A Genetic Algorithm Based Approach for the Maritime Inventory Routing Problem

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Abstract

The maritime inventory routing problem (MIRP) aims at satisfying the demands at different ports at a minimum cost during the planning horizon. In MIRP, a heterogeneous fleet of bulk ships with undedicated compartments is used to transport multiple non-mixable products from a production port to consumption ports located in several islands. Inventory constraints are present both at the factory and the silos, and there are upper and lower limits on the inventories. Besides, there are capacity constraints on the capacity of the ship compartments and the depth of ports. The problem's objective is to find a solution that minimized transportation costs while satisfying several technical and physical constraints within a given planning horizon. To solve MIRP, we propose a genetic algorithm (GA). The proposed GA is tested on several instances adopted from real-world problems with up to 6 consumption ports and 5 ships. The results indicate that the proposed GA effectively solves MIRP.

Keywords
Inventory Routing Problem, Maritime Transportation, Undedicated Compartment, Genetic Algorithm

1. Introduction
Inventory routing problem (IRP) is a combination and coordination problem between vehicle routing and inventory management. The maritime inventory routing problem is one IRP variation that applies in a maritime field or scope. Maritime transportation is the most significant transportation mode concerning large volumes and is responsible for over 80% of world trade volume. In 2018, the total volumes grew 2.7% and down from 4.1% in 2017. The slowdown was wide-ranging and affected almost all sea cargo segments (UNCTAD, 2019). In this situation, increasing the integration between the various stakeholders within the supply chain, reducing operational costs, and enhancing efficiency become necessary (Sanghikian, 2020). The importance of an adequate routing of these ships is highlighted to guarantee the efficient use of the maritime fleet, meeting deadlines, and competitive costs (Christiansen and Fagerholt, 2009). Thus, optimization strategies are critical in this sector since maritime logistics is a capital-intensive industry, where a relatively small improvement in its operation can provide a significant economy (Agra et al., 2013).

The optimization tools can be utilized so that various aspects within maritime transportation could be greater in impacts, such as cost reductions as well as a fleet or vehicle utilization.

In this paper, the focus of research will be on a definite problem of maritime transportation that deals with IRP at ports, known as the Maritime Inventory Routing Problem (MIRP). The problem is based on a real business case encountered by a national cement company in the Southeast Asia region which new players have succeeded in gaining 14% of the national cement market share (Indonesia Cement Association, 2020). As we know that cement is a cheap but heavy product. Therefore, its transportation costs are higher than that of other industries. It also means that transportation has a significant impact on the cement industry's bottom line and should increase its profits. The company has several cement grades produced in a production port that cannot be mixed and a consumption port set. Therefore, the products have to be treated separately in the ship's compartment and storage. Several heterogeneous ships in terms of capacity, time, number of compartments, and specific ship characteristics are utilized to distribute the products to the consumption ports during a given planning horizon. Hence one must decide for each ship is as follows: (i) the routing, (ii) the scheduling: times when it will travel between ports on the route, and the time at which loading and unloading operation will take place, and (iii) the amount to be loaded and discharged in each port visit. The capacity of the ship and port's storage should be taken into account. The goal of this problem is to minimize transportation costs during the given planning horizon. This study works the MIRP out by using a Mixed Integer Linear Programming (MILP) approach. Since MIRP is NP-hard, the heuristic approach is both an effective and efficient alternative to dig out the solution to solve such challenging problems. In this study, we develop a genetic algorithm (GA) to solve MIRP. The validation of methodology will be carried out by comparing it against the MILP solution that used CPLEX.

1.1 Objectives

The objective of this paper is threefold. First, we describe, model, and solve the MIRP exactly using MILP. We take some important things into account, such as a set of consumption ports with several products, the number of vehicles, and planning horizons size so that influence of every parameter can be calculated appropriately. Second, we develop a heuristic solution to solve the MIRP, GA, to get feasible solutions in which mathematical models cannot find an optimal solution with acceptable computational time. In the last contribution, we implement an instance that is adopted from a real-world problem to evaluate our algorithm. The testbed proposal is formulated to encompass a set of combinations regarding the number of consumption ports and the planning horizon length, ranging from relatively small and simple instances to very large and challenging ones.

2. Literature Review

2.1 Inventory Routing Problem

Several studies related to IRP have been conducted for the past over thirty years. The study itself was rooted in Bell et al. (1983) paper that proposes a method that integrates decisions for handling inventory, vehicle routing, and gas distribution for chemical products. IRP has been carried out several times to mine solutions for most maritime logistics problems. Literature reviews are provided in Christiansen (1999), Christiansen and Nygreen (1998a, b), Christiansen et al. (2013), Stålhane *et al.* (2012), Siswanto et al. (2011a,b), and Siswanto et al. (2019). The problems described in Christiansen (1999) and Christiansen and Nygreen (1998a, b) involve a many-to-many structure. Stålhane et al. (2012) solve a large-scale of IRP considering multiple products and heterogeneous ships for liquefied natural gas with direct deliveries to maximize the revenue generated. Siswanto et al. (2011a,b) develop a MIP and multi-heuristics to find a solution to maritime oil transportation with an undedicated ship's compartment. Siswanto et al. (2019) extend their previous research by reckoning the time window. Al-Khayyal & Hwang (2007), Grønhaug and Christiansen (2009), Song and Furman (2013) conducted other studies in the oil and gas industries, whereas Dauzère-Pérès et al., (2007) and Miller (1987) had solved problems arising in the chemical components industry.

