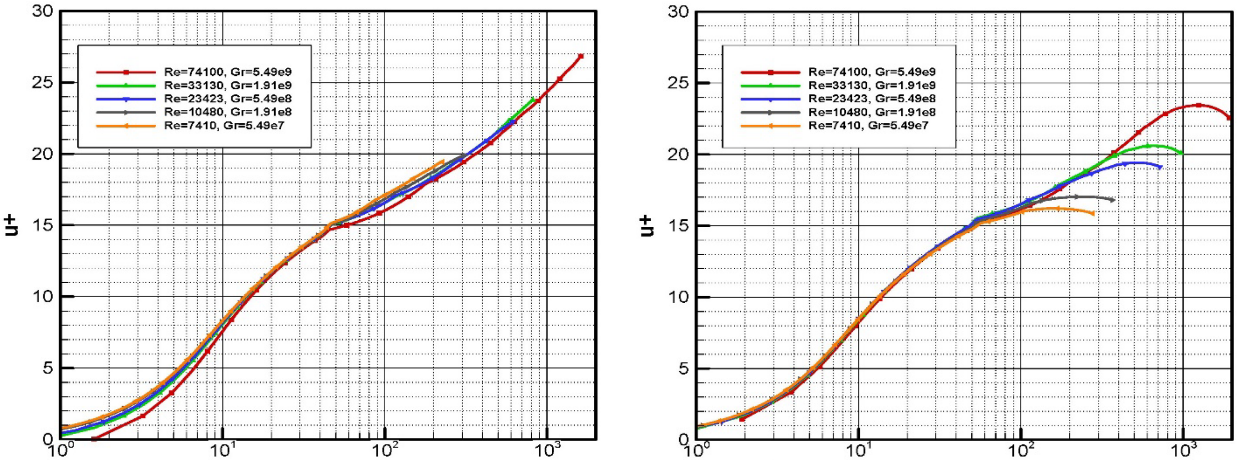


Figure 4: Effect of variation of Re & Gr number on the vertical axial velocity profile of a horizontal smooth pipe in the fully developed turbulent region (z/d = 61.1)

Dimensionless Velocity Profiles



(a) *(b)*

Figure 5: Effect of variation of Re & Gr number on u^+ profiles in the fully developed region at z/d=61.1 on the radius at (a) upper half of vertical diameter, (b) lower half of vertical diameter

In the inner region, with the decrease of Re & Gr number, the non-dimensional velocity increases but an opposite scenario can be found for the outer region (Fig. 5). For lower half, u^+ reaches a maximum value and the decreases. This indicates that maximum velocity occurs not in the center but lower than the center position. As the Reynolds number and Grashof number are decreasing, the location of the maximum velocity is moving further away from the center position because the viscous force is becoming dominant over inertia and buoyancy force. In the viscous sub layer for $y^+ \leq 5$ the profiles are mostly linear. They follow the normalized law of the wall: $u^+=y^+$, but in the overlap layer, $5 \leq y^+ \leq 30$ the velocity profiles are proportional with the logarithmic law: $u_z^+ = \frac{1}{\kappa} \ln (y^+) + B$. The values of κ & B are 0.4 and 0.5 respectively.

Dimensionless Temperature Profiles

In viscous sub layer for $y^+ \leq 5$ the temperature profile follows: $T^+ = Pry^+$ law. For $5 \leq y^+ \leq 30$, which is the overlap region, the temperature profiles follow a logarithmic expression: $T^+ = \frac{1}{\kappa_T} \ln (y^+) + \beta(\mathrm{Pr})$. In Fig.6, the effect of variation of Reynolds & Grashof number on nondimensional temperature profiles is seen at z/d=61.1 section. In the viscous sub layer, T^+ is higher for lower air velocity which enhances the heat transfer between layers. For lower half, $y^+ \geq 30$, T^+ reaches a maximum value and then decreases. This indicates that, at that location difference between wall temperature and local fluid temperature is maximum. As Reynolds and Grashof number is increasing, the maximum value of T^+ is moving towards the center of the pipe from the lower generatrix which can be further seen in Fig. 7.

