Economic Ordering Policy of Deterioration Item with Time Varying Holding Cost under Backlogging and Permissible Delay in Payments

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Abstract

Most of the items managed by a retailer are perishable and have a limited shelf life. The longer they are stored, the more their value decrease, where the holding cost is directly proportional to the value of the goods and deterioration is time proportional. If there is a shortage, it will be resolved backorder. The supplier allows payments within a certain period of time. This paper will try to combine and modify some basic inventory models to develop a model for deterioration items that have expiry date with varying holding cost to time where deterioration is time proportional, under the conditions of backlogging and allowing delays in payments. The model solution is carried out with an optimization approach based on the parameters that affect the model. The approximate optimal solution has been obtained. Numerical examples are given at the end of this paper to illustrate the model settlement algorithms.

Keywords
Inventory model, deteriorating items, backlogging, time-varying holding cost, permissible delay in payments

1. Introduction

Inventory of goods is one of the most important parts in the operation of a business unit. Every element in a company requires supplies in various forms and functions. Even though inventory is considered as a waste, it cannot be avoided. This is due the fact that it needs quite some time to wait for the goods arrived, from the first order until the goods arrived.

Inventory also has an important and strategic position, because it has a major effect on the company's performance, both because of its role and from the value of the investment that must be spent to meet its needs. The inventory investment value at the factory level ranges from 25 - 35% of the total assets owned. Meanwhile, at the wholesaler, distributor and retailer levels it is in the range of 15 - 90% of the total cost of the product (Tersine, 1994).

Most of the items managed by a retailer are perishable items and have a limited shelf life. The longer the goods are stored, the more their value decrease. As a result, the risk faced by losses due to expired goods is also large. As an illustration that is still relevant to consider, namely the 2005 National Supermarket Shrink Survey reported that deteriorating account for more than 54% of total store sales which constitute more than $ 200billion and approximately 57% of total store shrink (Nafisah et al., 2016a).

According to Wee (1993), deteriorating items refers to the items that become decayed, damaged, evaporative, expired, invalid, devaluation and so on through time. According to the definition, deteriorating items can be classified into two categories. The first category refers to the items that become decayed, damaged, evaporative, or expired through time, like meat, vegetables, fruit, medicine, flowers, film and so on. The other category refers to the items which lose part or total value through time because of new technology or the introduction of alternatives, like computer chips, mobile phones, fashion and seasonal goods, and so on. Both of the two categories have the characteristic of short life cycle. For the first category, the items have a short natural life cycle. After a specific period (such as durability), the natural attributes of the items will change and then lose useable value and economic value, for the second category, the items have a short market life cycle (Goyal dan Giri, 2001).

As an example, Chickenpedia is a culinary business which has three restaurant branches in Yogyakarta and has one inventory warehouse as a consolidated central warehouse to store all the needs of the three restaurants. Thus, the Chickenpedia central warehouse has the responsibility to fulfil the demand for all raw materials and other needs from branches. Most of the items managed by Chickenpedia's central warehouse are raw food materials, they are perishable and have a limited shelf life. The longer the raw material is stored, the more the quality decrease, where the storage cost is directly proportional to the value of the goods and the decline is directly proportional to time. Chickenpedia warehouse obtains raw materials from suppliers and most suppliers allow payment of raw materials within a certain period of time. Orders to suppliers are made when the stock of raw materials in the warehouse is running low and the order quantity is in accordance with the request of the three Chickenpedia branches. However, in fact there is often excessive stock in some raw materials and shortage of stock in some other raw materials. The remaining stock of raw materials will be stored and if they are in good condition, they will be used to fulfill further requests, but if they have expired or rot, the raw materials will be thrown away. Meanwhile, if there is a stockout, Chickenpedia warehouse will place an emergency order to the nearest shop, of course, the price of raw materials is more expensive and additional transport costsarise.

The more raw materials provided, the more invested capital cannot be used for, more profitable purposes. Thus the greater the risk of raw materials being damaged and expiring or rotting. The less raw materials provided, the greater the possibility of inventory shortage. As a result, the greater the loss of gaining profits opportunities or the greater the costs incurred for making emergency orders (Nafisah et al. 2, 2016b).
Therefore, inventory control of perishable in Chickenpedia's central warehouse is very necessary to avoid the risk of even greater losses due to product expiration.