Over the last three decades, many variants of the IRP have been described and could not have a standard version of the problem. Coelho et al. (2014) categorized the IRP into seven criteria, as seen in Table 1. Unfortunately, the optimal solution to a real problem is currently unreachable regardless of what type and characteristics of the IRP are used, which is caused by the complexity of the problem. A survey about the industrial aspects of combined inventory management and routing performed by Andersson et al. (2010) shows that only small IRP instances have optimal solutions. Most researchers introduce heuristics approaches to solve such problems. Therefore, we use a heuristic approach in our research to solve a problem on a larger scale.

2.2 The Use of Metaheuristic Approach

The metaheuristic methods applied to solve MIRP, namely heuristic based on Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Two-Stage Stochastic (TSS), Cuckoo Optimization Algorithm (COA), and so on. GA or a combination with other methods has been widely used to solve IRP. GA approach method is an effective approach and can produce close to optimal results on IRP. The variation of the multiproduct version with multiple suppliers but only one customer was researched by Moin et al. (2011). They extracted lower and upper bounds from the linear mathematical model and then calculated the better upper bounds using GA. Abdelmaguid et al. (2009) also use GA to answer the inventory problem with backlogging. GA-Taguchi Design combination method was used as a method in Azadeh et al. (2017). Taguchi Design in Azadeh et al. (2017) will produce optimal results by combining it with GA method.

Table 1. Structural variant of the IRP

Criteria	Alternatives		
Time Horizon	Finite	Infinite	
Structure	One-to-one	One-to-many	Many-to-many
Routing	Direct	Multiple	Continuous
Inventory Policy	Maximum Level	Order-up-to Level	
Inventory Decisions	Lost Sales	Backorder	Nonnegative
Fleet Composition	Homogeneous	Heterogeneous	
Fleet size	Single	Multiple	Unconstrained

PSO is an optimization approach inspired and influenced by the movement of bird flocks and fish to find the optimal place. PSO method has been developed into a discrete multi-swarm PSO used in Rau et al. (2018) research to solve the MIRP problem. Discrete multi swarm PSO is a combination of metaheuristic methods of discrete PSO and multi swarm PSO. Studies examining LNG shipments that using TSS method was conducted by Cho et al. (2018). This research aims to minimize the impact of extreme weather on the LNG supply chain. TSS method was developed, which considers the uncertainty of weather that interferes with LNG production, storage, and shipment. COA is an evolutionary algorithm inspired by the life of a bird family. COA was used as an approach method to solve MIRP in Sangaiah et al. (2020). The computational performance of this research is better and more effective than CPLEX.

3. Methods

The MIRP aims to minimize the total transportation cost while fulfilling the demand of each consumption port. This problem is subject to the following constraints: (i) the storage level at consumption port could not exceed its capacity, (ii) each ship route can serve at most one consumption port or direct routing that starts and ends at the production port, (iii) ship capacities cannot be exceeded which ship's compartment is not dedicated to a specific product but only one at a time, (iv) ship-port compatibility, and (v) satisfying all demand that is assumed to be deterministic during the planning horizon. The solution should determine which consumption port to serve during the planning horizon using ships, how much to deliver each product type to each visited, and which routes to choose. CPLEX solves the mathematical formulation for the MIRP using MILP. GA is developed due the exact solution methods are highly consuming for larger-scale problems. This method has been coded in MATLAB.

