# Some Forms of Single Stage Queuing Systems Resulted from The Decomposition of Complex Supply Chain

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## Abstract

The complexity of today's supply chain systems has increased as a result of technological breakthroughs and the changing nature of consumer demand. It's getting more difficult to solve supply chain issues. Appropriate supply chain modeling is needed for supply network analysis. Experts have proposed a number of different modeling approaches for the supply chain system. Supply chain system performance models can be in the form of discrete simulation or mathematical models based on the concept of queuing theory. Although it is an approach technique, the decomposition method will facilitate the completion of complex system analysis. The results of the decomposition will bring up various forms of single stage queue subsystems. In this paper, a qualitative overview is expressed on this topic in the hope that it will provide benefits in providing a description of how a complex supply chain can be modeled with a simple method. An example of a complex supply chain system consisting of two suppliers, one factory, two distribution centers, each of which has two channels and one retailer channel, is used as a case in this paper. This paper will summarize several single queuing systems as a result of the decomposition of complex supply chain systems.

## Keywords
Performance, models, supply chains, multi-channel, decomposition

## 1. Introduction
Business growth in the global sphere requires a good competitive climate. The current scope of competition involving competition between supply chains makes the competitive climate more complex (Lambert and Cooper 2000). On the other hand, people's economic growth can increase with collaboration between supply chains (Barratt 2004, Sulistyono et al. 2021). Over the passage of time, triggered by increasingly high industrial challenges, industry players are increasingly aware of the importance of collaboration between supply chains. A supply chain management emerged based on broader systemic insights to obtain a win-win solution between chains in the system. Thus, the efficiency and/or effectiveness of product distribution to end consumers can be achieved as much as possible.

Various forms of modeling efforts on the supply chain system have been put forward by experts and generally divided into two types: the optimization model and the performance model (Altiok 1997). Optimization model is used when a configuration of control variable values for a supply chain system is desired to be known optimally. Meanwhile, the performance model is more intended for the study of the behavior of the system. Supply chain system performance models can be in the form of discrete simulation models, or mathematical models based on the concept of queuing theory. In the case of supply chain systems, some of these models include the work of Karaman and Altiok (2009), Saetta et al. (2012), and Murdapa et al. (2020). The performance model revealed is straight (unbranched) multi-echelon model.

The difficulty of analyzing supply chain performance is proportional to the system's complexity. Mostly, as result, the more complicated the supply chain is, the more challenging it is to manage its performance. The adoption of performance model equality as a response to difficulties is a viable solution. Production line analysis systems are similar to supply chain performance systems. If the supply chain system consists of a series of echelons (supplier-factory-distribution center-retailer), the production line system consists of stations. This production line system is analogous to the queuing network system. This queuing network system was analyzed by the decomposition method. Performance models for the production line system have been revealed long ago by Gershwin (1994), Buzacott and Santhikumar (1993), Altiok (1997), Dallery and Gershwin (1992), Liberopoulos et al. (2006).

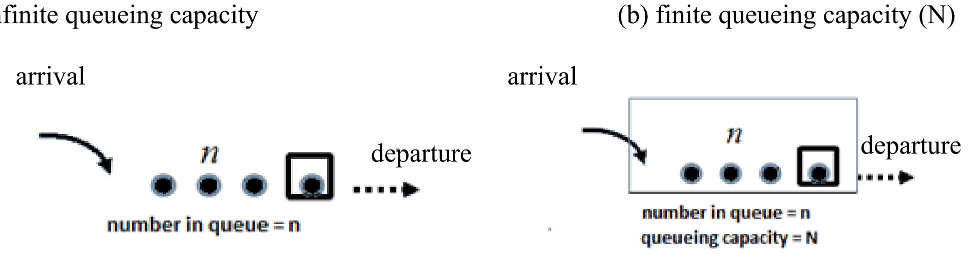



Figure 1. Two types of single stage queueing system (Source: Murdapa, 2019)

At first glance, this decomposition method is done by breaking down the whole system into several subsystems that are single queue systems. This single queuing system is simple enough so that its model can be handled with relatively simple as well. Figure 1 shows a single queuing system in general, which consists of two types of systems, i.e., queuing systems with infinite and finite capacity. A complex supply chain system that supplies a certain (assumed single) product generally consists of several suppliers, one factory, several distribution centers, and several retailers. The supply chain system can be described conceptually as in Figure 2 (Karaman and Altiok 2009, Murdapa 2020). Then, this paper will summarize several single queuing systems as a result of the decomposition of complex supply chain systems.