1.1 Objectives

Based on the phenomena of problems faced by Chickenpedia's central warehouse, this study will develop an inventory model for items that are easily damaged and have an expiration period by considering the storage costs that vary to time, in backlogging conditions and allowable delays in payments.

2. Literature Review

Research on the supply control model for deterioration item has been carried out intensively by several researchers. The inventory model developed by Goyal (1985) considers the existence of a payment delay in two cases, namely when the product reorder point is greater than or equal to the allowable delay payment time and when the product reorder point is smaller than the allowable delay payment time. While Goyal and Giri (2001) gave recent trends of modeling in deteriorating item inventory. They classified inventory models on the basis of demand variations and various other conditions or constraints. The research area of deteriorating inventory models from January 2012 until December 2015 has also been given an up-to-date review by Janssen et al. (2016). As for the discussion a comprehensive view of the past research dealing with the management of deteriorating items inventory modeling literature has been done by Perez and Torres (2020).

In Karmakar and Choudhury (2010) reviewed the inventory models with shortages of different types for deterioration items with different demand patterns and proposed future need of research in this direction. Research on deterioration item modelling has been also developed by Mishra et al., (2013). They considered a deterministic inventory model with time-dependent demand and time-varying holding cost where deterioration is time proportional. The model considered here allows for shortages, and the demand is partially backlogged. Jayashree (2013) developed a deteriorated inventory control model for pharmaceutical products with variable holding costs. Amutha and Chandrasekaran (2013) presents an inventory model for deteriorating products with constant demand and time varying deteriorating rate, under permissible delay in payments. It discussed in two cases whether the permissible periods are less than or equal to or greater than replenishment cycle. During the permissible period both supplier and retailer got some benefits. Shortages are allowed and are completely backlogged.

Several other researchers who developed deterioration item models by considering delayed payments included Kumar (2013), Amutha and Chandrasekaran (2013), Nafisah et al. (2016a), Uthayakumar and Karuppasamy (2017), and Pervin and Roy (2018).

Singh (2016) presents an inventory model for perishable items with constant demand, for which holding cost increases with time, the items considered in the model are deteriorating items with a constant rate of deterioration. Nafisah et al, 2016b developed an inventory model for deteriorated items taking into account the sales discounts given to consumers at certain levels. The closer to the expiration date, the greater the discount per unit given, so that with this discount the demand is different. In addition, the product return factor is also considered. Li et al. (2018) presented the liquid pharmaceutical preparations inventory problem of a hospital. The first establish a pharmaceutical inventory model with shelf life and service level constraints with a stochastic lead time. Muniappan et al. (2019) investigated an inventory model for deteriorating products with maximum lifetime and constant demand. Shortages are allowed and backlogged them completely.

3. Methods

3.1. Assumption and Notations

The notations used in developing this model are as follows:
D : annual demand rate, units/year
Q : order quantity per cycle, units
$Q*$: economic order quantity per cycle, units
S : initial inventory level in a cycle, units
A : ordering cost per order
P : unit purchase price
π : stockout cost per unit
h : holding cost per unit per year excluding interest charges
d : expired cost per unit per year
I_e : interest which can be earned per year
t : permissible delay in settling accounts
T : time interval between successive orders (the length of the cycle)
t_1 : time interval between ordered received to ordered lossed
t_2 : time interval during stockout
t_3 : time of allowance for delays in supplier payments
L : lead time
t_1 : cost of interest which can be earned per year
B_e : deterioration rate, $0 < \theta < 1$
PC : purchase cost per year
OC : ordering cost per year
HC : holding cost per year
SC : shortage cost per year
DC : expired cost per year
TC : total annual variable cost

The following assumptions are made in deriving the model:
(1) The demand for the item is constant with time.
(2) Shortages are allowed by using backorder that equal to the amount of expired products
(3) The supplier allows delays in payment, $T > t$
(4) During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. At the end of this period, the account is settled and we start paying for the interest charges on the items in stock.
(5) Lead time constant
(6) Expiration date known
(7) Returns are not permitted by the supplier. Products that have expired will be destroyed
(8) The material of this research has the characteristics of a deteriorated item with a constant rate of damage per year.
(9) Deterioration rate is time proportional
(10) Holding cost is variable to time.

