Robust Optimization of Multi Depot Vehicle Routing Problems with Simultaneous Deliveries and Pickups in an Uncertain Environment

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Abstract

This paper discusses the development of a robust optimization model of vehicle routing problems carried out from multiple depots to meet consumer requirements for simultaneous delivery and pickup of products from customer locations as implemented in closed loop logistics. In this proposed optimization model, we considered the uncertainty of travel time and service time at each consumer location. The proposed robust optimization model aims to generate travel routes and vehicle scheduling from multi-depot by anticipating travel time uncertainty between locations and the uncertainty of the number of products delivered and picked up, resulting in uncertainty of service time needed at each consumer location. The method for solving this robust optimization model applied two-stage methods, namely clustering the consumer locations served by each depot by using the set partitioning model. Furthermore, the vehicle routing and scheduling optimization were applied to each customer clustering of each depot and solved by using Gurobi-Python optimization. Numerical experiments have been conducted to evaluate the robust optimization model and the proposed solution method. The experimental results pointed out the effectiveness of the optimization model and the efficiency of the computational time of the proposed solution method for medium scale problems.

Keywords
Robust optimization, Vehicle routing problem, Simultaneous deliveries and pickups, Multi-depot, Uncertainty.

1. Introduction
During the COVID-19 pandemic, the community needs for delivery and pick-up services have increased rapidly to carry out work from home and health protocols, maintain physical distance, reduce mobility, and stay away from crowds. This opens business opportunities for e-commerce-based goods delivery and pickup and conventional business supports such as catering, delivery, and pickup services of LPG cylinders, gallons of mineral water, etc. Delivery and pick-up operations will be effective if the determination of the depot that serves consumers, the assignment of vehicles and the arrangement of routes are carried out properly. These problems can be solved by implementing one of the variants of the vehicle routing problem (VRP) models, namely the Multi-Depot Vehicle Routing Problem with Simultaneous Deliveries and Pickups (MDVRPSDP) model. However, the determination of depots that serve consumers and the arrangement of efficient vehicle routes becomes more complex if you consider the uncertainty that occurs in traffic density, vehicle reliability conditions, the number of demands for delivery and pickup of goods because these uncertainty factors will affect the length of travel time between locations, the service times for loading and unloading of goods at customer's locations, and the number of vehicle loads.

However, based on our knowledge from literature studies regarding MDVRPSDP models, no studies have considered these environmental uncertainties (Elshaer & Awad, 2020). Therefore, this study discusses the development of a robust optimization model for Multi Depot Vehicle Routing Problems with Simultaneous Deliveries and Pickups (MDVRPSDP) by considering the uncertainty of (1) the length of travel time between locations, (2) the service time at each consumer location, and (3) the number of products delivery and pick-up from and to consumer locations. From the point of view of the complexity of the optimization mathematical model, the MDVRPSDP model is an NP-Hard problem according to the combinatorial optimization problem (Li et al., 2015; Rajak et al., 2019). Moreover, the MDVRPSDP model considers the uncertainty that occurs in practical conditions, then the solution of the optimization model becomes harder to obtain. Therefore, in this study, we propose a solution methodology for solving the Robust Optimization (RO) model for MDVRPSDP under environmental uncertainty with a cluster-based optimization

approach, which is divided into two stages, namely constructing the consumer clusters using set partitioning problem and robust optimization for each consumer cluster handled by single depots using RO-VRPSDP model.
The structure for the next section is organized as follows. Section 2 will discuss the literature review related to the MDVRPSDP and robust optimization model. Followed by the problem statement dan model formulation of robust optimization in Section 3. Numerical examples and analysis of the results will be discussed in Section 4 and closed with Section 5 which concludes and some suggestions for further research.

