

# **A Green Inventory Management for Manufacturer-Retailer System with Stochastic Demand and Carbon Tax**

**Wakhid Ahmad Jauhari**

Department of Industrial and Systems Engineering  
Institut Teknologi Sepuluh Nopember  
Surabaya, Indonesia  
&

Department of Industrial Engineering  
Universitas Sebelas Maret  
Surakarta, Indonesia  
wachid\_aj@yahoo.com,

**I Nyoman Pujawan and Mokh Suef**

Department of Industrial and Systems Engineering  
Institut Teknologi Sepuluh Nopember  
Surabaya, Indonesia  
pujawan@ie.its.ac.id, mokhsuef@gmail.com

## **Abstract**

In this paper, a joint economic lot-sizing (JELS) model consisting of a manufacturer and a retailer is formulated. The retailer orders product to the manufacturer and then sell them to end customers. The manufacturer produces product and deliver them to the retailer with equal-sized shipment. A hybrid system made of a green production and a regular production is utilized by the manufacturer to produce product. The green production generates fewer emissions than the regular production, but it requires higher production cost. Three activities in the supply chain generate emissions, which are transportation, production and storage. A carbon tax policy is applied by the regulator to control the emissions. A mathematical inventory model is proposed to determine the optimal values of number of shipments, shipment lot, safety factor and production allocation so that the total cost can be minimized. An iterative procedure is also formulated to solve the model and a numerical example is presented to show the application of the model. The results show that by controlling the production allocation, the emissions can be reduced, thus minimizing the total cost. It is also observed that the changes in the carbon tax and emission parameters give significant impact on the optimal production allocation.

## **Keywords**

Inventory management, joint economic lot-sizing, green production, carbon tax and supply chain

## **1. Introduction**

Storage, transportation, production, logistics and warehousing activities are the main activities in the supply chain that generate carbon emissions. According to IPPC (2017), manufacturers are known to be the main contributors of the carbon emissions in the supply chain system. To lessen the carbon emissions, the government often uses a strict carbon regulation on carbon emitters. Facing this condition, the manufacturer should formulate a strategy to minimize the carbon emissions produced from the system. One strategy that can be applied is to adopt green technology. Green technology can be defined as technologies that produce lower emissions, conserve energy, use renewable energy or have a concern with health and safety aspects (Mishra et al., 2020). Production system that adopts the green technology is known as green production. The green production produces smaller emissions, but this type of technology will generally require higher investment and operating costs. The manufacturer who already have a regular production

system can adopt a green production system and operate them simultaneously to reduce the emissions. This kind of policy is commonly called as hybrid production system. This policy is considered more economical than replacing the existing production system with the green production (Entezaminia et al., 2020). In reality, however, the use of the hybrid production system will not be easy, because the manufacturer will face a trade-off between the production cost and the emissions.

The manufacturer's policy to employ the hybrid production influences the supply chain's inventory decisions. Thus, the manager needs to synchronize the decision on the production allocation with the member's decisions. To manage the inventories in the supply chain, the manager must choose a method that have proven to be effective. A joint economic lot sizing (JELS) problem is known to be an effective method to determine inventory decisions in a system containing of more than one parties. According to Havelaque and Bironneau (2015), a good inventory management is the main key to reducing the carbon emissions generated from the supply chain. Hence, increasing awareness of environmental impacts has driven the development of JELS towards a low carbon supply chain model. Some researchers, including Jaber et al. (2013), Castellano et al. (2019), and Marchi et al. (2019), have developed JELS models considering carbon emissions and proposed a carbon regulation to lessen the emissions. Some regulations on carbon reduction, such as carbon tax, carbon cap, carbon cap and trade and carbon penalties, are commonly used by scholars to control the emissions from some activities, i.e transportation, storage, production.

Our review to JELS model shows that the model has been developed into various ways by considering some different aspects. However, we observe that there is no study considering the use of the green technology in the production in an effort to reduce the emissions. In this study, we focus on developing a mathematical inventory model for the supply chain system comprising of a manufacturer and retailer. Thus, the objectives of the study are defined as follows:

- To determine the optimal production policy for the hybrid production system that faces the trade-off between the production cost and the emissions.
- To determine the optimal inventory decisions for the supply chain system such that the joint total cost and the emissions can be minimized.

## 2. Literature Review

JELS problem refers to the problem of determining joint lot-sizing in a supply chain system involving of more than one member. The objective of the problem is to minimize total inventory cost by finding the optimal values of decision variables. The objective can be achieved by synchronizing inventory and production decisions. Goyal (1976) was known as the first author who introduced the concept of JELS. He considered a situation in which the manufacturer produces items in an infinite rate and then transports the to the retailer with equal-sized shipment. Then, the model was developed into various conditions and assumptions, including stochastic environment. The research dealing with stochastic JELS was firstly introduced by Ben-Daya and Hariga (2004). They proposed an inventory problem for a supply chain system made of single vendor and single buyer with variable lead time. The lead time is determined by taking into account the productive time and delay time. Glock (2012) developed a similar model to the Ben-Daya and Hariga's work and studied the influence of the strategy to reduce lead on inventory decisions. The lead time is shortened by an additional cost and production time is controlled by making a production rate adjustment. Jauhari (2012) investigated the influence of raw procurement strategy in a three-echelon inventory system made of a supplier, a manufacturer and a retailer. Wangsa and Wee (2018) considered an explicit transportation cost on a JELS model. Islam et al. (2017) investigated the effect of a consignment policy on inventory management and showed how this policy can improve total cost of the supply chain. Castellano et al. (2019) and Herbon (2020) proposed a vendor-buyer inventory model and showed that the policy on production rate adjustment can benefit the supply chain system. Then, stochastic JELS model has been widely developed by taking into account some important factors, such as imperfect production (Khara et al., 2020), inspection errors (Jauhari, 2016), shipment policy (Giri et al., 2017) and learning process (Giri and Glock, 2017).