3.2. Mathematical Formulation

This research is the development of an inventory model based on a case study in Chickenpedia central warehouse. The basic model used in model development here is the model of Jayashree (2013) and Goyal (1985). In Jayashree model (2013) has been developed a deterministic deterioration item model that considers variable holding cost. In Goyal (1985) has been derived for obtaining the economic order quantity for an item for which the supplier permits fixed delay in settling the amount owed to him. This research will develop a model for deterioration items which have expiry date with varying holding cost to time where deterioration is time proportional, under conditions of backlogging and allowing delays in payments. This proposed model is expected to produce an appropriate inventory control policy in determining the optimal product order quantity and optimal ordering time in order to minimize the total annual inventory cost. The position pattern of the inventory model to be developed is shown in Figure 1. The initial inventory level is $I_0 = S$ unit at time $t = 0$; from $t = 0$ to $t = t_1$, the inventory level reduces, owingto both demand and deterioration, until it reaches zero level at time $t = t_1$. At this time, shortage is accumulated whichis backlogged. At the end of the cycle, the inventory reaches a maximum shortage level so as to clear the backlogged and again raises the inventory level to $I_0 = S$ (Figure 1).

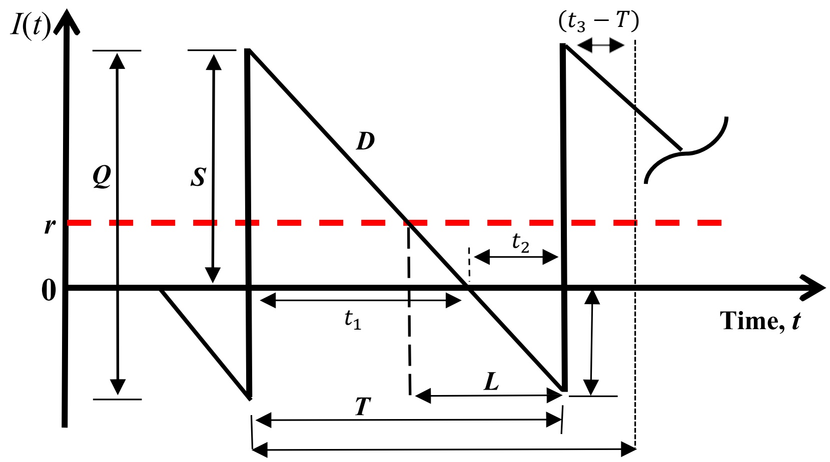

Figure 1. Position pattern of the developed inventory model

Based on the above notation and assumptions, in general, the level of inventory as long as can be explained as follows:

$$\frac{dI(t)}{dt} + \theta I(t) = -D \qquad 0 \le t \le t_1 \tag{1}$$

$$\frac{dI(t)}{dt} = -D \qquad t_1 \le t \le t_2 \tag{2}$$

While,

$t = 0 \;\; \rightarrow \;\; I(t = 0) = S$

$t = t_1 \;\; \rightarrow \;\; I(t = t_1) = 0$

$t = t_2 \;\; \rightarrow \;\; I(t = t_2) = -Q + S$

To differentiate equations (1) and (2), follow the following method:
General equation (Jayashree, 2013):

$$\frac{dy}{dx} + P_y = Q$$

$$ye^{\int pdx} = \int Q\, e^{\int pdx} dx + C$$

$$I(t)e^{\int \theta dt} = -D \int e^{\int \theta dt}\, dt + C$$

$$I(t)e^{\theta t} = -D\frac{e^{\theta t}}{\theta} + C$$

$$I(t) = -\frac{D}{\theta} + Ce^{\theta t} \tag{3}$$

At $t = 0$, the level of supply $I(t = 0) = S$, then based on equation (3), it is obtained

$$I(t = 0) = S \;\; \rightarrow \;\; S = -\frac{D}{\theta} + Ce^{\theta(0)}$$

$$S = -\frac{D}{\theta} + C$$

$$C = \frac{D}{\theta} + S \tag{4}$$

Substitute equation (4) to equation (3):

$$I(t) = -\frac{D}{\theta} + \left(\frac{D}{\theta} + S\right)e^{\theta t}$$

$$I(t) = \left(\frac{D}{\theta} + S\right) e^{\theta t} - \frac{D}{\theta} \qquad \text{at } 0 \le t \le t_1 \tag{5}$$