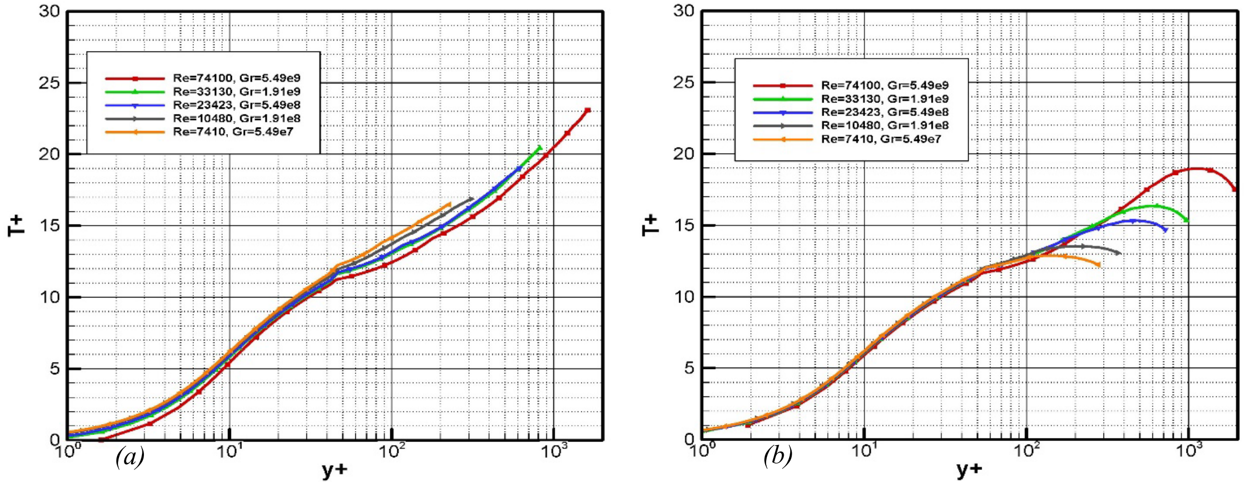


Figure 6: Effect of variation of Re & Gr number on T^+ profiles in the fully developed region at z/d=61.1 on the radius at (a) upper half of vertical diameter, (b) lower half of vertical diameter

Flow & Thermal Fields Visualization Along Pipe Cross Section

In the Fig.8, the influence of varying Reynolds & Grashof number with fixed Richardson number is seen for three different air velocity & wall heat flux. Axial velocity contour is obtained by the expression, $U_z = u_z/u_{in}$, in which u_z is the axial velocity. The isothermal contour is obtained by the expression: $T' = \frac{Tw-T}{Tw-Tb}$, in which T_w is the circumferentially averaged temperature of the wall (K) and T_b is the bulk temperature of the air (K), defined as the mass-weighted average air temperature over the cross section under consideration. From the contours it is seen that, axial velocity and isothermal contours are mostly concentrated towards the lower portion of the pipe, indicating steeper velocity and temperature there. The core velocity and temperature contour remain in the lower generatrix of the pipe due to the fixed value of Richardson number. It is seen from axial velocity contour Fig. 7 (a), (b) & (c) that the contour level 20, that is the maximum nondimensional velocity is moving further away from the center as the Reynolds number and Grashof number decrease. Similar scenario can be found for isothermal contour level 20.