## 2. System Description and Modeling

Figure 2 is a conceptual diagram of a complex supply chain system that will be an example of a study in this paper. Here, a supply chain system consists of two different suppliers, one factory that produces a particular single product, and two different distribution centers, each of which has two retailers and one retailer. The diagram is composed by the symbol O (circle) representing an echelon (supplier, production, distribution center, and retailer) and the symbol ∇ (inverted delta) representing the warehouse (SP Stock, RM Stock, FG Stock, RT Stock). Each echelon in the system (except for the production echelon) has a certain supply rate ($\mu_i$), which is approximately equal to the shipping lot size of the echelon, divided by its cycle time. In this case, the supplier always has materials on hand so that the cycle time of delivery to the factory is practically the same as the time of transportation from the supplier to the factory. The same is true for cycle times at distribution centers and retailers. However, as for the production echelon, the formulation of cycle times will be slightly different

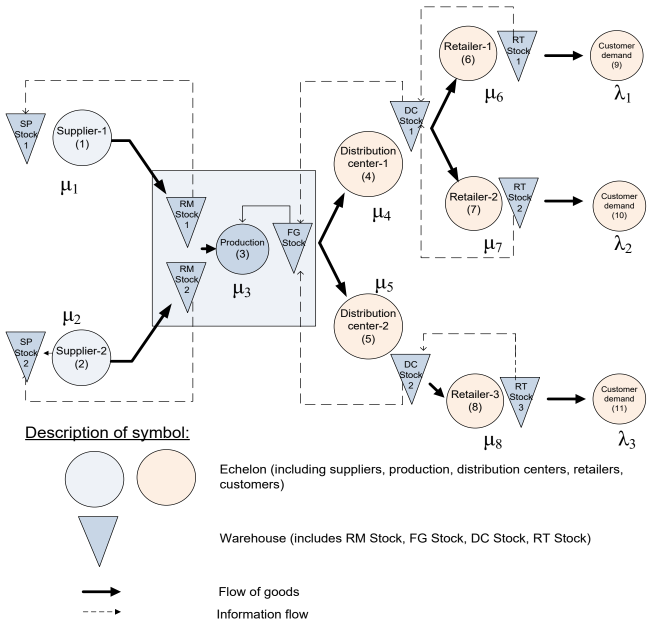



Figure 2. Conceptual diagram of a complex supply chain system

When an echelon sees that the inventory level in the warehouse has reached the reorder point (R), then an order to the echelon predecessor is immediately carried out. The order quantity is Q units. This is called the (R,Q)-mechanism, which will be assumed to be used in this paper. Besides this mechanism, other mechanisms are available such as Kanban (see Mascollo et al. 1996, Matta et al. 2005). In Figure 2, it is assumed that all echelons follow this mechanism, illustrated by a dashed line that leads from the echelon warehouse to the predecessor echelon warehouse. One exception is, in the FG Stock warehouse which uses the (r,R) mechanism. With this mechanism, when the inventory level on the FG Stock reaches r units (called reproduction points), the production echelon will immediately start production activities at the rate, say, p units per unit time, and stop when the inventory level has reached R units. Meanwhile, orders from end consumers to retailers occur at an average rate of $\lambda_i$ units per unit time. A simple model is generally taken with the assumption that the rate is a Poisson process. Hence, the conceptual diagram of the supply chain system in Figure 2 assumes that product fulfillment in each echelon occurs by make-to-stock (or rather deliver-to-stock). All requests in each echelon are fulfilled from the available inventory. Except for production echelons and retailers whose lot size is one, fulfillment of demand with lot size $Q_i$.