2. Literature Review

Eksioglu *et al.* (2009) have discussed a taxonomic review of the vehicle routing problem (VRP) models and Braekers *et al.* (2016) have conducted a state of the art classification and review of most variants of VRP models. One variant of the Vehicle Routing Problem (VRP) model is the Vehicle Routing Problem with Deliveries and Pickups (VRPDP) models. The VRPDP model itself has many variations, as discussed by Nagy *et al.* (2015) and Koç *et al.* (2020) have reviewed VRPSDP models. Based on the characteristics of delivery and pickup goods, VRPDP can be classified into four variants (Nagy *et al.,* 2015), namely:

1. The Vehicle Routing Problem with Backhaul (VRPB) model does not allow the fleet of consigned goods to be mixed with the goods picked up, such the vehicle will do a linehaul first followed by backhaul.
2. The Vehicle Routing Problem with Mixed Deliveries and Pickups (VRPMDP) model that allows linehaul and backhaul processes to occur in an arbitrary order.
3. The Vehicle Routing Problem with Simultaneous Deliveries and Pickups (VRPSDP) model, in which customers expect that the process of picking up goods can be done at the same time as the delivery of goods.
4. The Vehicle Routing Problem with Divisible Deliveries and pickups (VRPDDP) model that allows one customer to be visited twice if the customer needs delivery and pick-up services.

The development of the VRPSDP model in reverse logistics has been proposed by Dethloff (2001). Furthermore, Wang *et al.* (2013) proposed the VRPSDP model with Split Loads and Time Windows to minimize the total travel cost, the number of vehicles, and the loading rate. which was solved using a hybrid heuristic algorithm. Shi *et al.* (2020) has also addressed the VRPSDPTW model which was solved by using a lexicographic-based two-stage algorithm as a solution method.
The vehicle routing problem can be carried out in coordination with several depots. This problem can be solved by using the multi-depot vehicle routing problem (MDVRP) model. Dondo & Cerdá (2007) developed a cluster-based optimization approach for solving the multi-depot heterogeneous fleet, vehicle routing problem with time windows. While De Oliveira *et al.* (2016) proposed to solve the MDVRP by using a cooperative coevolutionary algorithm and Stodola (2018) applied metaheuristics algorithm for solving MDVRP with modified optimization criterion. Furthermore, Li *et al.* (2016) proposed the MDVRP model under shared depot resources with time windows. They formulated MILP to minimizing the total traveling cost by considering the constraints of route duration and capacity of the vehicle, time window, the fleet size, and the number of parking spaces of each depot, which was solved by using a Hybrid Genetic Algorithm with Adaptive Local Search. Bae & Moon (2016) also discussed MDVRPTW for service vehicles used for delivery and installation of electronics to minimize fixed costs of the depots and the expenses related to travel distances and labor as well as the installation and delivery vehicles, which were solved by using genetic algorithms. In addition, Zhen *et al.* (2020) also focus on MDVRPTW and release dates of customers' packages in the last mile distribution operation. Wang et al. (2019) discussed the multi-depot green vehicle routing problem with shared transportation resources by applying an integration of piecewise penalty cost and time-dependent speed. Soeanu *et al.* (2020) proposed a cost-effective learning-based heuristic technique for MDVRP with risk mitigation to minimize the routing cost along with the potential cost due to the cargo delivery failure and the risk of vehicle breakdown. Research related to the comparison of alternative formulations of the MDVRP model has been discussed by Ramos *et al.* (2020).
Few researchers have studied the robust optimization model for vehicle routing problems. João et al. (2020) proposed a robust metaheuristic approach to vehicle routing problems with selective backhauls under uncertain revenues. In this study, we focus on developing a robust optimization model for multi-depot vehicle routing problems with simultaneous deliveries and pickups under environmental uncertainty and a cluster-based optimization - two stages as a solution method.