The increasing of environmental awareness has prompted researchers to investigate the effect of carbon emissions on inventory decisions at the supply chain level. Wahab et al. (2011) considered carbon emission cost and unequal sized shipment policy in JELS model made of a manufacturer and a retailer. They compared the performance of the model in the domestic supply chain and international supply chain. Ghosh et al. (2017) used a strict carbon policy to lessen the emissions generated from supply chain activities. Wangsa (2017) applied a penalty and incentive scheme to control the emissions from both transportation and production in a supply chain system comprising of a manufacturer and a retailer. Panja and Mondal (2019) developed a three-stage JELS model by taking into account the imperfect

production, green degree of products and delayed payment scheme. Halat and Hafezalkotob (2019) investigated different types of carbon regulation in a three-stage supply chain system and compared the performance of coordinated inventory policy and non-coordinated inventory policy. Gautam et al. (2019) proposed a sustainable vendor-buyer system with investment on inspection process and imperfect production. The emissions cost is charged for the carbon emissions produced from waste disposal and transportation activities. Shuang et al. (2019) examined the influence of various transportation modes and carbon regulation in a closed-loop supply chain. Furthermore, Manupati et al. (2019) evaluated the performance of supply chain system under some carbon policies, which are strict carbon cap, carbon tax and carbon cap and trade.

### 3. Model Development

#### 3.1 Problem Description

The supply chain is made of a manufacturer and a retailer, where the manufacturer is product producer and the retailer is product distributor. The retailer faces demand that is normally distributed with mean  $D$  and standard deviation  $\sigma$ . To satisfy the demand from end customers, the retailer orders  $nQ$  units to the manufacturer. To respond to the retailer's order, the manufacturer produces a batch of  $nQ$  units and sent them to the retailer over  $n$  times. The retailer incurs ordering cost  $A$  for each order made and incurs transportation cost  $F$  for each shipment made. The demand from end customers that cannot be satisfied by the retailer is backordered.

A hybrid production system made of a green production and regular production is utilized by the manufacturer to produce products. The green production adopts green technology that results in lower emissions than the regular production. Although the regular production generates higher emissions than the green production, it results in cheaper production cost. The manufacturer incurs setup cost for each batch produced in the manufacturing system. For each  $nQ$  units,  $(1 - \alpha)nQ$  units are produced in the green production facility and  $\alpha nQ$  units are produced in the regular production facility. The production batch produced by the green production depends on the ratio of the green production rate to the total system production rate,  $(nQ_g = nQ \frac{P_g}{P})$  and the production batch produced by the regular production depends on the ratio of the regular production rate to the total system production rate,  $(nQ_r = nQ \frac{P_r}{P})$ . By considering the production allocation, the production rate of the green production and regular production can be expressed by  $P_g = (1 - \alpha)P$  and  $P_r = \alpha P$ , respectively.

Carbon emissions are generated from three activities which are transportation, production and storage. The amount of emissions resulted from production activity is influenced by the production rate. The higher production rate, the higher emissions generated by the system. The amount of emissions from transportations is divided into two types, which are direct emissions and indirect emissions. The amount of emissions resulted from storage activity depends upon the inventory level. In this study, a carbon tax is applied to control the overall emissions.

#### 3.2 Notations and Assumptions

To develop the mathematical model, we used the following notations:

$D$	demand per unit time (units/year)
$\sigma$	standard deviation of demand (units/year)
$A$	ordering cost (\$/order)
$F$	transportation cost (\$/shipment)
$h_b$	holding cost for retailer (\$/unit/year)
$\pi$	backorder cost (\$/unit)
$T_s$	delivery time (year)
$W_b$	amount of carbon emissions from retailer's storage (kg CO <sub>2</sub> /unit)
$E$	carbon emission tax (\$/kg CO <sub>2</sub> )
$\vartheta_{T1}$	indirect emission (kg CO <sub>2</sub> /liter)
$\vartheta_{T2}$	direct emission (kg CO <sub>2</sub> /kg)
$\varepsilon$	fuel consumption for truck (liters/km)
$J_B$	distance from manufacturer to retailer (km)
$x$	weight of product (kg/unit)
$f_s(k)$	probability density function of standard normal distribution
$F_s(k)$	cumulative density function of standard normal distribution
$K_g$	setup cost for green production (\$/setup)