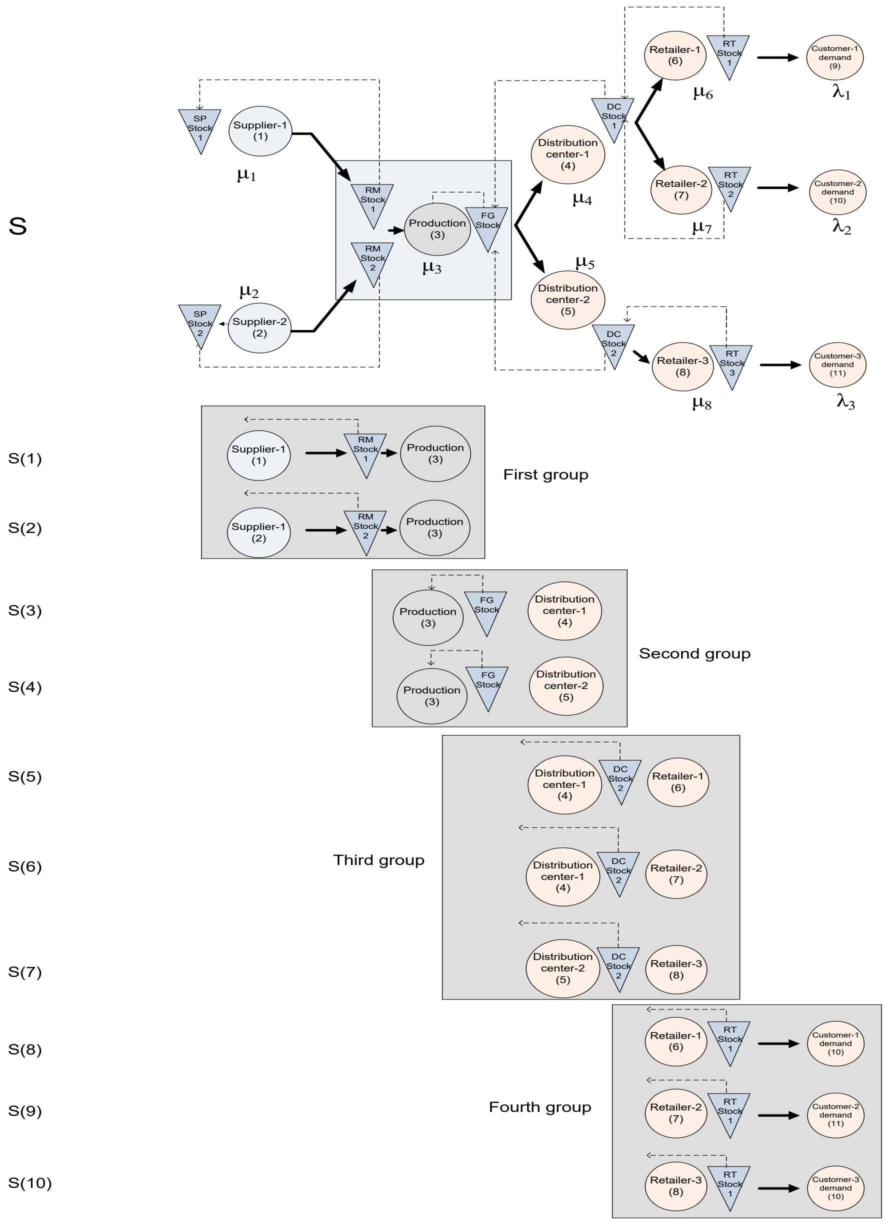



Figure 3. Conceptual system decomposition using the Dallery-Frein method

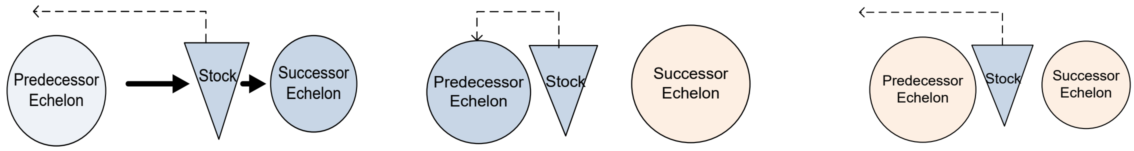
# 3. Decomposition Process

Dallery and Frein (1993) described a decomposition technique (Gershwin's decomposition) which is imposed on the case of a production line system (the flowline, to be precise). Meanwhile, Karaman and Altiok (2009), Saetta et al. (2012), and Murdapa (2019) revealed a straight supply chain system model in which mechanisms (R,Q) and production mechanisms (r,R) were applied. If the concepts of decomposition and mechanism (R,Q) and (r,R) are applied, then Figure 3 will be obtained (Murdapa 2019). The Dalley-Frein decomposition concept is carried out by "decomposing" the original system into several single queuing subsystems consisting of echelon - warehouse - echelon. In this case, what is meant by echelon in the subsystem is actually an aggregate echelon (Dallery and Frein 1993). This method is called decomposition because if all alternative aggregations are displayed, a total configuration that looks like decomposition will be obtained.