At $t = t_1$, the level of supply $I(t = t_1) = 0$, then based on equation (2), it is obtained

$$\frac{dI(t)}{dt} = -D$$

$$I(t) = -D \int_{t_1}^{t} dt$$

$$I(t) = -D(t - t_1) \tag{6}$$

Then based on equation (5), it is obtained

$$I(t = t_1) = 0 = \left(\frac{D}{\theta} + S\right) e^{\theta t} - \frac{D}{\theta}$$

$$\frac{D}{\theta} = \left(\frac{S\theta + D}{\theta}\right) e^{-\theta t_1}$$

$$e^{-\theta t_1} = \left(\frac{D}{\theta}\right)\left(\frac{\theta}{S\theta + D}\right)$$

$$\log e^{-\theta t_1} = \log\left(\frac{D}{S\theta + D}\right)$$

$$\theta t_1 = \log\left(\frac{S\theta + D}{D}\right)$$

$$t_1 = \frac{1}{\theta}\left\{\log\left(1 + \frac{S\theta}{D}\right)\right\} \tag{7}$$

Substitute equation (7) above into the general formula for the Maclaurin series as follows:

$$\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$$

$$t_1 = \frac{1}{\theta}\left[\frac{S\theta}{D} - \frac{S^2\theta^2}{2D^2} + \frac{S^3\theta^3}{3D^3} - \cdots\right]$$

The formula used is only up to the second order because the greater the order value, the less significant the result

$$0 = \frac{S\theta}{\theta D} - \frac{S^2\theta^2}{\theta 2D^2}$$

$$\frac{S}{D} = \frac{S^2\theta}{2D^2} \tag{8}$$

At $t = t_2$, the level of supply $I(t = t_2) = -Q + S$, then based on equation (6), it is obtained

$$I(t_2) = I(t)$$

$$-Q + S = -D(t_2 - t_1)$$

$$t_2 = \left(\frac{Q}{D} - \frac{S}{D}\right) + t_1 \tag{9}$$

Substitute equation (8) to equation (9), and take $t_1 = 0$, will get time interval during stockout interval:

$$t_2 = \left(\frac{Q}{D} - \frac{S^2}{2D^2}\right) \tag{10}$$

The objective function of the developed model is to minimize the total annual variable cost (*TC*) which consists of the purchase cost per year (*PC*), the ordering cost per year (*OC*), the holding cost per year (*HC*), the shortage cost of per year (SC), the expired cost per year (*DC*), and to maximize the cost of interest which can be earned per year (B_e).

The total annual variable cost consists of the following elements:

a) The purchase cost per year (*PC*)

$$PC = P.D \tag{11}$$

b) The ordering cost per year (*OC*)

$$OC = A.\frac{D}{Q} \tag{12}$$

c) The holding cost per year (*HC*)

The holding cost per cycle:

$$HC = h\int_0^{t_1} I(t)dt$$

Substitute $I(t)$ from equation (5),

$$HC = h\int_0^{t_1} \left(\left(\frac{D}{\theta}+S\right)e^{\theta t}-\frac{D}{\theta}\right)dt$$

$$HC = h\left\{\left(\frac{D}{\theta}+S\right)\int_0^{t_1} e^{\theta t}dt-\int_0^{t_1}\frac{D}{\theta}dt\right\}$$

$$HC = \frac{h}{\theta}\left\{\left(\frac{D}{\theta}+S\right)\left(1-e^{\theta t_1}\right)-Dt_1\right\}$$

$$HC = \frac{h}{\theta}\{S-Dt_1\}$$

Substitute t_1 from equation (7),

$$HC = \frac{h}{\theta}\left\{S-\frac{D}{\theta}\log\left(1+\frac{S\theta}{D}\right)\right\}$$

$$HC = \frac{h}{\theta}\left\{\frac{S^2\theta}{2D}-\frac{S^3\theta^2}{3D^2}\right\}$$