$K_r$	setup cost for regular production (\$/setup)
$P$	production rate (units/year)
$h_m$	holding cost for manufacturer (\$/unit/year)
$W_m$	amount of carbon emissions from production's storage (kg CO <sub>2</sub> /unit)
$X_{g1}$	green production's cost for running the machine (\$)
$X_{g2}$	the increase in green production's unit machining cost due to one unit increase in production rate (\$/unit)
$X_{r1}$	regular production's cost for running the machine (\$)
$X_{r2}$	the increase in regular production's unit machining cost due to one unit increase in production rate (\$/unit)
$a_g$	emission parameter for green production process (kg year <sup>2</sup> /unit <sup>3</sup> )
$b_g$	emission parameter for green production process (kg year/unit <sup>2</sup> )
$c_g$	emission parameter for green production process (kg /unit)
$a_r$	emission parameter for regular production process (kg year <sup>2</sup> /unit <sup>3</sup> )
$b_r$	emission parameter for regular production process (kg year/unit <sup>2</sup> )
$c_r$	emission parameter for regular production process (kg /unit)
$\alpha$	production allocation factor (decision variable)
$Q$	shipment size (decision variable)
$n$	number of shipments per production batch (decision variable)
$k$	safety factor (decision variable)

The following assumptions are utilized to formulate the mathematical model:

- The investigated system consists of a manufacturer who produce products and sell them to the retailer and a retailer who orders products to manufacturer and then sell them to end customers.
- The retailer faces a normal distribution demand and uses a continuous review to control the inventories.
- A hybrid system consisting of a green production and a regular production are employed by the manufacturer to produce products. Green production produces fewer emissions than regular production, but it requires higher production costs.
- The production cost and the amount of carbon emissions resulted from the manufacturing system depend on the production rate (Khouja and Mehrez,1994; Bogaschewsky,1995).
- The shortages are allowed and considered to be fully backordered.

### 3.3 Proposed Inventory Model

The retailer cost consists of ordering cost, transportation cost, holding cost, shortage cost and carbon emission cost. In our investigated system, the retailer order  $nQ$  units to manufacturer. The delivery is done with the quantity of  $Q$  units over  $n$  times. Equation (1) expresses the ordering cost and transportation cost.

$$OCT_b = \frac{D}{nQ} (A + nF) \quad (1)$$

To derive the retailer's inventory level, we refer to the formulation of Hadley Within (1963). The holding cost per unit time is formulated as follows

$$HC_b = h_b \left( \frac{Q}{2} + k\sigma \sqrt{\frac{Q}{P} + T_s} \right) \quad (2)$$

The shortage cost incurred by the retailer per unit time is

$$ES = \frac{\pi D \sigma}{Q} \sqrt{\frac{Q}{P} + T_s} \psi(k) \quad (3)$$

where,

$$\psi(k) = f_s(k) - k[1 - F_s(k)]$$

Storage and transportation activities done in the retailer generate carbon emissions. The emissions resulted from storage activity depends on inventory level while the emissions from transportation depends on fuel consumption, shipment frequency, product weight and the travel distance. The carbon cost is given by the following equation

$$EM_b = EW_b \left( \frac{Q}{2} + k\sigma \sqrt{\frac{Q}{P} + T_s} \right) + \frac{ED}{Q} \vartheta_{T1} \varepsilon J_b + E \vartheta_{T2} x D \quad (4)$$

Therefore, the retailer cost per unit time is given by

$$TCR = \frac{D}{nQ}(A + nF) + (h_b + EW_b)\left(\frac{Q}{2} + k\sigma Y_1\right) + \frac{\pi D\sigma}{Q}Y_1\psi(k) + E\left(\frac{D}{Q}\vartheta_{T_1}\varepsilon J_b + \vartheta_{T_2}x D\right) \quad (5)$$

where,

$$Y_1 = \sqrt{\frac{Q}{P} + T_s} \quad (6)$$

The manufacturer cost consists of setup cost, holding cost, production cost and emission cost. The manufacturer's setup cost consisting of setup cost for green production and setup cost for regular production, is given by

$$SC = \frac{(1 - \alpha)DK_g}{n(1 - \alpha)Q} + \frac{\alpha DK_r}{n\alpha Q} \quad (7)$$

The average inventory level can be calculated by considering the accumulative shipment and the manufacturer's accumulated production, that is

$$INV = \frac{(1 - \alpha)Q}{2}\left(n\left[1 - \frac{D}{P}\right] - 1 + \frac{2D}{P}\right) + \frac{\alpha Q}{2}\left(n\left[1 - \frac{D}{P}\right] - 1 + \frac{2D}{P}\right) \quad (8)$$

By referring to the basic formulation of Khouja and Mehrez (1994), the formulation of production cost is presented by the following expression

$$PC = \left(\frac{X_{g1}}{P_g} + X_{g2}P_g\right)(1 - \alpha)D + \left(\frac{X_{r1}}{P_r} + X_{r2}P_r\right)\alpha D \quad (9)$$

The carbon cost is incurred by the manufacturer for each emission generated from the production and storage activities. Equation (10) shows the manufacturer's carbon cost per unit time.

$$EMM = E(a_g P_g^2 - b_g P_g + c_g + a_r P_r^2 - b_r P_r + c_r) + EW_m \frac{(1 - \alpha)Q}{2}\left(n\left[1 - \frac{D}{P}\right] - 1 + \frac{2D}{P}\right) + EW_m \frac{\alpha Q}{2}\left(n\left[1 - \frac{D}{P}\right] - 1 + \frac{2D}{P}\right) \quad (10)$$

