A Model of Multiple Items Batch Scheduling for *m* Batch Processors Flow shop to Minimize Total Actual Flowtime

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Abstract

This research was motivated by a real problem in an aircraft company which has *m* batch processors (*BP*) flow shop in this machine order; $BP_1 - BP_2 - \cdots - BP_m$. The number of parts in a batch can be processed on a *BP* simultaneously. Under multiple items condition, the demanded part requires a different processing time which it depends on the type of item. Completed parts will be delivered simultaneously at a common due date. We developed a batch scheduling model for this problem by adopting the total actual flowtime part (F^a) as an objective function. Numerical examples are done by using the Lingo software, and the results showed that the model can guarantee F^a is minimum, the due date and the flow shop rules are fulfilled under flow shop condition.

Keywords
Actual Flowtime, Batch scheduling, Batch processor, Flow shop

1. Introduction

The problem in this research was motivated by a real problem in an aircraft which has *m BP* flow shop. Hidayat et al. (2021) have developed the model of the flow shop batch scheduling on *m BP*s for single item parts with common due date (abbreviated to FsSiCdd). Under condition of single item all parts come from the same item type, and each part has the same processing time. Thus, the processing time of each formed batch equals to the part processing time. In fact, the condition of the parts being processed is multiple items, whereas the processed parts come from *x* different types of items with the part processing time of one item being different from the other items. It is clear that the model in Hidayat et al. (2021) need to be developed from single item into multiple items.

The problem of the flow shop scheduling model on *m BP*s for multiple items with common due date can be described as follows. There are *n* parts consisting of *x* different items that must be processed in a flow shop of *m* batch processors with the order $BP_1 - BP_2 - BP_3 - \ldots - BP_m$ and the completed parts must be sent simultaneously at the same time, *d*. The processing time of the part from item g in a BP_k is t_{kg} for *k* = 1,2,…, *m* and *g* = 1,2,…, *x*. Each BP_k machine takes a set up time (s_{kg}) each time it will process a batch that contains parts of item *g*. Set up time is independent of batch size and batch processing order.

Figure 1 can be explained as follows, n parts consisting of $n_1, n_2, \ldots, n_x$ respectively are part of items 1, 2, ..., *x*. Grouping of parts into a batch is carried out based on the same processing time. All parts in the batching phase are grouped into batches to form $N_1, N_2, \ldots, N_x$ and the number of formed batches is $N = N_1 + N_2 + N_x$. Next is the scheduling phase, all the formed batches are arranged in a certain order to minimize the total actual flowtime of parts through the shop.