Thus, holding cost per cycle:

$$HC = h\left\{\frac{S^2}{2D}-\frac{S^3\theta}{3D^2}\right\} \tag{13}$$

The holding cost per year:

$$HC = h\left\{\frac{S^2}{2D}-\frac{S^3\theta}{3D^2}\right\}\frac{D}{Q}$$

$$HC = h\left\{\frac{S^2}{2Q}-\frac{S^3\theta}{3DQ}\right\} \tag{14}$$

d) The shortage cost per year (*SC*)

Shortage cost per cyle:

$$SC = \pi\int_{t_1}^{t_2} -I(t)dt$$

Substitute $I(t)$ from equation (6),

$$SC = \pi\int_{t_1}^{t_2} D(t-t_1)dt$$

$$SC = \pi\left[\frac{Dt_2^2}{2}\right]$$

Substitute t_2 from equation (9),

$$SC = \pi\left[\frac{D}{2}\left(\left(\frac{Q}{D} - \frac{S}{D}\right) + t_1\right)^2\right]$$

$$SC = \pi\left[\frac{D}{2}\left(\frac{(Q-S)^2}{D^2} + \frac{2(Q-S)}{D}t_1 + t_1^2\right)\right]$$

Thus, shortage cost per cycle is

$$SC = \pi\left[\frac{(Q-S)^2}{2D}\right] \tag{15}$$

The shortage cost per year is

$$SC = \pi\left[\frac{(Q-S)^2}{2D}\right]\frac{D}{Q}$$

$$SC = \pi\left[\frac{(Q-S)^2}{2Q}\right] \tag{16}$$

e) The expired cost per year (*DC*)

Expired cost per cycle:

$$DC = d.S - \int_0^{t_1} D dt$$

$$DC = d.S - Dt_1$$

Substitute t_1 from equation (7),

$$DC = d.S - \frac{D}{\theta}\left\{\log\left(1 + \frac{S\theta}{D}\right)\right\}$$

$$DC = \frac{d\theta S^2}{2D} \tag{17}$$

The expired cost per year is

$$DC = \frac{d\theta S^2}{2Q} \tag{18}$$

f) The income of interest which can be earned per year (B_e)

$T < t$ case:

According to Goyal (1985) that when items are sold, and before the replenishment account is settled, the sales revenue is used to earn interest.

Income of interest earned during t_3 is income of interest earned during T plus income of interest earned during $(t_3 - T)$.

B_e = {average income during period T + average income during period $(t_3 - T)$} × interest which can be earned

$$B_e = \left\{\frac{DT^2}{2} + DT(t_3 - T)\right\}P.I_e$$

Income of interest earned during t_3 is

$$B_e = DTPI_e\left\{t_3 - \frac{T}{2}\right\} \tag{19}$$

The income of interest which can be earned per year is

$$B_e = DPI_e\left\{t_3 - \frac{T}{2}\right\} \tag{20}$$

If $T = \frac{Q}{D}$,

$$B_e = DPI_e\left\{t_3 - \frac{Q}{2D}\right\} \tag{21}$$

The total annual variable cost (*TC*):

$TC = PC + OC + HC + SC + DC - B_e$

$$TC = PD + \frac{AD}{Q} + h\left\{\frac{S^2}{2Q} - \frac{S^3\theta}{3DQ}\right\} + \pi\left[\frac{(Q-S)^2}{2Q}\right] + \frac{d\theta S^2}{2Q} - DPI_e\left\{t_3 - \frac{Q}{2D}\right\} \quad (22)$$

To determine the optimal order quantity and the optimal time between orders, equation (22) is derived respectively with respect to *Q* and *T* such that $\frac{\partial TC}{\partial Q} = 0$ and $\frac{\partial TC}{\partial T} = 0$.

Economic order quantity per order is

$$Q^*(PI_eQ^* - 2\pi) = 2AD + hS^2 - \frac{2hS^3}{3D} - 2\pi S + d\theta S^2 \quad (23)$$

Economic order interval per cycle is

$$T^* = \sqrt{\frac{2A + \frac{hS^2}{D} - \frac{2hS^3\theta}{3D^2} + \frac{2\pi(Q^*-S)D}{D} + \frac{d\theta S^2}{D}}{DPI_e}} \quad (24)$$

The solving of problem using heuristic approach by trial and error. By solving (23), the optimal of order quantity (Q^*) can be obtained,and with the use of this optimal value, the optimal of order interval (T^*) can be obtained by equation (24).