The manufacturer cost can be expressed as follows

$$TCM = \frac{DK_g}{nQ} + (h_m + EW_m)\frac{(1 - \alpha)Q}{2}Y_2 + E(1 - \alpha)D(a_g P_g^2 - b_g P_g + c_g) + \left(\frac{X_{g1}}{P_g} + X_{g2}P_g\right)(1 - \alpha)D + \frac{DK_r}{nQ} + (h_m + EW_m)\frac{\alpha Q}{2}Y_2 + E\alpha D(a_r P_r^2 - b_r P_r + c_r) + \left(\frac{X_{r1}}{P_r} + X_{r2}P_r\right)\alpha D \quad (11)$$

where,

$$Y_2 = \left(n\left[1 - \frac{D}{P}\right] - 1 + \frac{2D}{P}\right) \quad (12)$$

Thus, joint total cost per unit time is given by

$$JTC = TCR + TCM \quad (13)$$

#### 4. Solution Method

For fixed values of  $n$  and  $\alpha$ , the optimal values of  $Q$  and  $k$  can be determined by obtaining the first partial derivatives of  $JTC$  with respect to  $Q$  and  $k$ , respectively. By setting the results to zero we have

$$F_s(k) = 1 - \frac{(h_b + EW_b)Q}{\pi D} \quad (14)$$

$$Q = \sqrt{\frac{2D\left\{A/n + F + \pi\sigma\psi(k)Y_1 + E\vartheta_{T_1}\varepsilon J_b + \frac{K_g}{n} + \frac{K_r}{n}\right\}}{(h_b + EW_b) + \frac{k\sigma(h_b + EW_b)}{PY_1} + \frac{\pi D\sigma\psi(k)}{PQY_1} + (h_g + E_g W_g)Y_2 + (h_r + E_r W_r)Y_2}} \quad (15)$$

To solve the proposed inventory problem, we proposed an algorithm based on the procedure suggested by Ben-Daya and Hariga (2004). The proposed algorithm is as follows

1. Set  $\alpha = 0.01$ ,  $z_\alpha = 1$  and  $JTC(Q, k, n, z_\alpha - 1) = \infty$
2. Set  $n=1$  and  $JTC_{n-1}(Q_{n-1}, k_{n-1}, n - 1, z_\alpha) = \infty$

3. Calculate  $Q$  by using Equation (15).
4. Calculate  $k$  by substituting  $Q$  into Equation (14).
5. Repeat steps 3–4 until no change occurs in the values of  $Q$  and  $k$
6. Set,  $Q_n = Q$  and  $k_n = k$ . Calculate  $JTC_n(Q_n, k_n, n, z_\alpha)$  by using Equation (13).
7. If  $JTC_n(Q_n, k_n, n, z_\alpha) \leq JTC_{n-1}(Q_{n-1}, k_{n-1}, n-1, z_\alpha)$  repeat steps 2–6 with  $n=n+1$ , otherwise go to step 8.
8. Calculate  $JTC(Q, k, n, z_\alpha) = JTC_{n-1}(Q_{n-1}, k_{n-1}, n-1, z_\alpha)$ .
9. If  $JTC(Q, k, n, z_\alpha) \leq JTC(Q, k, n, z_\alpha - 1)$  repeat steps 1–8 with  $\alpha = \alpha + 0.01$  and  $z_\alpha = z_\alpha + 1$ , otherwise go to step 10.
10. Set  $P, Q, k, n, \alpha$  as the solutions of the above problem and  $JTC(P, Q, k, n, z_\alpha - 1)$  is the minimum value of joint total cost.

## 5. Results and Discussion

### 5.1 Numerical Example

In this section, to show the application of the proposed model we present a numerical example with the following data:  $D=1000, \sigma=5, A=100, F=50, h_b=4, \pi=50, T_s=0.05, W_b=8, \vartheta_{T1}=2.6, \vartheta_{T2}=2.5, \varepsilon=0.3, J_b=400, x=0.05, K_g=500, K_r=500, h_m=2, W_m=5, E=0.0518, X_{g1}=2500, X_{r1}=2000, X_{g2}=0.0008, X_{r2}=0.0004, a_g=0.0000007, b_g=0.0012, c_g=1.4, a_r=0.0000012, b_r=0.0008, c_r=8.4$ . The values of truck fuel consumption and carbon tax are taken from Volvo Truck report (2019) and Chan (2019), respectively. Table 1 presents the optimal inventory decisions of the proposed problem. It shows that the delivery should be made 6 times per production run with size of 317.29 units. The optimal production batch for the green production and the regular production are 837.64 units and 1,066.09 units, respectively. Thus, the cost incurred by the retailer, manufacturer and supply chain are \$516.2, \$3,711.44 and \$4,227.64, respectively.

Table 1. Optimization results

Decision variables and costs	Optimum values
Production allocation	0.56
Number of shipments	6
Delivery quantity	317.29
Green's production rate	1,408
Regular's production rate	1,792
Safety factor	2.3
Retailer cost	516.2
Manufacturer cost	3,711.44
Joint total cost	4,227.64

### 5.2 Analysis of Carbon Tax

Table 2 shows how the change in carbon tax gives significant influence on model's behavior. Clearly, if the tax becomes more expensive, the manager should give more production allocation to the cleaner production system. By allocating more production to the greener one, the system can reduce the emissions, thus minimizing the impact of the carbon tax increase. Facing an increase in carbon tax, the manager should reduce the shipment frequency to cut down the emissions resulted from transportation activity. In addition, the production batch and safety stock should be reduced to lessen the emissions from holding items. The results from Figure 1 show that the carbon emissions produced by the system decreases due to the increase in the carbon tax. Furthermore, as the production allocation to the green production increases, the manager should adjust its production rate to the higher level to balance the production capacity with the production allocation. We further observe that if carbon tax increases, the green's production rate increases while the regular's production rate decreases. Finally, the carbon tax gives significant impact on the member cost and the total cost. The costs incurred by the retailer, manufacturer and supply chain drastically increase due to the increase in carbon tax.