3. Model Development

The development of a robust optimization model for MDVRPSDP model is integration between Robust Optimization - RO, Multi Depot Vehicle Routing Problem – MDVRP model, and Vehicle Routing Problem with Simultaneous Deliveries and Pickups - VRPSDP model based on robust optimization of large scale system of Mulvey *et al.* (1995), MDVRP model of Ramos *et al.* (2020), by using a cluster based optimization approach of Dondo & Cerdá (2007), and VRPSDP model of Dethloff (2001) with some modifications and adjustments. Each depot has a limited storage capacity and several assigned vehicles with heterogeneous capacities. Each assigned vehicle can only operate within the available time limit. Each customer must be served by one depot and visited by one vehicle for delivery and pick up of goods simultaneously. In addition, at each consumer location, there are demands for delivery and pick up of goods such that the vehicle load after serving the consumer will fluctuate and it is necessary to check to ensure the vehicle load does not exceed the vehicle's load capacity. The optimal baseline route solution is robust so that it can absorb uncertainties in the length of travel times between locations, service time at each customer location, and the number of demands for delivery and pick up of goods. The mathematical formulation of the robust optimization model for MDVRPSDP will be discussed in the next section.

3.1. Model Formulation

Indices:
i, j: the index of location, $i = 1, 2, \dots, N$, where: $N = D \cup C$; D: set of all depots and C: set of all customers
k: the index of vehicles, $k = 1, 2, \dots, K$.
s: the index of discrete scenarios, $s = 1, 2, \dots, S$.

Model Parameters:
t_{ij}: The travel time from location *i* to location *j*.
ts_i: The service time at customer location *i*.
D_i: The number of delivery units at customer location *i*.
P_i: The number of pickup units at customer location *i*.
t_{ijs}: The travel time from location *i* to location *j* in scenario *s*.
ts_{is}: The service time at customer location *i* in scenario *s*.
D_{is}: The number of delivery units at customer location *i* in scenario *s*.
P_{is}: The number of pickup units at customer location *i* in scenario *s*.
$capd_i$: The storage capacity of depot *i*.
$capv_k$: The capacity of vehicle *k*.
T_{max}: Maximum of available time.
p_s: The probability value of scenario *s*.
Cd_i: Fixed cost for opening depot *i*.
FC_k: Fixed cost for using vehicle *k*.
Ct: Transportation cost per unit time
M: a sufficient large positive number.

Decision Variables:
x_{ijk}: set to 1 if vehicle *k* travels directly from location *i* to location *j*, and 0 otherwise.
z_{ij}: set to 1 if customer location *j* is served by depot *i*, and 0 otherwise.
f_i: set to 1 if depot *i* is opened, and 0 otherwise.
l_{ijk}: the load of vehicle *k* on arc (i, j).
f_{is}: set to 1 if depot *i* in scenario *s* is opened, and 0 otherwise.
y_k: set to 1 if vehicle *k* is used, and 0 otherwise.
y_{ks}: set to 1 if vehicle *k* in scenario *s* is used, and 0 otherwise.
u_{ik}: the order of location visits *i* in the route of vehicle *k*.
l_{ijks}: the vehicle load *k* in the path of location *i* to location *j* in scenario *s*.

Objective Function:

$$Min. ETC = \sum_{i \in D} Cd_i f_i + \sum_{k \in K} FC_k y_k + Ct \sum_{i \in N} \sum_{j \in N} \sum_{k \in K} t_{ij} x_{ijk} + \sum_{s \in S} p_s \{TC_s - \lambda |TC_s - \sum_{s' \in S} p_{s'} TC_{s'}|\} \quad (1)$$

Where: $TC_s = \sum_{s \in S} p_s \left(\sum_{i \in D} Cd_i f_{is} + \sum_{k \in K} FC_k y_{ks} + Ct \sum_{i \in N} \sum_{j \in N} \sum_{k \in K} t_{ijs} x_{ijk} \right)$ (2)