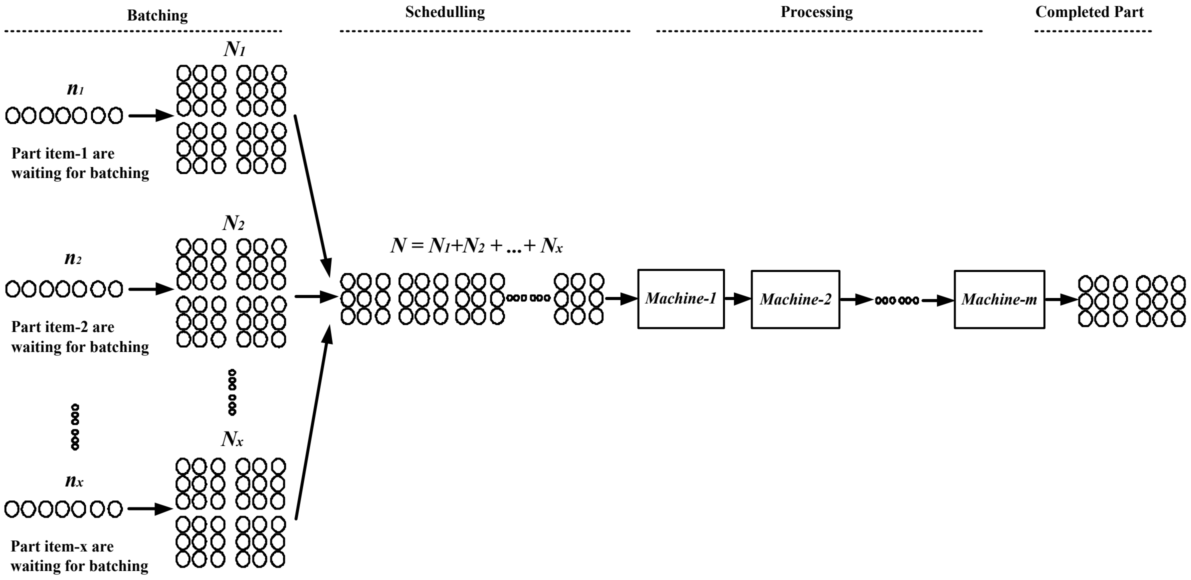

Figure 1: The problem of FsMiCdd

1.1 Objectives

In this study we developed the flow shop scheduling model on *m BP*s for multiple items with common due date (abbreviated to FsMiCdd) by changing the number of items of parts processed. This development is needed because in fact the condition of part being processed is multiple items. Total actual flowtime of part through the shop is adopted as an objective function to guarantee that the shop time will be minimized, and the due date will be fulfilled.

2. Literature Review

The research on a flow shop with batch processor have interested many researchers, one of them is Rosi, et.al (2013) who have developed a hybrid model of a flow shop for a sterilisation plant. The job in a sterilization plant is surgical kit, every tardy job will cause surgery to be rescheduled which brings economic consequences. This is the reason why their aim is to minimize the number of tardy jobs. Pèrés and Mönch (2013) have discussed scheduling jobs on a single BP with incompatible job families and the objective is weighted number of tardy jobs. Chen et.al (2014) and Matin et.al (2017) also dealt with the problems of BPs flowshop, Chen et al. (2014) considered the dynamic job arrivals at the first BP. In Matin et al. (2017) the batch size and parts in a batch may change when the batch is processed on different machines. However, all of these studies adopted forward scheduling approach. In this study, backward scheduling approach is adopted because the schedule must meet the due date. Halim and Ohta (1993) developed the actual flowtime as an objective with adopting a backward scheduling approach. The actual total flowtime of parts through the shop (F^a) is defined by Halim et al. (1994) as the total time that the parts spend in the shop from the starting times of processing until the delivery times of the completed parts.

The problem of batch scheduling on a *BP* using the total actual flowtime as an objective was first introduced in Hidayat, et al. (2013). Furthermore, the condition of a *BP* was developed into *m* heterogeneous *BP*s in Hidayat et al. (2014), the common due date condition was developed into multiple due dates in Hidayat et al. (2016), and the single item condition was developed into multiple items in Hidayat et al. (2018). Halim et al. (2018) have developed a model of batch scheduling for a single batch processor which needs two kinds of setups. The first setup is required before processing any batches, and the second setup is required after completing a certain number of batches. Hidayat, et al. (2020*a*) deals with a batch scheduling problem which the parts to be processed are multiple items. All of parts will be processed on one of *m* heterogeneous batch processors. Hidayat et al. (2020*b*) also discussed a problem of scheduling batches consisting of multiple item parts. The difference lies on a batch in Hidayat et al. (2020) can be formed of the parts with different type of item. It is assumed that no defect will be occurred even though the duration of processing the parts is excessive. This is the reason why the batch processing time will be the longest processing time of parts in the batch.

Hidayat et al. (2021a) have discussed the batch scheduling problem on a BP for the etching process in a craft company. The condition of the processed parts are multiple items, and each type requires different processing and setup time. The problem is formulated as mixed integer nonlinear programming model. The single stage condition in Hidayat, et al. (2013) was developed into a flow shop in Hidayat et al. (2021b). However, the condition of the part that is processed remains single item. It seems we need to develop the model in Hidayat et al. (2021b) from the single item of parts item into multiple items.

3. Problem Formulation

The notations used in the proposed models are as follows.