4. Results and Discussion

4.1 Numerical Results

In this section, numerical examples were given to illustrate the proposed model and its solution procedure. Numerical examples based on the existing problems in the Chickenpedia's central warehouse. The parameters are as follows: Suppose $D = 2500$, $S = 100$, $A = 50$, $P = 14.385$, $\pi = 50$, $h = 100$, $d = 50$, $I_e = 0.15$, $t_3 = 0.11$, and $\theta = 0.01$.
To determine the optimal ordering quantity Q^* using equation (23) in such a way that the calculation result on the right side must be the same as the calculation result on the left side by trial and error.
The calculation results obtained from the above case are the optimal value Q^* is used as input to find the optimal time interval between orders T^* by equation (24). The results obtained are $Q^* = 577$ unit, $T^* = 5$ weeks, and total annual variable cost, $TC = 35.633.600$.

4.2. Proposed Improvements

This model is a simple model of the real conditions in Chickenpedia's central warehouse which still uses many assumptions. That is why, there are still many weaknesses in this model. The development for further research can consider several aspects, including the fact that the rate of demand is a function of price, discount from suppliers, deteriorated rates that are not constant, multi item that have different expiration dates.

4.3. Validation

Based on the results of validation through the implementation of the data model, it is proven that the model is valid against the real system in place of the research object. Model validation uses cases in Chickenpedia's central warehouse with data for January - December 2020, if there is a shortage of inventory, it will be fulfilled in the future order period (backorder). According to the agreement between the warehouse and the supplier that the allowable payment period is t_3 which $t_3 > T$. Based on the calculation results, the Q optimal value is 577 units and the the optimal of order interval value is 0,094 years or 5 weeks, and the total cost of inventory per year (TC) incurred was 35.633.600.

In addition, a sensitivity analysis was carried out on the model that has been developed on several parameters they are in the total inventory cost of -20% to + 50%. Sensitivity analysis is carried out to see the effect that will occur on the value of the decision variable and total annual cost if these parameters are changed in value. The parameters used to perform the sensitivity analysis are $\theta, \pi, A, h,$ and d. The results of the calculation of the sensitivity analysis can be seen in Table 1 and Table 2. The results obtained in Table 1 show that with changes in the parameters $\theta, \pi, A, h,$ and d, the value of the decision variables and the total cost of inventory did not change significantly. Even by changing the two parameters at once (Table 2), the result is that the model remains in a valid condition and is not sensitive to these changes.

Table 1. Sensitivity analysis on changes of one parameter

Parameter	Δ %	Parameter value	Q^*	T^*	TC
θ	-50%	0,005	576	0,094	35.633.575,51
	-20%	0,008	577	0,094	35.633.609,10
	20%	0,012	578	0,094	35.633.623,12
	50%	0,015	578	0,094	35.633.611,38
π	-50%	25	580	0,06	35.536.997,16
	-20%	40	578	0,08	35.593.885,17
	20%	60	576	0,10	35.651.727,50
	50%	75	575	0,11	35.681.615,10
A	-50%	25	519	0,08	35.594.438,80
	-20%	40	554	0,09	35.622.254,18
	20%	60	600	0,09	35.623.369,57
	50%	75	635	0,10	35.651.192,59
h	-50%	50	346	0,06	35.536.413,38
	-20%	80	485	0,08	35.593.621,61
	20%	120	670	0,10	35.652.046,20
	50%	150	809	0,11	35.682.427,82
d	-50%	25	576	0,09	35.622.786,64
	-20%	40	577	0,09	35.622.810,30
	20%	60	578	0,09	35.622.834,42
	50%	75	579	0,09	35.622.858,07