$$\sum_{i \in N} \sum_{k \in K} x_{ijk} = 1 \quad ; \forall j \in C \quad (3)$$

$$\sum_{j \in N} \sum_{k \in K} x_{ijk} = 1 \quad ; \forall i \in C \quad (4)$$

$$\sum_{\substack{i\in N\\ i\neq l}} x_{ilk} - \sum_{\substack{j\in N\\ j\neq l}} x_{ljk} = 0 \quad ; \forall l \in N, k \in K \tag{5}$$
$$\sum_{\substack{j\in N\\ j\neq i}} x_{ijk} - \sum_{\substack{j\in N\\ j\neq i}} x_{jik} = 0 \quad ; \forall i \in C, k \in K \tag{6}$$
$$u_{ik} - u_{jk} + |N|x_{ijk} \leq |N| - 1 \quad ; \forall i,j \in C, i \neq j, k \in K \tag{7}$$
$$\sum_{i\in D}\sum_{j\in C} x_{ijk} \leq 1 \quad ; \forall k \in K \tag{8}$$
$$\sum_{i\in C}\sum_{j\in D} x_{ijk} \leq 1 \quad ; \forall k \in K \tag{9}$$
$$\sum_{i\in N}\sum_{j\in N} t_{ij} + ts_i x_{ijk} \leq T_{max} \quad ; \forall k \in K \tag{10}$$
$$\sum_{j\in C} D_j z_{ij} \leq capd_i f_i \quad ; \forall i \in D \tag{11}$$
$$\sum_{k\in K} x_{ijk} \leq z_{ij} \quad ; \forall i \in D, j \in C \tag{12}$$
$$\sum_{i\in D}\sum_{j\in D} x_{ijk} = 0 \quad ; \forall k \in K \tag{13}$$
$$\sum_{j\in N}\sum_{k\in K} l_{ijk} - D_i + P_i = \sum_{j\in N}\sum_{k\in K} l_{jik} \quad ; \forall i \in C \tag{14}$$
$$l_{ijk} \leq capv_k y_k \quad ; \forall k \in K \tag{15}$$
$$\sum_{i\in N}\sum_{j\in N} t_{ijs} + ts_{is} x_{ijk} \leq T_{max} \quad ; \forall k \in K, s \in S \tag{16}$$
$$\sum_{j\in C} D_{js} z_{ij} \leq capd_i f_{is} \quad ; \forall i \in D, s \in S \tag{17}$$
$$\sum_{j\in N}\sum_{k\in K} l_{ijks} - D_{is} + P_{is} = \sum_{j\in N}\sum_{k\in K} l_{jiks} \quad ; \forall i \in C, s \in S \tag{18}$$
$$l_{ijks} \leq capv_k y_{ks} \quad ; \forall k \in K, s \in S \tag{19}$$
$$f_i \geq f_{is} \quad ; \forall i \in D, s \in S \tag{20}$$
$$y_k \geq y_{ks} \quad ; \forall k \in K, s \in S \tag{21}$$
$$x_{ijk} \in \{0,1\} \quad ; \forall i,j \in N, k \in K \tag{22}$$
$$z_{ij} \in \{0,1\} \quad ; \forall i \in D, j \in C \tag{23}$$
$$f_i, f_{is} \in \{0,1\} \quad ; \forall i \in D, s \in S \tag{24}$$
$$y_k, y_{ks} \in \{0,1\} \quad ; \forall k \in K, s \in S \tag{25}$$
$$l_{ijk}, l_{ijks} \geq 0 \quad ; \forall i,j \in N, k \in K \tag{26}$$

Equation (1) describes the objective function to minimize the expected total cost, which consists of the total cost of the baseline route and the expected total cost of all scenarios, and the robustness of the solution and optimization model. Robust optimization of the expected total cost for all scenarios is expressed in equation (2). The total cost of the baseline route consists of the total fixed costs of the depots, the total fixed costs of assigned vehicles, and the total variable transportation costs. Constraints (3) and (4) guarantee that every customer is visited by one vehicle from another location and goes to another location. Constraints (5) and (6) ensure flow in the route of each vehicle. Constraint (7) is a sub-tour elimination constraint. Constraints (8) and (9) guarantee that every vehicle assigned from a depot will return to the home base depot. Constraint (10) states that the total travel time and service times at all consumer locations visited by each vehicle do not exceed the maximum available time limit. Constraint (11) shows that the total customer demand handled by each depot does not exceed the storage capacity of the opened depot. Constraint (12) shows that there are vehicle routes from consumers that are handled by the depot. Constraint (13) guarantees no routes between depots. Constraint (14) shows the load balance of each vehicle after delivering and picking up goods from the consumer's location. Constraint (15) ensures that the vehicle load at arc (i,j) does not exceed the vehicle's capacity. Constraints (16) – (19) ensure compliance with available time limits, depot storage capacity, and vehicle capacity used for each scenario. Constraints (20) and (21) ensure that the opened depots and assigned vehicles are feasible for all scenarios. Decision variable domains are expressed in the constraints (22)-(26).