$b_{k(i)}$: The batch sequenced at position *i* processed on BP_k
$B_{k(i)}$: The starting time of processing batch $b_{k(i)}$
$C_{k(i)}$: The completion time of batch $b_{k(i)}$
d : The common due date
F^a : The total actual flow time of parts through the shop
g : The index for the item of part, $g = 1,2, \dots, x$
i : The index for the batch's position, in backward manner, $i = 1,2, \dots, N$.
k : The index for the machine's number, $k = 1,2, \dots, m$
n : The number of parts in the shop
n_g : The number of part item *g* in the shop
N : The number of batches in the shop
N_g : The number of batches which consists part item *g* in the shop
$Q_{k(i)}$: The size of batch $b_{k(i)}$
s_{kg} : Set up time batch which consists part item *g* on BP_k
$s_{k(i)}$: The set-up time before processing a batch $b_{k(i)}$ on BP_k;
t_{kg} : The part processing time of item *g* on BP_k
$t_{k(i)}$: The processing time of batch $b_{k(i)}$

In this study the size and the arrangement of the formed batches will be determined so that $\boldsymbol{F^a}$ is minimized. Hidayat et al. (2021) has developed the formulation of $\boldsymbol{F^a}$ for FsSiCdd problem as follows.

$$F^a = \sum_{i=1}^{N} \left(d - B_{1(i)}\right) Q_{1(i)} \tag{1}$$

The similarity between the FsMiCdd and FsCdd problems is that batches are processed in a flowshop *m* batch processors and the completed parts are delivered simultaneously at common due date, *d*. While the difference lies in the number of items of the processed part. Equation (1) shows that the number of part items does not effect $\boldsymbol{F^a}$. Thus equation (1) also suits for FsMiCdd model eventhough the part characteristics is change from single items to multiple items.

Under condition of multiple items, the batch processing time depends on the type of item of the part that is in the batch. For this reason, a binary number variable is needed to declare the item type of the part that is in the batch $b_{k(i)}$, i.e. $z_{g(i)}$. The binary variable $z_{g(i)}$ will be 1 if the part in batch $b_{k(i)}$ is item *g*, and 0 if not item *g*. This forms several conditions that must be met as follows.

a) The processing time of batch $b_{k(i)}$ is $t_{k(i)} = \sum_{g=1}^{x} z_{g(i)} . t_{kg}$ for *k* = 1, ...,*m* and *i* = 1, ..., *N*
b) The set up time of batch $b_{k(i)}$ is $s_{k(i)} = \sum_{g=1}^{x} z_{g(i)} . s_{kg}$ for *k* = 1, 2, ..., *m* and *i* = 1,2,...,*N*
c) The number of batches which consist the part of item *g* must be equal to the sum of the binary variable $z_{g(i)}$, i.e. $\sum_{i=1}^{N} z_{g(i)} = N_g$ for *g*=1, 2, ..., *x*.
d) The number of batches processed on each machine is equal to $N = \sum_{g=1}^{x} N_g$

e) The number of parts of item *g* must be equal to the multiplication of the binary variable z_{ig} with the size of batch $b_{k(i)}$, i.e. $n_g = \sum_{i=1}^{N} Q_{(i)} . z_{g(i)}$ for *g* =1, 2, …, *x*.

f) The number of processed parts on the shop is equal to $n = \sum_{g=1}^{x} n_g$

Model FsMiCdd

Minimize

$$F^a = \sum_{i=1}^{N} \left(d - B_{1(i)}\right) Q_{(i)} \tag{1}$$

subject to

$$B_{m(1)} + t_{m(1)} = d \tag{2}$$

$$B_{k(1)} = B_{k+1(1)} + t_{k(1)} \qquad \forall k = 2, 3, \dots, (m-1) \tag{3}$$

$$B_{k(i)} = Max\left\{B_{k-1(i)} + t_{k-1(i)}\,; B_{k(i+1)} + t_{k(i+1)} + s_{k(i)}\right\}\ \forall i = 2, \dots, (N-1), \forall\, k = 2, \dots, m \tag{4}$$

$$B_{k(N)} = B_{k-1(N)} + t_{k-1(N)} \qquad \forall k = 2, 3, \dots, m \tag{5}$$

$$B_{1(i)} \geq B_{1(i+1)} + t_{1(i+1)} + s_{1(i)} \qquad \forall i\ = 1, 2, \dots, (N-1) \tag{6}$$