Table 2. The impact of carbon tax on the model

% change in carbon tax	-80%	-40%	0%	40%	80%	120%	160%
Production proportion ( $\alpha$ )	0.64	0.6	0.56	0.54	0.51	0.49	0.47
Number of shipments ( $n$ )	7	6	6	5	5	5	4
Shipment lot size ( $Q$ )	337.34	343.21	317.29	340.65	322.28	307.21	346.72
Safety factor ( $k$ )	2.39	2.33	2.30	2.23	2.21	2.19	2.11
Green's production rate ( $P_g$ )	1,152	1,280	1,408	1,472	1,568	1,632	1,696

Regular's production rate ( $P_r$ )	2,048	1,920	1,792	1,728	1,632	1,568	1,504
Retailer total cost	393.61	458.35	516.20	583.73	640.23	697.01	772.84
Green's total cost	1,485.35	1,638.93	1,803.89	1,914.06	2,064.10	2,187.20	2,303.48
Regular's total cost	1,718.45	1,828.16	1,907.55	2,011.19	2,071.13	2,147.00	2,200.90
Joint total cost	3,597.41	3,925.43	4,227.64	4,508.98	4,775.46	5,031.20	5,277.23

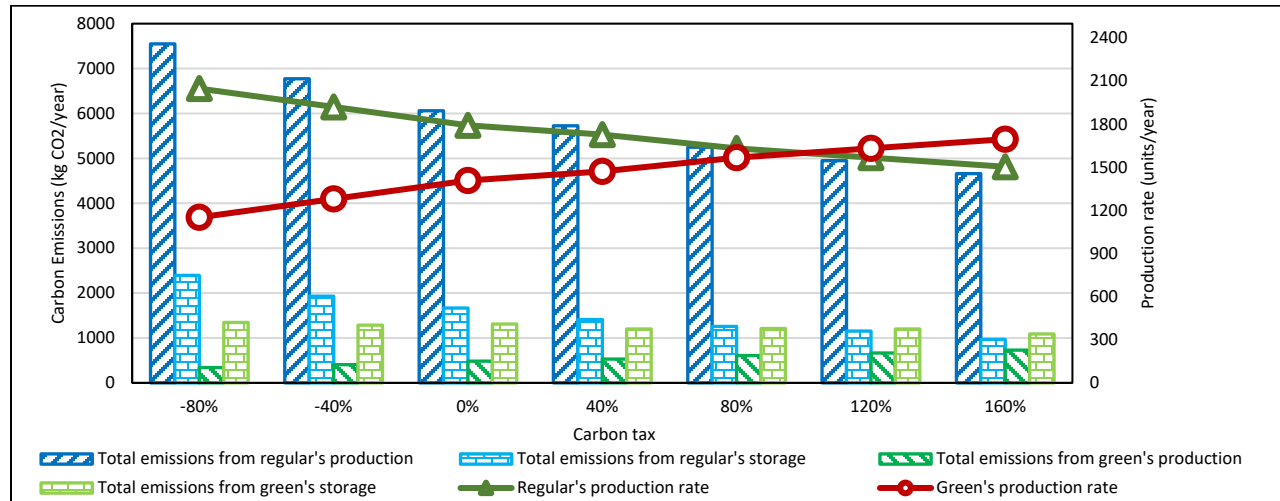


Figure 1. The impact of carbon tax on carbon emissions and production rate

### 5.3 Analysis of Emission Parameter

We further investigate the influence of regular's emission parameter on the model's behavior. Table 3 presents the results of the investigation. We observe that when the emission level of the regular production gets higher, it is beneficial for the system to allocate more production to the green production. This makes sense because by giving a larger production allocation to the greener system, the manufacturer has an opportunity to control the amount of the emissions produced. However, the overall emissions generated by the system rise due to the increase in the emission parameter (see Figure). The results from table 3 indicate that the number of shipments, shipment quantity and safety factor are insensitive to the change of the emission parameter. As a result, the cost incurred by the green production increases and the cost incurred by the regular production decreases. However, the retailer cost remains unchanged.

Table 3. The impact of emission parameter on the model

% change in $a_2$	-80%	-40%	0	40%	80%	120%	160%
Production proportion ( $\alpha$ )	0.62	0.59	0.56	0.54	0.52	0.51	0.49
Number of shipments ( $n$ )	6	6	6	6	6	6	6
Shipment lot size ( $Q$ )	317.29	317.29	317.29	317.29	317.29	317.29	317.29
Safety factor ( $k$ )	2.30	2.30	2.30	2.30	2.30	2.30	2.30
Green's production rate ( $P_g$ )	1,216	1,312	1,408	1,472	1,536	1,568	1,632
Regular's production rate ( $P_r$ )	1,984	1,888	1,792	1,728	1,664	1,632	1,568
Retailer total cost	516.20	516.20	516.20	516.20	516.20	516.20	516.20
Green's total cost	1,639.48	1,719.15	1,803.89	1,863.23	1,924.88	1,956.56	2,021.69
Regular's total cost	1,966.09	1,943.16	1,907.55	1,891.26	1,868.13	1,871.18	1,837.60
Joint total cost	4,121.77	4,178.51	4,227.64	4,270.70	4,309.21	4,343.95	4,375.49