Seen in Figure 3, As seen in Figure 3, the original system, S, is decomposed into ten subsystems, namely, subsystems S (1), S (2), S (3), S (4), S (5), S (6), S (7), S (8), S (9), S (10), each of which is a single queuing system consisting of two aggregate echelons flanking one warehouse. The presence of dash lines and the size of the shipping lot size will distinguish the characteristics of the subsystem. The same position on the original system will give the same subsystem character. Thus, the ten subsystems in Figure 3 above can be categorized into four groups. The first group consists of S (1) and S (2). The second group consists of S (3) and S (4). The third group consists of S (5), S (6), S (7) and the last group, which is the fourth group, consists of S (8), S (9), S (10). However, some groups have the same characteristics even though the context is different. For example, the first group is a subsystem where the procurement lot is $Q_i$ (ordering raw materials, with *i* = 1, 2) and the shipping lot (to production) is one unit. This is exactly the same as the fourth group (i.e., the size of the shipping lot to the final customer is one unit).

The third group is different from the first group or the fourth group although it has a diagram that looks exactly the same. The difference is in the size of the shipping lot. Both procurement and shipping in this third group are above one unit (batch processing). Meanwhile, the second group is different from the other three groups. In this second group, the procurement lot size (production) is one unit with a mechanism (r,R) and the shipping lot size is $Q_i$ (the product shipping to the distribution center, with *i* = 3, 4). Thus, in net terms, there will be three groups of types of single queuing systems which are distinguished by procurement mechanisms and the amount of lot size (procurement, or delivery). Figure 4 visually displays the three groups of single queuing types.

(a) Group A (b) Group B (c) Group C




Procurement lotsize: $Q_1$ units
Shipping lotsize: 1 unit

Procurement lotsize: 1 unit
Shipping lotsize: $Q_2$ units

Procurement lotsize: $Q_{3a}$ units
Shipping lotsize: $Q_{3b}$ units

Figure 4. Three groups of single queue systems distinguished by procurement mechanisms, and procurement/shipping lotsizes

# 4. Discussion

The three groups of single queuing systems in Figure 4 can be studied with a model that can simply be arranged based on the Markovian principle. This principle is derived from the assumption that all activities (production, transportation) take place in exponentially distributed duration. More complex (also Markovian) assumptions have been made by Karaman and Altiok (2009) and Saetta et al. (2012), where the aggregate duration of activity will follow a phase-type distribution. One form of phase-type distribution is the Erlang distribution which is a series of processes in which the duration of time is exponentially distributed.

Exponential assumptions will greatly simplify the problem, but from here it will be able to obtain an easy general picture, which will be very useful for people as a first step to understanding the problem. With the help of a state transition diagram, state transition equations can be arranged when the steady state is reached. Transition diagrams are arranged to include lot sizes according to the context of the original system behavior. Then, a suitable analytical formula for each of these single stage subsystems can be obtained. But the formula must then be reassembled into a unity with the decomposition equations. For this purpose, Dallery and Frein (1993) revealed very useful concepts to be applied.

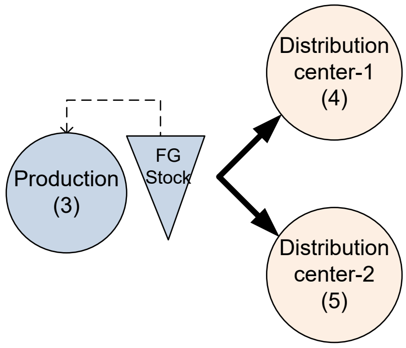



Figure 5. Letting the branching subsystem as a multi server single stage system

There is one last thing to note in the discussion of group B's single stage queuing system (Figure 4). Compared with the case in the decomposition of Helber (2000), the difference is at the branching point, either at echelon or warehouse. In the supply chain system presented in this paper, branching occurred in the warehouse (FG Stock). Hence, instead of decomposing the system into two subsystems in group B, it would actually be more profitable if the analyst kept leaving the subsystem in a branched form (Figure 5). Murdapa et al. (2019) used this concept to solve two channel problems.