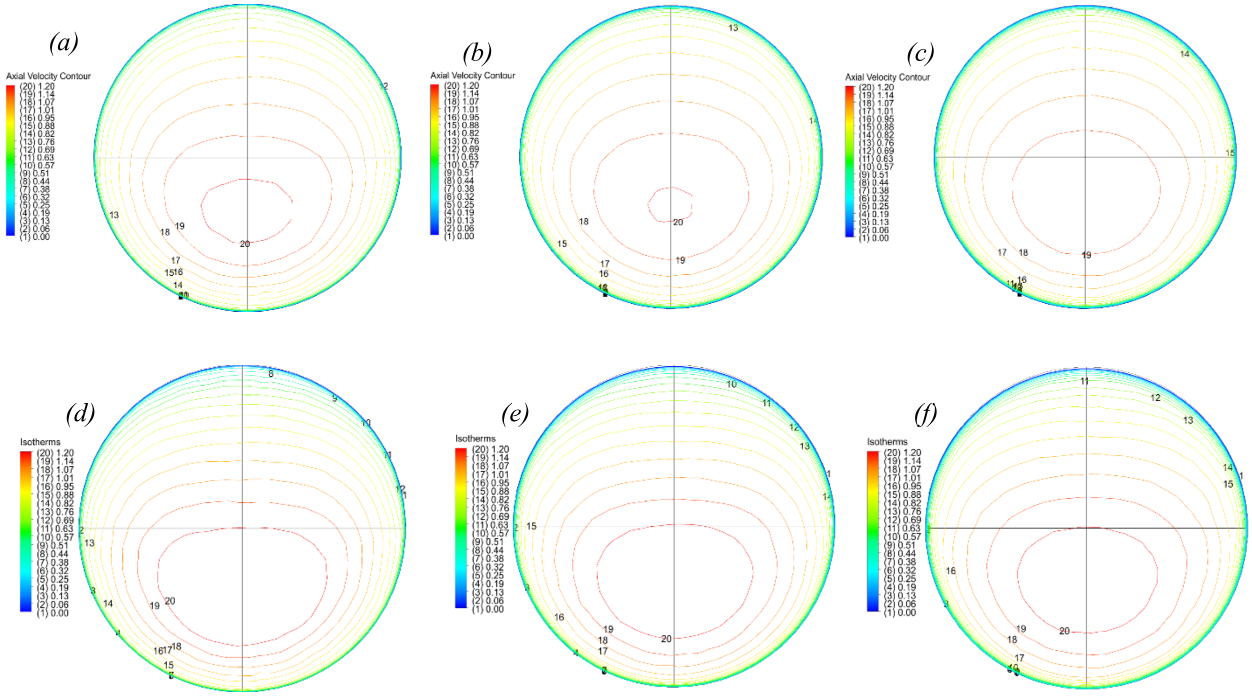
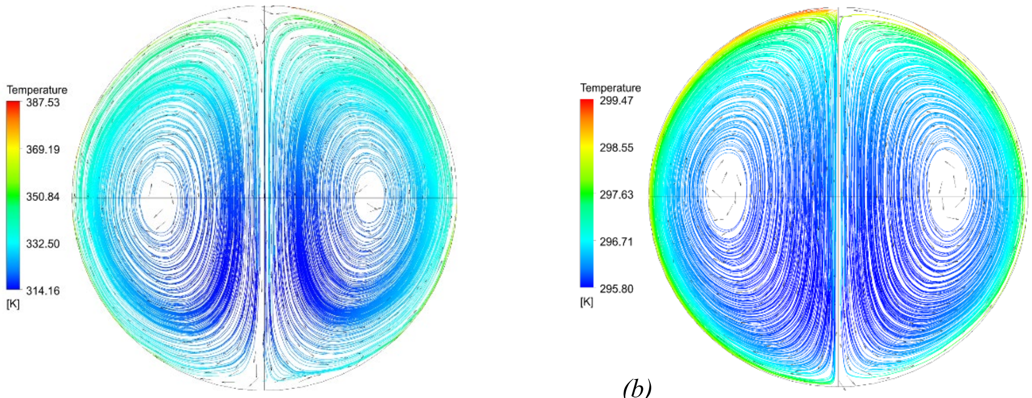


Figure 7: Comparison of velocity & isothermal contours for Constant Ri=1 (a), (d) Re=74100, Gr=5.49×10^9; (b), (e) Re=33130, Gr=1.91×10^9; (c), (f) Re=23423, Gr=5.49×10^8 in the fully developed region at z/d=61.1, whereas, the top row presents velocity contours, and the bottom row presents isotherm contours.

Secondary Velocity Vector & Streamline

The pipe's wall is subjected to varying heat flux. Wall temperature is higher than air temperature. For this reason, the air adjacent to the wall becomes less dense and rises upward along the sides to top. At the same time, the relatively cool air at the center of the pipe descends, leading to the formation of two counter rotating vortices on the cross section. From the Fig. 8, it is seen that this secondary flow is symmetrical about the vertical diameter and asymmetrical about the horizontal diameter. It has no effect on varying Reynolds & Grashof number. For Re=7410 & 74100 similar secondary velocity vector & streamline are obtained.



(a) *(b)*

Figure 8: Representative flow patterns in the fully developed region at z/d=61.1; (a) Re=74100, Gr=5.49×10^9, (b) Re=7410, Gr=5.49×10^7

Conclusion

In this paper, the combined influence of Reynolds & Grashof number is studied numerically for purely turbulent mixed convection in a horizontal smooth pipe. The results are shown in terms of axial velocity and isothermal contours, secondary velocity vector with streamlines, u^+ and T^+ profiles. From this study, it is seen that, u^+ profiles are linear in viscous sub layer and follows a logarithmic law in overlap region. Axial velocity profiles are almost similar for all Reynolds & Grashof due to fixed Richardson number. The u^+ & T^+ values is highest in the viscous sub layer for lowest Reynolds number but lowest in the outer region. Location of the maximum non dimensional temperature and velocity can be found from Fig.5,6 which will provide information in the design of the heat transfer equipment. Varying Reynolds & Grashof number has no effect on the secondary flow. The core region of the velocity and temperature contours remain in the lower generatrix of the pipe due to the fixed value of Richardson number.