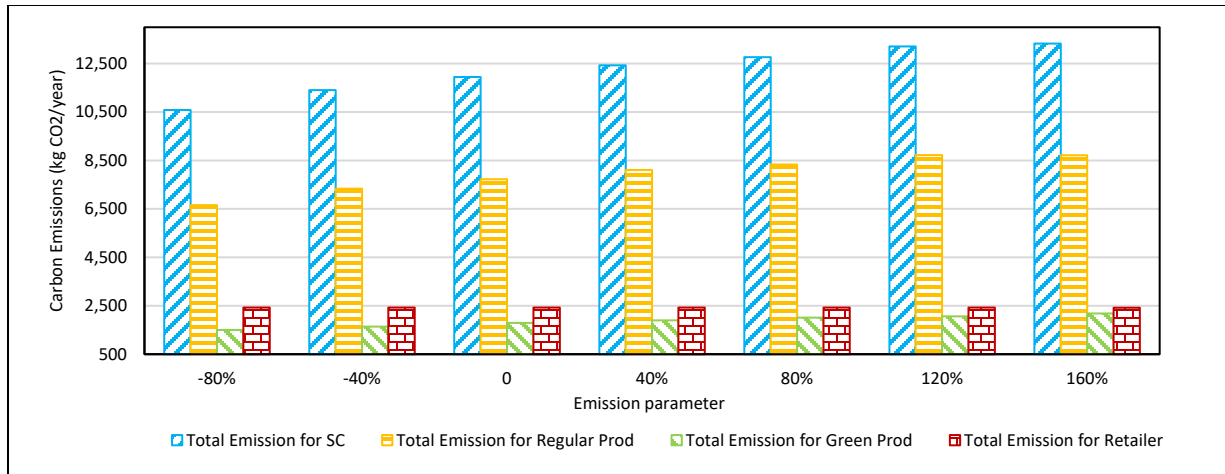


Figure 2 The impact of emission parameter on carbon emissions

### 5.4 Analysis of Demand

For the final investigation, we focus on studying the effect of the change in demand on the decision variables and costs. The results from Table 4 shows that the shipment quantity, number of shipments and safety factor are sensitive to the change in the demand. Facing a larger demand, the manufacturer should increase the production batch and the retailer should increase the safety stock. This makes perfect sense since increasing both production batch and safety stock will naturally increase the inventories kept by the system. Furthermore, the emissions generated from supply chain activities increases significantly which leads to the increase in the total cost (see Figure 3).

Table 4. The impact of demand on the model

% change in demand	-80%	-40%	0%	40%	80%	120%	160%
Production proportion ( $\alpha$ )	0.56	0.56	0.56	0.56	0.56	0.56	0.56
Number of shipments ( $n$ )	4	5	6	7	8	10	14
Shipment lot size ( $Q$ )	181.28	271.17	317.29	353.13	389.10	404.87	408.95
Safety factor ( $k$ )	1.90	2.18	2.30	2.38	2.43	2.48	2.53
Green's production rate ( $P_g$ )	1,408	1,408	1,408	1,408	1,408	1,408	1,408
Regular's production rate ( $P_r$ )	1,792	1,792	1,792	1,792	1,792	1,792	1,792
Retailer total cost	245.17	407.47	516.20	604.41	679.71	744.52	802.60
Green's total cost	503.13	1,189.92	1,803.89	2,376.67	2,916.28	3,420.92	3,879.36
Regular's total cost	539.40	1,264.09	1,907.55	2,504.26	3,062.75	3,583.10	4,052.06
Joint total cost	1,287.70	2,861.48	4,227.64	5,485.34	6,658.74	7,748.54	8,734.02