## 5. Conclusion

In real conditions, supply chain systems are more often composed by several suppliers. Each of them supplies a portion of raw materials to a factory to be processed into products that will be needed by the final consumer. To get to the hands of consumers, the products are then distributed by the factory to several distribution centers and to several retailers. Thus, the supply chain will have a branching form. The conceptual diagram of the supply chain system can be arranged so that the configuration will repeatedly take the form echelon-warehouse-echelon.

Each warehouse is assumed to use a mechanism (R,Q) in the recovery of its stock of goods, except for finished product warehouse. It relates to the production echelon that use the (r,R) mechanism. The decomposition technique can then be applied for easy analysis of the supply chain system. The decomposition produces simple subsystems in various characteristics, depending on the relative position in the system and the size of the procurement and shipping lot. The various forms of the single stage queuing system can be analyzed simply if the Markovian concept is assumed in it.

## References

Lambert, D.M., and Cooper, M.C., Issue in Supply Chain Management, *Industrial Marketing Management*, vol. 29, no. 1, pp.65-83, 2000.

Barratt, M., Understanding the meaning of collaboration in the supply chain, *Supply Chain Management*, vol. 9, no. 1, pp. 30-42, 2004.

Sulistyono, S.W., Suliswanto, M.S.W., Dewa, P.K., Santosa, S., Astina, C., Revenue optimization strategy through digitizing retribution parking in kota Batu, *Journal of Revenue and Pricing Management*, 2021

Altiok, T., *Performance analysis of manufacturing systems*, Springer, New York, 1997.

Karaman, A., and Altiok, T., Approximate analysis and optimization of batch ordering policies in capacitated supply chains, *European Journal of Operational Research*, vol. 193, no. 1, pp. 222-237, 2009.

Saetta, S., Paolini, L., Tiacci, L., and Altiok, T., A decomposition approach for the performance analysis of a serial multi-echelon supply chain, *International Journal of Production Research*, vol. 50, no. 9, pp. 2380-2395, 2012.

Murdapa, P. S., Pujawan, I. N., Karningsih, P.D., and Nasution, A.H., Incorporating carbon emissions in queuing models to determine lot sizes and inventory buffers in a supply chain, *International Journal of Intelligent Enterprise*, vol. 7, no. 4, pp. 373-390, 2020.

Gershwin, S.B., *Manufacturing systems engineering,* NJ: Prentice Hall, 1994.

Buzacott, J.A, and Shanthikumar, J.G., *Stochastic models of manufacturing systems*, NJ: Prentice Hall, 1993.

Dallery, Y., and Gershwin, S.B., Manufacturing flow line systems: a review of models and analytical results, *Queueing systems*, vol.12, pp. 3-94, 1992.

Liberopoulos, G., Papadopoulos, C.T., Tan, B., Smith, J.M., and Gershwin, S.B., *Stochastic Modeling of Manufacturing Systems: Advances in Design, Performance Evaluation, and Control Issues,* Springer-Verlag, Berlin, 2006.

Murdapa, P.S., *Performance Model of supply chain system that involves carbon emission variable*, PhD Dissertation Industrial Engineering Department ITS Indonesia, 2019.

Mascolo, M.D., Frein, Y., and Dallery, Y., An analytical method for performance evaluation of kanban controlled production systems, *Operations Research*, vol. 44, no. 1, pp. 50-64, 1996.

Matta, A., Dallery, Y., and Mascolo, M., Analysis of assembly systems controlled with kanbans, *European Journal of Operational Research*, vol. 166, no. 2, pp. 310–336, 2005.

Dallery, Y., and Frein, Y., On decomposition methods for tandem queueing networks with blocking, *Operations Research*, vol. 41, no. 2, pp. 386–399, 1993.

Helber, S., Approximate analysis of unreliable transfer lines with splits in the flow of material, *Annals of Operations Research*, vol. 93, no. 1-4, pp. 217-243, 2000.

Murdapa, P.S., Pujawan, I.N., Karningsih, P.D., and Nasution, A.H., A numerical analysis model involving carbon emissions in a single echelon supply chain systems with two distribution channels: proposed preliminary model, *Proceeding of 2019 IEEE International Conference on Industrial Engineering and Applications, ICIEA 2019*, pp. 748-752, 2019.

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