3.2. Solution Method

For solving the proposed RO-MDVRPSDP model, we apply a cluster-based optimization approach of Dondo & Cerdá (2007), with two stages, namely constructing consumer clusters which are handled by the depots that are opened, and for each consumer cluster, a route arrangement will be made by applying the robust optimization for VRPSDP model. For constructing the consumer clusters, a set partitioning problem approach of Vemuganti (1998) with bi-objective optimization is used. The first objective function is to minimize the maximum total travel time of all consumer clusters and the second is to minimize the number of consumer clusters by considering the demand uncertainty for each consumer. Each consumer cluster will be handled by one depot with available storage capacity. Model formulation of a bi-objective optimization model for constructing customer clusters is described as follows.

$$Min.\ MaxTT \tag{27}$$
$$Min. TD = \sum_{i\in D} f_i \tag{28}$$

$MaxTT \geq t_{ijs}z_{ij}$; $\forall i \in D, j \in C, s \in S$ (29)
$\sum_{i \in D} z_{ij} = 1$; $\forall j \in C$ (30)
$z_{ij} \leq f_i$; $\forall i \in D, j \in C$ (31)
$\sum_{j \in C} D_{js}z_{ij} \leq capd_i f_i$; $\forall i \in D, s \in S$ (32)
$z_{ij} \in \{0,1\}$; $\forall i \in D, j \in C$ (33)
$f_i \in \{0,1\}$; $\forall i \in D$ (34)

Equation (27) is the first objective function, which is to minimize the maximum travel time from the depot to each customer. Equation (28) shows the second objective function, which is to minimize the number of the opened depots. Constraint (29) ensures that the travel time from the depot to each served customer in each scenario does not exceed the maximum travel time. Constraint (30) ensures that each customer is served by one depot. Constraints (31) and (32) state that a depot will serve consumers if the depot is opened and the total demand for all the served customers in each scenario does not exceed the storage capacity of the depot. Constraints (33) and (34) are binary decision variable domains.

After the consumer clusters are formed, each consumer cluster served by a single depot will be solved by using the robust optimization model for VRPSDP as follows.