$$B_{1(N)} \geq 0 \tag{7}$$

$$\sum_{i}^{N} Q_{(i)} = n \tag{8}$$

$$0 < Q_{(i)} \leq c\ \ \forall\, i = 1,2, \dots, N \tag{9}$$

$$N = \sum_{g=1}^{x} N_g \tag{10}$$

$$t_{k(i)} = \sum_{g=1}^{x} z_{g(i)} . t_{kg} \qquad \forall i = 1, 2, \dots, N \tag{11}$$

$$s_{k(i)} = \sum_{g=1}^{x} z_{g(i)} . s_{kg} \qquad \forall i = 1, 2, \dots, N \tag{12}$$

$$N_g = \sum_{i=1}^{N} z_{g(i)} \qquad \forall g = \ 1, 2, \dots, x \tag{13}$$

$$n_g = \sum_{i=1}^{N} Q_{(i)} . z_{g(i)} \qquad \forall g = 1, 2, \dots, x \tag{14}$$

$$z_{g(i)} = \begin{cases} 1 \\ 0 \end{cases} \qquad \forall g = 1,2, \dots, x\,;\ \ \forall i = 1,2, \dots, N \tag{15}$$

$$\sum_{g=1}^{x} z_{g(i)} = 1 \qquad \forall i = 1,2, \dots, N \tag{16}$$

Equation (2) shows that the completion time of the last batch processed on the last machine must coincide with the due date, *d*. Equation (3) shows the starting time of the first batch on BP_2 until BP_{m-1}. Equation (4) determines the start time of all other batches. A batch can only be started on BP_k after it is completed on the previous machine and after the batch previously processed on BP_k is completed. As soon as the batch processing is completed, the batch processing on the next machine can be carried out and so on until the last machine BP_m, as shown in Equation (5). Equation (6) determines the relationship between the start time of consecutive batches on the first machine BP_1. Equation (7) guarantees the feasibility of the schedule that the processing of the first batch on BP_1 will occur on after time zero. Equation (8) ensures the material balance in the shop. Equations (9) and (10) restrict the batch sizes and number of batches.

Equation (11) states that the processing time of batch $b_{k(i)}$ depends on the item type of the part in batch $b_{k(i)}$. This also applies to the set-up time in equation (12). Equation (13) to ensure the number of batches that are part of item *g* is equal to the sum of the binary variable for item *g*. Equation (14) to ensure the number of parts of item *g* processed is the same as the number of parts of item *g* requested by the customer. Equation (15) is a binary variable to express the item type of the part in batch $b_{k(i)}$. Equation (16) guarantees the parts in each batch are the same type of item.

4. Numerical Examples

There are *n* parts of 2 different items that must be processed in a flow shop of 3 batch processors with the order $BP_1 - BP_2 - BP_3$ and the completed parts must be sent simultaneously at due date, *d* = 100. The number of required parts must be processed (in units) and the part processing time (in units of time) for each item *g* in each BP_k is listed in table 1. While the set-up time for each item part on each machine is the same, namely 1 (in units of time).

Table 1: The part processing time

Case	Item (g)	n_g	The part processing time		
			BP_1	BP_2	BP_3
a.	1	20	5	4	6
	2	12	3	6	2
b.	1	14	3	5	4
	2	16	5	2	6
c.	1	20	6	5	7
	2	18	6	4	3

5. Results and Discussion

The calculation of numerical example is done by using software Lingo.

5.1 Numerical Results

The results of calculation are as shown in the table 2.

Table 2. The result of calculation of numerical example.

