# A NEW APPROACH TO TWO STAGE CAPACITATED WAREHOUSE LOCATION PROBLEM (TSCWLP)

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## Abstract

In this paper, we give a new method for solving the MID CPLP problem that results when vertical decomposition is applied to two-stage capacitated warehouse location problem (see Sharma and Agarwal (2014)). In the MID CPLP problem, warehouses are locatable in stage 1 and stage 2 (thus incurring fixed costs), and we incur transportation costs as goods are transported from stage 1 to stage 2. Here in MID CPLP, we minimize the sum total of the cost of warehouse location (stage 1 and stage 2) and transportation. In the method due to Sharma and Agarwal (2014), the decomposition is followed where location variables at one of the stages are relaxed, and it is reduced to LHS CPLP (Left Hand Side Capacitated Plant/Warehouse Location Problem), and reasonable bounds are obtained by the procedures given by Verma and Sharma (2007).

Thus, in the method, due to Sharma and Agarwal (2014), the problem TSCWLP is reduced to LHS CPLP, MID CPLP (that is again reduced to LHS CPLP) and an RHS CPLP. This paper reduces the problem TSCWLP to LHS LP (linear program), MID CPLP and RHS LP (linear program). It is expected to offer good computational advantages. This procedure is outlined in detail in this paper.

**Keywords**
Plant Location Problem, Warehouse Location Problem, Two Stage Warehouse Location problem and Multi-echelon warehouse location problem.

## 1. Introduction

In real life situations single and two stage plant/warehouse location problems are frequently encountered. With increasing globalization we are encountering multi-echelon supply chains with location decision encountered in each stage. Hence research into this problem is very important. Even the simplest location problem, the SPLP (simple plant location problem) is NP-Hard; and so are many multistage warehouse location problem. In literature, a new approach has been advocated and that is called as VERTICAL DECOMPOSITION (Sharma and Agarwal (2014) and Verma and Sharma (2007)). We give a new vertical decomposition approach in this paper that reduces computational burden. We hope this approach will have potential to be extended to multi-echelon location-allocation problems.

## 2. Literature Review

Warehouse/Plant location problems are well known in literature. Here we give only the most relevant references and latest ones only. The beginning to location theory was made by work on simple plant location problem (SPLP) and capacitated plant location problem (CPLP), see Sharma and Muralidhar (2009) and Cornuejols et. al (1991) for latest works. Particularly Cornuejols et. al (1991) gave strengths of different lagrangian relaxations; and some of these are used in our current paper. Geoffrion and Graves (1974) and Sharma (1991) were among the first to consider single and two stage warehouse location problem and successfully applied Bender's decomposition. Later Verma and Sharma (2007) gave the concept of vertical decomposition that was applied to single stage warehouse location problem and they gave the concept of LHS and RHS CPLP and facilitated extension of concepts given by Cornuejols et. al to single stage capacitated warehouse location problem (SSCWLP). Later Sharma and Agarwal (2014) extended the idea of vertical decomposition to two stage capacitated warehouse location problem. In this paper we give an improvement to the method of Sharma and Agarwal (2014).

## Formulation and Relaxations of TSCWLP

### 2.1. Problem Formulation of TSCWLP

In this section, we propose the formulation of TSCWLP using the style of (Sharma1991;Sharma and Sharma, 2001). Verma and Sharma (2007) developed a variety of constraints that link real and 0-1 integer variables. They have also developed some strong constraints based on Sharma and Berry(2007).

#### 2.1.1. Index Used

h: set of supply points (plants); h = 1,..,H
i: set of potential warehouse points at stage 1; i = 1,.., I
j: set of potential warehouse points at stage 2; j = 1,.., J
k: set of markets; k = 1,..,K

#### 2.1.2. Definition of Constants

D(k): Demand for the commodity at market "k"
d(k): D(k)/(sum(k),D(k)), Demand at market "k" as a fraction of total market demand
S(h): Supply available at plant "h"
s(h): S(h)/(sum(k),D(k)), supply available at plant 'h' as a fraction of the total market demand
fws1(i): Fixed cost of locating a warehouse at "i"

fws2(j): Fixed cost of locating a warehouse at “j”
CAPWS1(i): Capacity of a stage 1 warehouse “i”
CAPWS2(j): Capacity of a stage 2 warehouse “j”
capws1(i): CAPWS1(i)/(sum(k),D(k)), Capacity of whs-1 at “i”as a fraction of the total market demand
capws2(j): CAPWS2(j)/(sum(k),D(k)), Capacity of whs-2 at “j” as a fraction of the total market demand
cpws1(h,i): Cost of transporting (sum(k),D(k)) goods from plant “h” to warehouse1 “i”
cws1ws2(i,j): Cost of transporting (sum(k),D(k)) goods from warehouse1 “i” to warehouse2 “j”
cws2m(j,k): Cost of transporting (sum(k),D(k)) goods from warehouse2 “j” to market “k”