$Min. ETC' = \sum_{k \in K} FC_k y_k + Ct \sum_{i \in N_0} \sum_{j \in N_0} \sum_{k \in K} t_{ij} x_{ijk} + \sum_{s \in S} p_s \{TC'_s - \lambda |TC'_s - \sum_{s' \in S} p_{s'} TC'_{s'}|\}$ (35)
Where: $TC'_s = \sum_{s \in S} p_s \left(\sum_{k \in K} FC_k y_{ks} + Ct \sum_{i \in N_0} \sum_{j \in N_0} \sum_{k \in K} t_{ijs} x_{ijk}\right)$ (36)
$\sum_{i \in N_0} \sum_{k \in K} x_{ijk} = 1$; $\forall j \in C$ (37)
$\sum_{j \in N_0} \sum_{k \in K} x_{ijk} = 1$; $\forall i \in C$ (38)
$\sum_{i \in N_0} x_{ilk} - \sum_{j \in N_0} x_{ljk} = 0$; $\forall l \in C, k \in K$ (39)
$\sum_{j \in N_0, j \neq i} x_{ijk} - \sum_{j \in N_0, j \neq i} x_{jik} = 0$; $\forall i \in C, k \in K$ (40)
$\sum_{j \in C} x_{0jk} \leq 1$; $\forall k \in K$ (41)
$\sum_{i \in C} x_{i0k} \leq 1$; $\forall k \in K$ (42)
$\sum_{i \in N_0} \sum_{j \in N_0} t_{ijs} + ts_{is} x_{ijk} \leq T_{max}$; $\forall k \in K, s \in S$ (43)
$l_{ks} = \sum_{i \in N_0} \sum_{j \in C} D_{js} x_{ijk}$; $\forall k \in K, s \in S$ (44)
$l_{ks} \leq capv_k y_{ks}$; $\forall k \in K, s \in S$ (45)
$l_{jks} \geq l_{ks} - D_{js} + P_{js} - M(1 - x_{0jk})$; $\forall j \in C, k \in K, s \in S$ (46)
$l_{jks} \geq l_{iks} - D_{js} + P_{js} - M(1 - x_{ijk})$; $\forall i, j \in C, i \neq j, k \in K, s \in S$ (47)
$l_{jks} \leq capv_k y_{ks}$; $\forall j \in C, k \in K, s \in S$ (48)
$y_k \geq y_{ks}$; $\forall k \in K, s \in S$ (49)
$u_{ik} + 1 - N(1 - \sum_{k \in K} x_{ijk}) \leq u_{jk}$; $\forall i, j \in C, i \neq j, k \in K$ (50)
$x_{ijk} \in \{0,1\}$; $\forall i, j \in N_0, i \neq j, k \in K$ (51)
$y_k, y_{ks} \in \{0,1\}$; $\forall k \in K, s \in S$ (52)
$l_{ks} \geq 0;\ l_{jks} \geq 0$; $\forall j \in C, k \in K, s \in S$ (53)
$u_{ik} \geq 0\ \&\ integer$; $\forall i \in C, k \in K$ (54)

Equation (35) describes the objective function to minimize the total expected cost, which consists of the total cost of the baseline route of VRPSDP and the total expected cost of all scenarios, and the robustness of the solution and optimization model. Robust optimization of the expected total cost for all scenarios is expressed in equation (36). The total cost of the baseline route of VRPSDP consists of the total fixed costs of assigned vehicles, and the total variable transportation costs. Constraints (37) and (38) guarantee that every customer is visited by one vehicle from another location and goes to another location. Constraints (39) and (40) ensure flow in the route of each vehicle. Constraints (41) and (42) guarantee that every vehicle assigned from a depot will return to the depot. Constraint (43) states that the total travel time and service times at all consumer locations visited by each vehicle in each scenario do not exceed the maximum available time limit. Equation (44) and constraint (45) describe the total load of each vehicle in each scenario when departing from the depot and do not exceed the capacity of the vehicle used. Constraints (46), (47), and (48) show the vehicle load after visiting the first customer and subsequent customers in each scenario that should not exceed the capacity of the vehicle used. Constraint (49) describes the relationship between assigned vehicle decisions in each scenario and at the baseline route. Constraint (50) is a sub-tour elimination constraint. Constraints (51)-(54) are the decision variable domains.

4. Parameter Settings, Results and Discussion

The proposed robust optimization model for MDVRPSDP is applied to a numerical example. Seven potential depots will serve 68 consumers using the twelve available heterogeneous vehicles. Depots have various storage capacities that will affect their fixed opening costs. The fixed cost of vehicle assignment also varies depending on the vehicle's payload capacity. There are 10 scenarios to describe the uncertainty of the length of travel time between locations, the length of service time at each customer location, and the number of delivery and pickup demands for each consumer. The variable transportation cost per minute is $ 0.75. Maximum of available time for each vehicle is 480 minutes. Each scenario has an equal probability of occurrence. Table 1 shows data on the storage capacity and fixed costs of opening each depot. The payload capacity and fixed costs of each vehicle are found in Table 2. All parameter settings are listed in Table 3.