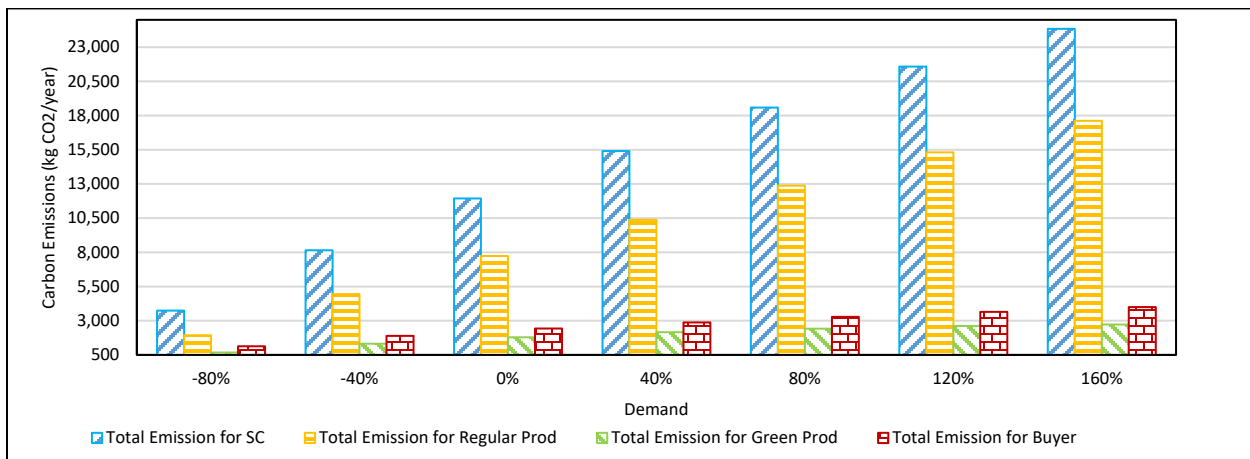


Figure 3. The impact of demand on carbon emissions



## 6. Conclusions and Future Research Directions

In this paper, a mathematical inventory model for a supply chain system under stochastic demand and carbon reduction is proposed. A hybrid production system is adopted by the manufacturer to lessen the emissions generated from the production. A carbon tax is implemented by the regulator to restrict the emissions from supply chain. An efficient procedure is suggested to solve the model. In addition, a numerical example along with sensitivity analysis is provided to show the model applicability and to investigate the impact of the change in some key parameters on model's behavior. The results of the proposed study are summarized as follows. First, by adjusting the production allocation, the manager can control the emissions and minimize the joint total cost of the supply chain. We observe that the production allocation is very sensitive to the change in the carbon tax and emission parameters. Second, facing the increase in the demand, the manager should increase the inventory level by adjusting the safety factor, delivery frequency and delivery quantity.

In our proposed model, the system production rate is assumed to be fixed. The model can be extended by allowing the inclusion of adjustable production rate. Future study can also be done by investigating the impact of different carbon policies, i.e carbon cap, carbon cap and trade, on inventory decisions. Other extension can also be made by relaxing the assumption of perfect manufacturing system. In reality, the manufacturing system is imperfect, so it results some defective products.

## References

- Ben-Daya, M. and Hariga, M., Integrated single vendor single buyer model with stochastic demand and variable lead time, *International Journal of Production Economics*, vol. 92, no. 1, pp.75-80, 2004.
- Castellano, D., Gallo, M., Grassi, A., and Santillo, L. C., The effect of GHG emissions on production, inventory replenishment and routing decisions in a single vendor-multiple buyers supply chain, *International Journal of Production Economics*, vol. 218, pp. 30-42, 2019.
- Chan, Y. Taiwan plans taxes for energy and CO2 emissions by 2011, Available: <https://www.businessgreen.com/bg/news/1800579/taiwan-plans-taxesenergy-co2-emissions-2011>. May, 12, 2019.
- Entezaminia, A., Gharbi, A. and Ouhimmou, M., Environmental hedging point policies for collaborative unreliable manufacturing systems with variant emitting level technologies, *Journal of Cleaner Production*, vol. 250, 119539, 2020.
- Gautam, P., Kishore, A.S., Khanna, A. and Jaggi, C.K., Strategic defect management for a sustainable green supply chain, *Journal of Cleaner Production*, vol. 233, pp. 226-241, 2019.
- Goyal, S.K., An integrated inventory model for a single supplier-single customer problem, *International Journal of Production Research*, vol. 15, no.1, pp.107-111.
- Ghosh, A., Jha, J.K. and Sarmah, S.P., Optimal lot-sizing under strict carbon cap policy considering stochastic demand, *Applied Mathematical Modelling*, vol. 44, pp. 688-704, 2017.
- Giri, B., Chakraborty, A., Maiti, T., Consignment stock policy with unequal shipments and process unreliability for a two-level supply chain, *International Journal of Production Research*, vol. 55, pp. 2489–2505, 2017.
- Giri, B.C. and Glock C.H., A closed-loop supply chain with stochastic product returns and worker experience under learning and forgetting, *International Journal of Production Research*, pp.1-19, 2017.
- Glock, C.H., Lead time reduction strategies in a single-vendor-single buyer integrated inventory model with lot size-dependent lead times and stochastic demand, *International Journal of Production Economics*, vol. 136, no. 1, pp. 37-44, 2012.
- Halat, K. and Hafezalkotob, A., Modeling carbon regulation policies in inventory decisions of a multi-stage green supply chain: A game theory approach, *Computers & Industrial Engineering*, vol. 128, pp. 807-830, 2019.
- Herbon, A., An approximated solution to the constrained integrated manufacturer-buyer supply problem, *Operation Research Perspectives*, vol. 7, pp. 1-9, 2020.
- Hovelaque, V., and Bironneau, L., The carbon-constrained EOQ model with carbon emission dependent demand, *International Journal of Production Economics*, vol. 164, pp. 285-291, 2015.
- Islam, S.M.S., Hoque, M.A. and Hamzah, N., Single-supplier single-manufacturer multi-retailer consignment policy for retailers' generalized demand distributions, *International Journal of Production Economics*, vol. 184, no. 1, pp. 157-167, 2017.
- IPPC, Climate Change 2017 Mitigation, Contribution of Working Group III to the Fourth Assessment Report, Cambridge University Press. New York, 2017.