				$Z_{g(i)}$		$t_{k(i)}$			$B_{k(i)}$			$C_{k(i)}$			
				g		k			k			k			
Case	N	i	$Q_{(i)}$	1	2	1	2	3	1	2	3	1	2	3	F^a
a.	4	1	10	0	1	3	6	2	89	92	98	92	98	100	600
		2	10	1	0	5	4	6	82	87	91	87	91	97	
		3	10	1	0	5	4	6	75	80	84	80	84	90	
		4	2	0	1	3	6	2	70	73	79	73	79	81	
b.	4	1	10	1	0	3	5	4	88	91	96	91	96	100	574
		2	10	0	1	5	2	6	82	87	89	87	89	95	
		3	6	0	1	5	2	6	75	80	82	80	82	88	
		4	4	1	0	3	5	4	69	72	77	72	77	81	
c.	4	1	10	0	1	6	4	3	87	93	97	93	97	100	900
		2	8	0	1	6	4	3	80	86	93	86	90	96	
		3	10	1	0	6	5	7	73	79	85	79	84	92	
		4	10	1	0	6	5	7	66	72	77	72	77	84	

5.2 Graphical Results

Figures 2, 3 and 4 are the gantt chart for case *a*, *b* and *c* respectively.

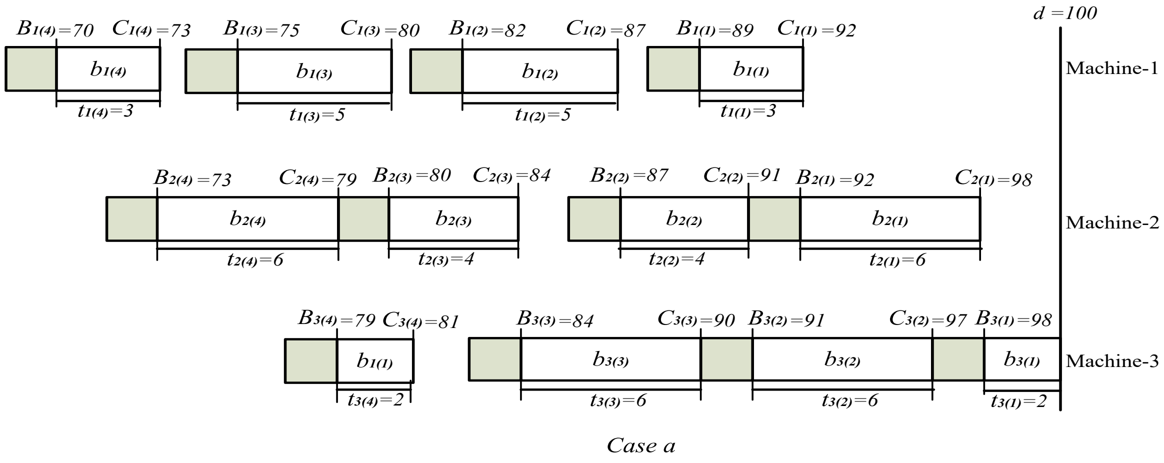

Figure 2: The Gantt Chart for Case *a*

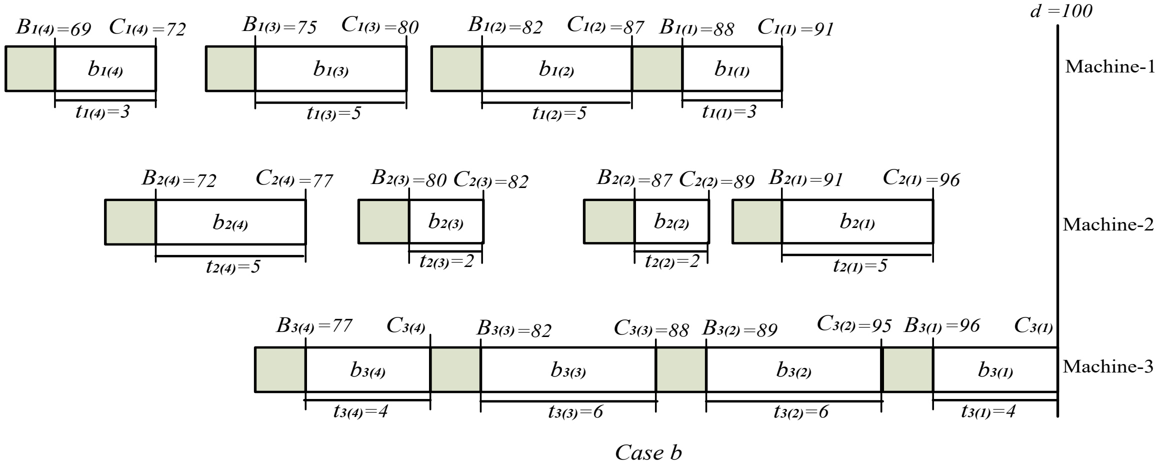

Figure 3: The Gantt Chart for Case *b*

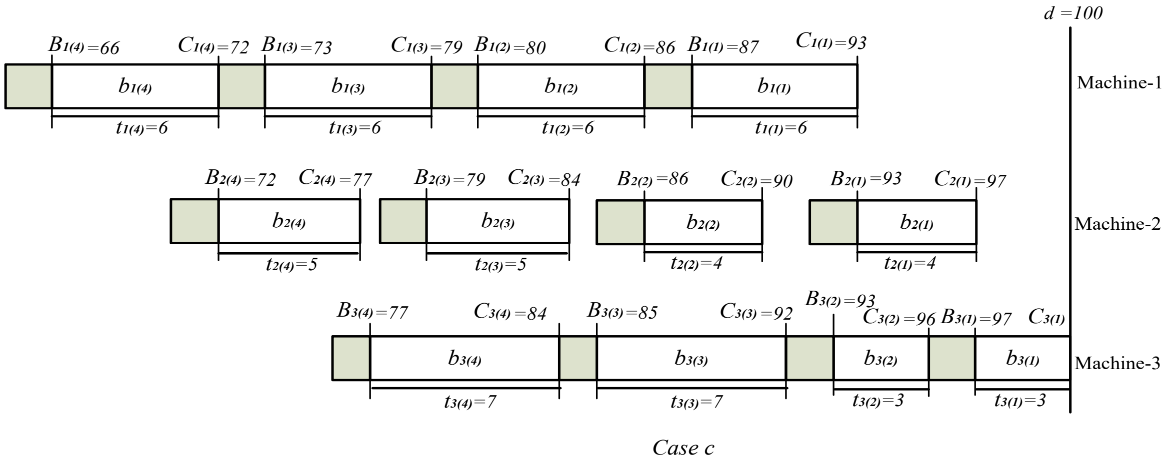

Figure 4: The Gantt Chart for Case *c*

5.3 Discussion

A batch is called a full batch capacity if the size of the batch equals to the BP's capacity, otherwise it is called an idle batch capacity. It can be seen that the maximum number of idle batch capacity for each item is one. This happens when the number of parts to be processed is not a multiple of the BP's capacity so that the result of n_g/c is not an integer number. In case a, the demanded part of item 2 is $n_2 = 12$ and the BP's capacity is $c = 10$ so that $n_g/c = 12/10 = 1{,}2$ is not an integer number. The number of formed batch is rounded up from 1.2 to be equal to 2 batches (1 full batch capacity and 1 idle batch capacity).

In all cases the completion time of the first batch on BP_3 is equal to the due date ($C_{3(1)} = d$). This is a consequence of the backward scheduling approach as stated in equation (2). Regarding the sequence of batch on each BP, it appears that a batch is processed on BP_2 when the processing on BP_1 has been completed. As in case a, $B_{2(1)} = 92$ and $C_{1(1)} = 92$, $B_{3(1)} = 98$ and $C_{2(1)} = 98$; in case b, $B_{2(1)} = 91$ and $C_{1(1)} = 91$, $B_{3(1)} = 96$ and $C_{2(1)} = 96$; in case *c,* $B_{2(1)} = 93$ and $C_{1(1)} = 93$, $B_{3(1)} = 97$ and $C_{2(1)} = 97$. This shows that the results obtained meet the flowshop rules with a backward scheduling approach.

6. Conclusion

It can be concluded:

a) The maximum number of an idle batch capacity for each item is 1. This is due to rounding up on the result of n_g/c which is not an integer.

b) The completion time of the last batch, $b_{m(N)}$ on the last machine BP_m will be the same as the due date. This is due to in the backward scheduling approach.

c) Each batch can be processed on the next machine if the process on the previous machine has been completed and the next machine is not processing another batch. This is due to flow shop condition.

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