### 2.1.3. Definition of Variables

XPWS1(h,i): Quantity of commodity transported from plant “h” to whs-1 “i”
xpws1(h,i): XPWS1(h,i)/ (sum(k),D(k)), Quantity transported from “h” to “i” as fraction of total demand
XWS1WS2(i,j): Quantity of commodity transported from whs-1 “i” to whs-2 “j”
Xws1ws2(i,j): 1 2ij k XWS WS ΣD , Quantity transported from “i” to “j” as fraction of total demand
XWS2M(j,k): Quantity of commodity transported from whs-2 “j” to market “k”
Xws2m(j,k): XWSM2(j,k)/ (sum(k),D(k)), Quantity transported from “j” to “k” as fraction of total demand
yws1(i): 1 if stage 1 warehouse is located at “i”, 0 otherwise
yws2(j): 1 if stage 2 warehouse is located at “j”, 0 otherwise

### 2.1.4. Mathematical Formulation

The cost minimization problem for the TSCWLP can be written as mixed 0-1 integer linear programming problem as given in the formulation below.

**Objective Function:**

**Min Z =** sum(h,i), xpws1(h,i)*cpws1(h,i) + sum(i,j), xws1ws2(i,j)*cws1ws2(i,j) +
sum(j,k), xws2m(j,k)*cws2m(j,k) +
sum(i), fws1(i)*yws1(i) + sum(j), fws2(j)*yws2(j) (1)

**s.t.** sum(h,i), xpws1(h,i) = 1 (2)

sum(i,j), xws1ws2(i,j) = 1 (2a)

sum(j,k), xws2m(j,k) = 1 (2b)

xpws1(h,i) <= yws1(i)*s(h) for all i,h (3a)

xws2m(j,k) <= yws2(j)*d(k) for all j,k (3b)

xpws1(h,i) <= yws1(i) for all i,h (4a)

xws2m(j,k) <= yws2(j) for all j,k (4b)

sum(h), xpws1(h,i) <= yws1(i)*capws1(i) for all i (5)

sum(j), xws1ws2(i,j) <= yws1(i)*capws1(i) for all i (5a-i)

sum(i), xws1ws2(i,j) <= yws2(j)*capws2(j) for all j (5a-ii)

sum(k), xws2m(j,k) <= yws2(j)*d(k) for all j and k (5b) sum(i), xpws1(h,i) <= s(h)   for all h   (6)

sum(j), xws1xws2(i,j) <= capws1(i)   for all i   (6a-i)

sum(i), xws1xws2(i,j) <= capws2(j)   for all j   (6a-ii)

sum(j), xws2m(j,k) = d(k)   for all k   (6b)

xpws1(h,i) >= 0   for all h, i   (7)

xws1ws2(i,j) >= 0 for all i, j   (7a)

xws2m(j,k) >= 0   for all j, k   (7b)

yws1(i) = (0,1)   for all i   (8a)

yws2(j) = (0,1)   for all j   (8b)

sum(i), yws1(i)*capws1(i) >= 1   (9a)

sum(j), yws2(j)*capws2(j) >= 1   (9b)

sum(i), capws1(i) >= 1   (10a)

sum(j), capws2(j) >= 1   (10b)

sum(h), xpws1(h,i) = sum(j), xws1ws2(i,j) for all i   (11a)

sum(i(, xws1ws2(I,j) = sum(k), xws2m(j,k) for all j   (11b)

Since (sum(k),D(k)) = 1, constraints 2, 2(a) and 2(b) ensure that flow across stages is equal to total demand by all the markets. Constraints 3(a) and 3(b) are strong linking constraints (see Sharma and Berry 2008 and Verma and Sharma 2007). Equations 4(a) and 4(b) are weak linking constraints. Equations 5, 5(a-i), 5(a-ii) and 5(b) strong capacity constraints (see Sharma and Berry 2008). Equation 6 ensures the flow from plant is less than supply available at that plant. Equations 6(a-i) and 6(a-ii) ensure that throughput form a warehouse is less than or equal to its capacity. Equation 6(b) ensures that the quantity received at a market is equal to its demand. Equations 7, 7(a) and 7(b) are non-negativity restrictions on real variables. Equations 8(a) and 8(b) are 0-1 restrictions on binary location variables. Equations 9(a) and 9(b) ensure that located capacity is more than or equal to the total demand of the markets. Equations 10(a) and 10(b) ensure that total average capacity is more than market demand at each stage. Equations 11(a) and 11(b) are flow balance constraints (inflow is equal to outflow) at each of the warehouses.