- Jaber, M.Y., Glock, C.H and El-Saadany, A.M.A., Supply chain coordination with emissions reduction incentives, *International Journal of Production Research*, vol. 51, no. 1, pp. 69-82, 2013.
- Jauhari, W.A., Integrated inventory model for three-layer supply chains with stochastic demand, *International Journal of Operational Research*, vol. 13, no. 3, pp. 295-317.
- Jauhari, W.A., Integrated vendor-buyer model with defective items, inspection error and stochastic demand, *International Journal of Mathematics in Operational Research*, vol. 8, no. 3, pp.342–359, 2016.
- Khara, B., Dey, J.K., Mondal, S.K., Sustainable recycling in an imperfect production system with acceptance quality level dependent development cost and demand, *Computers and Industrial Engineering*, vol. 142,106300, 2020.
- Manupati, V.K., Jedidah, S.J., Gupta, S., Bhandari, A. and Ramkumar, M., Optimization of a multi-echelon sustainable production-distribution supply chain system with lead time consideration under carbon emission policies, *Computers & Industrial Engineering*, Vol. 135, pp. 1312–1323, 2019.
- Mishra, U., Wu, J.Z. and Sarkar, B., A sustainable production -inventory model for a controllable carbon emissions rate under shortages, *Journal of Cleaner Production*, Vol. 256, 120268, 2020.
- Marchi, B., Zanoni, S., Zavanella, L.E. and Jaber, M.Y., Supply chain models with greenhouse gasses emissions, energy usage, imperfect process under different coordination decisions, *International Journal of Production Economics*, vol. 211, pp. 145-153, 2019.
- Panja, S. and Mondal S.K., Analyzing a four-layer green supply chain imperfect production inventory model for green products under type-2 fuzzy credit period, *Computers & Industrial Engineering*, vol. 129, pp. 435–453, 2019.
- Shuang, Y. Diabat, Ali and Liao, Y., A stochastic reverse logistics production routing model with emissions control policy selection, *International Journal of Production Economics*, vol. 213, pp. 201–216, 2019.
- Volvo Truck, Emissions from Volvo's truck, Available: [https://www.volvotrucks.com/content/dam/volvo/volvo-trucks/markets/global/pdf/our-trucks/Emis\\_eng\\_10110\\_14001.pdf](https://www.volvotrucks.com/content/dam/volvo/volvo-trucks/markets/global/pdf/our-trucks/Emis_eng_10110_14001.pdf), May, 15, 2019.
- Wahab, M. I. M., Mamun, S. M. H., and Ongkunaruk, P., EOQ models for a coordinated two-level international supply chain considering imperfect items and environmental impact, *International Journal of Production Economics*, vol. 134, no. 1, pp. 151-158, 2011.
- Wangsa, I. D., and Wee, H. M., An integrated vendor-buyer inventory model with transportation cost and stochastic demand, *International Journal of Systems Science: Operations & Logistics*, vol. 5, no. 4, pp.1-15, 2018.
- Wangsa, I. D., Greenhouse gas penalty and incentive policies for a joint economic lot size model with industrial and transport emissions, *International Journal of Industrial Engineering Computations*, vol. 8, pp. 453-480, 2017.

## Biographies

**Wakhid Ahmad Jauhari** is currently a doctoral student in Industrial Engineering Department, Sepuluh Nopember Institute of Technology (ITS), Surabaya, Indonesia and a senior researcher in Production System Laboratory, Universitas Sebelas Maret, Surakarta, Indonesia. His research's interests include modelling inventory, supply chain management, energy management and manufacturing design. He has published many articles in international journals and proceedings, including Operations Research Perspectives, Journal of Industrial Engineering International, Energy System, Production and Manufacturing Research, Journal of Industrial Engineering and Management, International Transactions on Electrical Energy System, Journal of Industrial and Production Engineering, Advances in Operation Research, Cogent Engineering, International Journal of Logistics and Systems Management and Journal of Control and Decisions.

**I Nyoman Pujawan** is Professor of Supply Chain Engineering at the Department of Industrial Engineering, Sepuluh Nopember Institute of Technology (ITS), Surabaya, Indonesia. He is currently the President of the Indonesian Supply Chain and Logistics Institute (ISLI). He received a Bachelor degree in Industrial Engineering from ITS, Indonesia, Master of Engineering in Industrial Engineering from Asian Institute of Technology (AIT) Bangkok, Thailand, and PhD in Management Science from Lancaster University, UK. He also holds Certified Supply Chain Professional (CSCP) from APICS (USA). He was a Lecturer in Operations Management in Manchester Business School, The University of Manchester, UK in 2003 – 2004. His papers have appeared in many international journals including the European Journal of Operational Research, International Journal of Production Economics, International Journal of Production Research, Production Planning and Control, International Journal of Physical Distribution and Logistics Management, Supply Chain Management: An International Journal, International Journal of Logistics Research and Applications, Business Process Management Journal, among others. Professor Pujawan worked in industry before moving to the academia. While his academic background is very strong, he is equally well experienced in handling

industry problems. He is an active consultant for various supply chain and logistics related industry problems (has involved in over 40 consulting projects).

**Mokh Suf** is a senior lecturer in the Department of Industrial Engineering, Sepuluh Nopember Institute of Technology (ITS), Surabaya, Indonesia. He received a Bachelor degree in Mechanical Engineering from Sepuluh Nopember Institute of Technology, Surabaya, Indonesia in 1989 and Master of Science in Engineering from Birmingham University in 1997 with specialization in Integrated Quality Management. He also received a Ph.D degree in Industrial Engineering from Institut Teknologi Sepuluh Nopember in 2016. His research interests include quality management, manufacturing process and maintenance management. He has published articles in international journals and proceedings, including TQM Journal, International Journal of Technology and International Journal of Mechanical Engineering and Technology.