Table 2. Sensitivity analysis on changes of two parameter

Parameter	Δ_1 %	Δ_2 %	Q^*	T^*	TC
θ and π	$\theta = -50\%$	$\pi = -50\%$	577	0,06	35.536.964,36
		$\pi = -20\%$	577	0,08	35.593.865,60
		$\pi = 20\%$	575	0,10	35.651.698,24
		$\pi = 50\%$	573	0,11	35.681.544,14
	$\theta = -20\%$	$\pi = -50\%$	579	0,06	35.536.986,08
		$\pi = -20\%$	578	0,08	35.593.884,36
		$\pi = 20\%$	576	0,10	35.651.726,68
		$\pi = 50\%$	575	0,11	35.681.614,28
	$\theta = 20\%$	$\pi = -50\%$	580	0,06	35.536.997,98
		$\pi = -20\%$	579	0,08	35.593.903,75
		$\pi = 20\%$	577	0,10	35.651.755,52
		$\pi = 50\%$	575	0,11	35.681.615,93
	$\theta = 50\%$	$\pi = -50\%$	581	0,06	35.537.009,47
		$\pi = -20\%$	579	0,08	35.593.904,73
		$\pi = 20\%$	577	0,10	35.651.756,75
		$\pi = 50\%$	576	0,11	35.681.651,64
θ and A	$\theta = -50\%$	$A = -50\%$	518	0,08	35.594.414,55
		$A = -20\%$	553	0,09	35.622.229,82
		$A = 20\%$	599	0,09	35.623.345,08
		$A = 50\%$	634	0,10	35.651.168,07
	$\theta = -20\%$	$A = -50\%$	519	0,08	35.594.437,90
		$A = -20\%$	554	0,09	35.622.253,32
		$A = 20\%$	600	0,09	35.623.368,78
		$A = 50\%$	608	0,09	35.623.370,15
	$\theta = 20\%$	$A = -50\%$	520	0,08	35.594.461,68
		$A = -20\%$	555	0,09	35.622.277,27
		$A = 20\%$	600	0,09	35.623.370,37

		$A = 50\%$	636	0,10	35.651.216,01
	$\theta = 50\%$	$A = -50\%$	521	0,08	35.594.485,05
		$A = -20\%$	555	0,09	35.622.278,55
		$A = 20\%$	602	0,09	35.623.416,51
		$A = 50\%$	636	0,10	35.651.217,13
θ and h	$\theta = -50\%$	$\pi = -50\%$	345	0,06	35.536.390,09
		$\pi = -20\%$	484	0,08	35.593.597,44
		$\pi = 20\%$	669	0,10	35.652.021,63
		$\pi = 50\%$	808	0,11	35.682.403,10
	$\theta = -20\%$	$h = -50\%$	345	0,06	35.536.392,21
		$h = -20\%$	484	0,08	35.593.598,92
		$h = 20\%$	670	0,10	35.652.045,50
		$h = 50\%$	808	0,11	35.682.403,96
	$\theta = 20\%$	$h = -50\%$	346	0,06	35.536.414,79
		$h = -20\%$	485	0,08	35.593.622,60
		$h = 20\%$	670	0,10	35.652.046,89
		$h = 50\%$	809	0,11	35.682.428,38
	$\theta = 50\%$	$h = -50\%$	347	0,06	35.536.436,66
		$h = -20\%$	486	0,08	35.593.641,77
		$h = 20\%$	671	0,10	35.652.070,75
		$h = 50\%$	810	0,11	35.682.452,51
θ and d	$\theta = -50\%$	$d = -50\%$	576	0,09	35.622.785,67
		$d = -20\%$	576	0,09	35.622.786,32
		$d = 20\%$	576	0,09	35.622.787,19
		$d = 50\%$	577	0,09	35.622.810,20
	$\theta = -20\%$	$d = -50\%$	576	0,09	35.622.786,25
		$d = -20\%$	577	0,09	35.622.809,66
		$d = 20\%$	577	0,09	35.622.811,04
		$d = 50\%$	578	0,09	35.622.811,04
	$\theta = 20\%$	$d = -50\%$	576	0,09	35.621.956,95
		$d = -20\%$	577	0,09	35.622.809,39
		$d = 20\%$	578	0,09	35.622.865,40
		$d = 50\%$	579	0,09	35.622.859,32
	$\theta = 50\%$	$d = -50\%$	589	0,09	35.623.078,02
		$d = -20\%$	587	0,09	35.623.036,01
		$d = 20\%$	578	0,09	35.623.836,89
		$d = 50\%$	580	0,09	35.623.883,57
π and A	$\pi = -50\%$	$A = -50\%$	522	0,08	35.590.240,32
		$A = -20\%$	556	0,09	35.617.623.83
		$A = 20\%$	603	0,06	35.537.276,64
		$A = 50\%$	638	0,07	35.564.674,07
	$\pi = -20\%$	$A = -50\%$	520	0,07	35.565.783,15
		$A = -20\%$	555	0,08	35.593.439,45
		$A = 20\%$	601	0,08	35.594.331,97
		$A = 50\%$	636	0,08	35.595.012,90
	$\pi = 20\%$	$A = -50\%$	519	0,09	35.623.102,02
		$A = -20\%$	553	0,10	35.651.059,25
		$A = 20\%$	600	0,10	35.652.464,78
		$A = 50\%$	634	0,10	35.653.458,79
	$\pi = 50\%$	$A = -50\%$	517	0,10	35.652.582,89
		$A = -20\%$	552	0,11	35.680.779,93
		$A = 20\%$	598	0,11	35.682.452,37
		$A = 50\%$	633	0,12	35.710.735,84
π and h	$\pi = -50\%$	$h = -50\%$	348	0,04	35.480.300,01
		$h = -20\%$	487	0,06	35.535.877,10