Acknowledgements

The authors gratefully acknowledge the logistic support provided by Department of Mechanical Engineering, Bangladesh University of Engineering and Technology (BUET) during this research work.

References

Metais, B., Criteria for mixed convection, Technical report no. 51, Heat Transfer Laboratory, University of Minnesota, Minneapolis, MN 1963.

Kakac¸, S., Shah, R.K. and Aung, W., *Handbook of Single-Phase Convective Heat Transfer*, chapter 15 1987 (John Wiley and Sons: NY).

Petukhov, B. S., and Polyakov, A. F., *Heat Transfer in Turbulent Mixed Convection*, Hemisphere, New York,1988.

Carr, A., Connor, M., Buhr, H., *Velocity, Temperature, and Turbulence measurements in Air for Pipe Flow With Combined Free and Forced Convection,* Journal of heat transfer, 1973.

Grassi, W., Testi, D., *Evaluation of two RANS turbulence models in predicting developing mixed convection within a heated horizontal pipe,* International Journal of Computational Fluid Dynamics, 2007.

Faheem, A., Ranzi, G., Fiorito, F., Lei, C., *A Numerical Study of Turbulent Mixed Convection in a Smooth Horizontal Pipe*, Journal of Heat Transfer, 2016.

Mohammed, S., Bhattacharjee, A., Saha, S., *Numerical Simulation of Turbulent Mixed Convective Pipe Flow*, International Conference on Mechanical Engineering, 2018.

Piller, M., *Direct Numerical Simulation of Mixed Convection in Horizontal Pipe Flow*, University of Trieste, Department of Naval and Marine Engineering for the Environment, via A. Valerio, 10 - 34127 Trieste, ITALY.

Li, L., Lin, C., Ebadian, M., *Turbulent mixed convective heat transfer in the entrance region of a curved pipe with uniform wall-temperature*, Int. J. Heat Mass Transfer, pp. 3793–3805, 1998.

You, J., Yoo,J., Choi, H., *Direct numerical simulation of heated vertical air flows in fully developed turbulent mixed convection*, International Journal of Heat and Mass Transfer, vol. 46,no. 9, pp. 1613-1627, 2003.

Santis, D., Santis, A., Shams, A., Kwiatkowski, T., *The influence of low Prandtl numbers on the turbulent mixed convection in a horizontal channel flow: DNS and assessment of RANS turbulence models,* International Journal of Heat and Mass Transfer, pp. 345-358, 2018.

Guo, W., Prasser, H., *Mixed convection study on the influence of low Prandtl numbers and buoyancy in turbulent heat transfer using DNS,* Annals of Nuclear Energy, 2021.

Faheem, A., Ranzi,G., Fiorito,F., Lei,C., *A Numerical Study of Turbulent Mixed Convection in a Smooth Horizontal Pipe*, Journal of Heat Transfer, 2016.

Biographies

Kazi Afif Salam received his B.Sc. in Mechanical Engineering from Military Institute of Science and Technology (MIST) on 2018. He is now continuing in M.Sc in Mechanical Engineering from Bangladesh University of Science and Technology (BUET). His research interest involves computational fluid mechanics, computational heat transfer and aerodynamics. He has recently published one research paper in international conference proceedings.

Dr. Sumon Saha received his PhD in Engineering from the University of Melbourne, Victoria, Australia in 2014. He completed his B.Sc. and M.Sc. in Mechanical Engineering from Bangladesh University of Engineering and Technology (BUET), Dhaka, Bangladesh on 2004 and 2007, respectively. His major field of study is numerical analysis on problems of thermo-fluid. He is now working as a Professor in the Department of Mechanical Engineering of Bangladesh University of Engineering and Technology (BUET). He already published more than 140 research papers in International Journals and Conference Proceedings and coauthor of two books in engineering field. His fields of interests are turbulent flows, computational fluid mechanics, computational heat transfer and thermal postbuckling analysis. Dr. Saha is the editor of one international journal and reviewers of several international conference proceedings and international journals. He is currently senior member of International Association of Computer Science and Information Technology (IACSIT), Singapore. Moreover, he is a life member of Bangladesh Solar Energy Society. He has received many professional awards like International Postgraduate Research Scholarship by the Australian federal government; Melbourne International Research Scholarship by the University of Melbourne; RHD Studentship by University of Melbourne, and so on.