Table 1. The storage capacity and fixed cost for each depot

Depot	Storage Capacity (units)	Fixed Cost ($)
Depot 1	3600	760
Depot 2	5000	950
Depot 3	3200	780
Depot 4	4500	860
Depot 5	3900	820
Depot 6	4100	840
Depot 7	3000	730

Table 2. The payload capacity and fixed cost for each vehicle

Vehicle	Payload Capacity (units)	Fixed Cost ($)
V1	600	40
V2	600	40
V3	900	50
V4	900	50
V5	900	50
V6	1200	65
V7	1200	65
V8	1200	65
V9	1200	65
V10	1500	80
V11	1500	80
V12	1500	80

Table 3. Parameter settings for each data

Model Parameters	Data Distributions
Number of consumers' delivery and pickup demands, D_i and P_i(units)	D_i and $P_i \sim U[120,150]$
Number of consumers' delivery and pickup demands for each scenario, D_{is} and P_{is} (units)	$D_{is} \sim N(D_i, 0.2D_i)$ $P_{is} \sim N(P_i, 0.2P_i)$
Travel times between locations, $t_{ij} = t_{ji}$ (mins)	$t_{ij} \sim U[30,180]$
Travel times between locations for each scenario, $t_{ijs} = t_{jis}$ (mins)	$t_{ijs} \sim N\left(t_{ij}, 0.4t_{ij}\right)$
Service time for each customer, ts_i(mins)	$ts_i = 5\left\lceil\frac{D_i}{10}\right\rceil + 2\left\lceil\frac{P_i}{10}\right\rceil$
Service time for each customer in scenario s, ts_{is}(mins)	$ts_{is} \sim N(ts_{is}, 0.1ts_{is})$

The robust optimization model for MDVRPSDP was solved by applying two stages of a cluster-based optimization approach and using Gurobi-Python 9.0. The solution result is to open three depots, namely Depot 1, Depot 3, and Depot 4. The assignment of vehicles from each depot and the vehicle routes are shown in Table 4. with the expected total cost of $ 10113.25.

Table 4. Results of two stages of a cluster-based optimization.

Depot	Assigned Vehicles	The route of each vehicle
Depot 1	V7	Depot 1 - C13 - C40 - C57 - C43 - C12 - C49 - C5 – Depot 1
	V8	Depot 1 - C35 - C8 - C27 - C66 - C18 - C32 - C55 - C20 – Depot 1
	V9	Depot 1 - C47 - C59 - C19 - C38 - C42 - C48 - C51 - C23 – Depot 1
Depot 3	V3	Depot 3 - C26 - C30 - C36 - C54 - C7 - C15 – Depot 3
	V4	Depot 3 - C39 - C63 - C16 - C62 - C21 - C53 – Depot 3
	V6	Depot 3 - C24 - C44 - C41 - C3 - C29 - C22 - C25 - C67 – Depot 3
Depot 4	V10	Depot 4 - C65 - C31 - C33 - C6 - C14 - C11 - C46 - C58 - C34 – Depot 4
	V11	Depot 4 - C9 - C56 - C61 - C68 - C4 - C37 - C1 - C10 – Depot 4
	V12	Depot 4 - C17 - C28 - C45 - C2 - C64 - C60 - C52 - C50 – Depot 4

5. Conclusion

This study has proposed a robust optimization model for MDVRPSDP by considering the uncertainty of the length of travel time between locations, service times for each customer, and the number of demands for simultaneous delivery and pickup of goods that are often encountered in real practice. The mathematical formulation of the proposed robust optimization model for MDVRPSDP is included in the NP-Hard problem category in the combinatorial optimization problem which is hard to solve using exact methods. Therefore, in this study, we have developed a cluster-based optimization in two stages as a solution methodology for solving this RO-MDVRPSDP model. In the first stage, the consumer clusters are formed by applying a bi-objective optimization model with a set partitioning problem approach. Then in the second stage, route optimization is carried out using the robust optimization model for VRPSDP on each consumer cluster that has been formed. The results of the optimal solutions indicate a good solution quality within a reasonable computation time. For further research development, it can be considered the application of multi-objective optimization for robust MDVRPSDP as needed and the application of heuristic or metaheuristic algorithms as an alternative solution methodology to obtain near-optimal solutions in a reasonable computation time.

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