By relaxing different constraints, various relaxations can be obtained as Lagrangian relaxation (LR). LR is a relaxation technique, which works by moving hard constraints into the objective to impose a penalty on the objective if they are not satisfied. This is easier to solve than the original problem. An optimal objective value of the Lagrangian relaxed problem, for a given set of multipliers, provides a lower bound (in the case of minimization) for the optimal solution to the original problem. The best lower bound can be derived by updating the multipliers by a dual ascent procedure. An upper bound on the optimal solution of the original problem can be derived by using the information obtained from the LR to construct a feasible solution to the original problem. This is normally done by applying some heuristic.

In the next section, we present vertical decomposition approach for solving TSCWLP as given by Sharma and Agarwal (2014).

## 3 Methodology:

Before we give the best method described in literature, we give below a brief description of LR (Lagrangian Relaxation).

Problem P

Min **cx**

**s.t. Ax = b**
**Dx >= e**

**x >= 0 where c, A, b, D and e are matrices of conformable dimensions.**

In general problem P is intractable, however if we dualize **Dx >= e** and put it in the objective function, then the remaining problem is easily solvable (typically in polynomial time), that is,

Problem DP

ZD(**mu**) = min (**mu**), **cx + mu*(e-Dx)**

**Ax = b**

**x >= 0**

**mu** are known as Lagrangian multipliers (LMs) and optimal LMs can be easily determined by a suitable sub gradient procedure (see Sharma (1991)). Thus we have the following:

Z(*) = max (**mu**), ZD(**mu**)

Thus for a minimization problem, by using LR we get a good lower bound to the problem P.

In literature Sharma and Agarwal (2014) gave such a LR scheme. We describe it below in words and later in pure mathematical format. Sharma and Agarwal (2014) relaxed constraints (11a) and (11b); and transferred half of location cost sum(i), yws1(i)*fws1(i) to LHS CPLP and half to MID CPLP; and for the location cost sum(j), yws2(j)*fws2(j) half of it was transferred to MID CPLP and other half to RHS CPLP. Thus by deploying LR (Lagrangian Relaxation) they obtained good lower bounds. It is explained in mathematical terms as given below.

Z1 = sum(h,i), xpws1(h,i)*cpws1(h,i) + 0.5*{sum(i), fws1(i)*yws1(i)}

Z2 = sum(i,j), xws1ws2(i,j)*cws1ws2(i,j) +
0.5*{sum(i), fws1(i)*yws1(i) + sum(j), fws2(j)*yws2(j)}

Z3 = sum(j,k), xws2m(j,k)*cws2m(j,k) + 0.5*{sum(j), fws2(j)*yws2(j)}

Then LHS CPLP is:

Min Z1, s.t. (2), (3a), (4a), (5), (6), (7), (8(a)), (9(a)), (10(a)).

MID CPLP is:

Min Z2, s.t. (2(a)), (5(a-i)), (5(a-ii)), (6(a-i)), (6(a-ii)), (7(a)), (8(a)), (9(a)), (10(a)), (8(b)), (9(b)), (10(b)).

RHS CPLP is:

Min Z3, s.t. (2(b)), (3(b)), (4(b)), (5(b)), (6(b)), (7(b)), (8(b)), (9(b)) and (10(b)).

LHS CPLP (see Sharma and Verma (2007)) and RHS CPLP (see Cornuejols et al. (1991)) are well researched in literature. Building on these Sharma and Agarwal (2014) gave a good LR based procedure to get a lower bound.

Now we give the new LR procedure in the section given below.

## 4 Discussion

### 4.1. Theoretical Contribution

Constraints (3a), (3b), (4a), (4b), (5) and (5b) are relaxed along with (11a) and (11b). Now we have following reduced problems.

Z1_New = sum(h,i), xpws1(h,i)*cpws1(h,i)

Z2_New = sum(i,j), xws1ws2(i,j)*cws1ws2(i,j) +
sum(i), fws1(i)*yws1(i) + sum(j), fws2(j)*yws2(j)

Z3_New = sum(j,k), xws2m(j,k)*cws2m(j,k)

Now we have LHS LP as:

Min Z1_New, s.t. (2), (6), (7)

MID CPLP NEW as:

Min Z2_New, s.t. (2(a)), (5(a-i)), (5(a-ii)), (6(a-i)), (6(a-ii)), (7(a)), (8(a)), (9(a)), (10(a)), (8(b)), (9(b)), (10(b)).