		$h = 20\%$	672	0,07	35.565.090,23
		$h = 50\%$	811	0,08	35.593.767,12
	$\pi = -20\%$	$h = -50\%$	347	0,06	35.535.554,06
		$h = -20\%$	486	0,07	35.565.138,56
		$h = 20\%$	671	0,09	35.622.891,18
		$h = 50\%$	810	0,10	35.652.367,48
	$\pi = 20\%$	$h = -50\%$	345	0,06	35.535.523,7
		$h = -20\%$	484	0,07	35.565.104,74
		$h = 20\%$	669	0,11	35.681.415
		$h = 50\%$	808	0,12	35.712.506,49
	$\pi = 50\%$	$h = -50\%$	343	0,08	35.592.449,82
		$h = -20\%$	482	0,10	35.651.284,55
		$h = 20\%$	668	0,12	35.711.981,32
		$h = 50\%$	807	0,14	35.771.039,15
π and d	$\pi = -50\%$	$d = -50\%$	579	0,06	35.536.984,62
		$d = -20\%$	579	0,06	35.536.985,92
		$d = 20\%$	580	0,06	35.536.997,91
		$d = 50\%$	581	0,06	35.537.009,47
	$\pi = -20\%$	$d = -50\%$	577	0,08	35.593.865,36
		$d = -20\%$	578	0,08	35.593.884,19
		$d = 20\%$	579	0,08	35.593.903,44
		$d = 50\%$	579	0,08	35.593.904,73
	$\pi = 20\%$	$d = -50\%$	575	0,10	35.651.698
		$d = -20\%$	576	0,10	35.651.726,51
		$d = 20\%$	577	0,10	35.651.755,45
		$d = 50\%$	578	0,10	35.651.783,96
	$\pi = 50\%$	$d = -50\%$	574	0,11	35.681.578,36
		$d = -20\%$	575	0,11	35.681.614,12
		$d = 20\%$	575	0,11	35.681.615,86
		$d = 50\%$	576	0,11	35.681.651,64

4.2. Proposed Improvements

This model is a simple model of the real conditions in Chickenpedia's central warehouse which still uses many assumptions. That is why, there are still many weaknesses in this model. The development for further research can consider several aspects, including the fact that the rate of demand is a function of price, discount from suppliers, deteriorated rates that are not constant, multi item that have different expiration dates.

5. Conclusion

Inventory model is developed base on the existing problems in the Chickenpedia's central warehouse considering factors deterioration, variable holding cost to time, and permissible delay in payment. Demand rate and deterioration rate are constant. Validation of the model using case T < t and product data in January to December of 2020 and other inventory costs. Solution model used a heuristic approach by trial and error. This model produces inventory policy in terms of how the optimal order quantity and when the optimal order interval is made by considering the minimum total annual variable costs and maximum revenue due to allowing delaying payments.

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