RHS LP as:

Min Z3_New, s.t. (2(b)), (6(b)), (7(b)).

Now in this scheme of things, our MID_CPLP (given here) is same as MID_CPLP given in Sharma and Agarwal (2014). Hence all the nice results for MID_CPLP given in Sharma and Agarwal (2014) are valid for us also. But advantage is now we just need to solve two LPs LHS and RHS; and these are simpler problems without binary variables (instead of two NP-Hard problems LHS_CPLP (Not yet proved) and RHS_CPLP which is proved to be a NP-Hard problem); and it is easy to see it could offer computational advantages. This is a useful contribution we make in this paper.

Since the LHS LP and RHS LP problems devoid of all binary variables need to be solved with sum(k), d(k) = 1 and equations (2), (2a) and (2b) it can be easily solved by a greedy (polynomial time) procedure which may give upper bound generally. This simple procedure is given below as procedure A.

**Procedure A:**

The LHS LP problem is devoid of all binary variables and needs to be solved with sum(h), s(h) >= 1, and sum(i), capws1(i) >= 1, and equation (2) of Sharma and Agarwal (2014) can be easily solved as LP. Below we give a simple algorithm/heuristic to solve this problem.

Step 0: Sort all arcs cpws1(h,i) in an increasing order. Obviously sum (h), s(h) >= 1 and sum(i), capws1(i) >= 1 for the problem to be feasible. QTY = 0. Set all xpws1(h,i) = 0.

Step 1: if QTY < 1 then {Remove the arc (h1,i1) from consideration & go to step 2} else go to step 3.

Step 2: choose the min cpws1(h1,i1) for all eligible h & i and allot xpws1(h1,i1) = qty = min (s(h1), capws1(i1)). And set QTY = QTY + qty. set s(h1) = s(h1) – qty and capws1(i1) = capws1(i1) – qty.

Step 3: If QTY = 1 then GO TO STEP 4; else set current xpws1(h,i) = xpws1(h,i) - (QTY – 1) & GO TO STEP 4.

Step 4: Compute GOOD_SOLN_VAL = sum(h,i), xpws1(h,i)*cpws1(h,i) & STOP.

It is easy to see that solution given by primal based procedure A is an upper bound. Similar procedure can be used for solving RHS LP. From this good solution one can proceed to optimality easily. It is to be noted that complexity of procedure A is O(n*ln(n)) if we use binary sort (at Step 0).

Now we give below a dual based procedure to obtain a lower bound to LHS LP and RHS LP that have similar structure. LHS LP and RHS LP is similar to as given below.

**Procedure B**

Min sum(h,i), C(h,i)*x(h,i) (12)

s.t. sum(h,i), x(h,i) = 1 (13)

-sum(h), x(h,i) >= - capws1(i) for all i (14)

-sum(i), x(h,i) >= -s(h) for all h (15)

x(h,i) >= 0 (16)

(14) follows from (6a-i) and (11a).

Where we have in general sum (h), s(h)) >= 1 and sum(i), capws1(i) >= 1 (17).

This is similar to problem P in Sharma and Sharma (2000). We associate dual variables vo with equation (13), v1(i) with equations (14) and v2(h) with equations (15). Then its dual is written as given below.

Max vo – sum(i), capws1(i)*v1(i) – sum(h), s(h)*v2(h) (18)

s.t. vo – v1(i) – v2(h) <= C(h,i) for all h, i (19)

v0 uis, v1(i) >= 0 for all i, v2(h) >= 0 for all h (20)

Although in Sharma and Sharma (2000) it is assumed that sum (h), s(h)) = 1 and sum(i), capws1(i) = 1 (18); but it causes no problems. For a given value of v2(h) optimal value of the remaining dual problem can be easily found (see p. 219 of Sharma (1996) (problem DRP3)).

Thus, very easily a lower bound to problem LHS LP can be found by solving dual problem (18), (19) and (20) by using method given in Sharma and Sharma (2000).

These lower and upper bounds to LHS LP and RHS LP can be fruitfully used to quickly determine the optimal solution.

## 5 Conclusion

In this paper we give a new relaxation scheme for solving problem TSCWLP that leads to 2 LPs and one MID CPLP problem (that is intractable). In the earlier method due to Sharma and Agarwal (2014) the problem TSCWLP was decomposed into one LHS CPLP, one MID CPLP and one RHS CPLP (all three of them are intractable). The method given in this paper could offer computational advantages (as only one problem is computationally intractable; and other two LPs (RHS LP and LHS LP) can be solved by polynomial time procedure to get a good upper bound and lower bound).

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