3.1 Mathematical Model

In this study, MIRP is formulated as a MILP model that is able to provide a solution for small-scale problems and verify the performances of heuristic solutions. This problem consists of a set of ships, *V,* to be routed and scheduled. A ship $v \in V$ begins its journey at an artificial origin port, *o(v).* Ships serve a set of ports, defined as *H.* The subset of ports that can be visited by ship *v* is defined as $H_v \subseteq H$. Thus, the set of all possible locations for ship *v* is $H_v \cup \{o(v)\}$. During a given planning horizon, TH, a port can be visited and served several times until it reaches the maximum number of visits, denoted by MX_i. However, an artificial port can only visit at most one. The problem may be defined on a graph $G = (N, A)$, where N=$\{(i, m): i \in H, m = 1, \dots, MX_i\} \cup \{o(v), 1\}$ is the node set. A is the arc set. *N* is represented by a pair of a port and a number of ship visits at that port. Moreover, the arc set *A* is defined as all possible connections between two nodes. A subset of all feasible arcs for ship *v* is given as $A_v \subseteq A$. As we assume that storage can specify only one product, the notation and index of products also refer to their storage. Let $k \in K$ is the set of all possible products. The inventory level of product *k* at port *i* must lie between an upper and a lower bound, defined as SMN_{ik} and SMX_{ik}, consecutively. When a ship arrives at node *(i,m)* denoted by t_{im}, the inventory level of the storage, S_{imk}, should be calculated as well as updated. Each port has an initial inventory level denoted as IS_{ik}, and a constant production or consumption rate of R_{ik}. Ports are associated with parameter J_{ik} that indicates its type, which is equal to +1 for production port and -1 for consumption port. Moreover $K_i \subseteq K$ is the subset of products produced or consumed in port *i* and $P_v \subseteq K$ is the subset of products that ship *v* can carry. Each ship *v* has its compartment *c,* which has its maximum capacity of compartments is represented by CM_{vc} and $C_v \subseteq C$ is the subset of compartments that ship *v* has. At the beginning of the period, there is a possibility of the initial quantity of product *k* loaded in compartment *c* of ship *v* which is denoted as QQ_{vkc}. For the loading and unloading process, the loading (or unloading) time of one unit of product k at port i is given by the parameter TQ_{ik}. We define binary decision variables $x_{imjnv}, y_{im}, z_{imv,} o_{imvkc}$ as follows: $x_{imjnv} = 1$ if ship *v* routes from node *(i,m)* to node *(j,n)*; $y_{im} = 1$ if no ship takes node call *(i,m)*; $z_{imv} = 1$ if ship *v* finishes its route at node *(i,m)*; $o_{imvkc} = 1$ if ship *v* visits node *(i,m)* and loads or unloads product *k* into or from compartment *c.* The quantity of a specific product loaded or discharged into or from a particular compartment of ship at the port is indicated by Q_{imvkc}, whereas the quantity of product loaded in the compartment of the ship after departing from the current port is denoted by I_{imvkc}. The formulation of MILP for the MIRP is as follows:

$$Min \sum_{(i,m)\in N} \sum_{v \in V} \sum_{k\in P_v} \sum_{c\in C_v} TC_{ijv} q_{imvkc}, \qquad i \neq j \tag{1}$$

subject to

$$\sum_{(j,n)\in N} x_{o(v)1jnv} + z_{o(v)1v} = 1, \forall v \in V \tag{2}$$

$$\sum_{(j,n)\in N\cup\{o(v),1\}} x_{imjnv} - \sum_{(j,n)\in N} x_{jnimv} - z_{imv} = 0, \forall (v,i,m) \in V \times N, i \neq j \tag{3}$$

$$\sum_{(i,m)\in N} z_{imv} = 1, \forall v \in V \tag{4}$$

$$\sum_{v\in V} \sum_{(j,n)\in N\cup\{o(v),1\}} x_{imjnv} + y_{im} = 1, \forall (i,m) \in N, i \neq j \tag{5}$$

$$y_{im} - y_{i(m-1)} \geq 0, \forall (i,m) \in N, m \neq 1 \tag{6}$$

$$x_{imjnv} = 0 \quad, \forall v \in V, \forall (i,m,j,n) \in A_v, i \neq j, i > 1, j > 1 \tag{7}$$

$$x_{imjnv} = 0 \quad, \forall v \in V, \forall (i,m,j,n) \in A_v, j \notin H_c, i \neq j \tag{8}$$

$$x_{imjnv}\left(I_{imvkc} + J_{ik} - I_{jnvkc}\right) = 0, \forall v \in V, \forall (i,m) \in N, \forall (j,n) \in N, \forall (k,c) \in P_v \times C_v, i \neq j \tag{9}$$

$$x_{imjnv}(I_{imvkc}) = 0 \quad, \forall v \in V, \forall (i,m) \in N \cup \{o(v),1\}, \forall (j,n) \in N_p, \forall (k,c) \in P_v \times C_v, i \neq j \tag{10}$$

$$QQ_{vkc} = I_{o(v)1kc}, \forall v \in V, \forall (k,c) \in P_v \times C_v \tag{11}$$

$$q_{imvkc} \leq \sum_{(j,n)\in N\cup\{o(v),1\}} CM_{vc} x_{jnimv}, \forall v \in V, \forall (i,m) \in N, \forall (k,c) \in P_v \times C_v, i \neq j \tag{12}$$

$$I_{imvkc} \leq \sum_{(j,n)\in N\cup\{o(v),1\}} CM_{vc} x_{jnimv}, \forall v \in V, \forall (i,m) \in N, \forall (k,c) \in P_v \times C_v, i \neq j \tag{13}$$

$$q_{imvkc} \leq CM_{vc} o_{imvkc}, \forall v \in V, \forall (i,m) \in N, \forall (k,c) \in P_v \times C_v \tag{14}$$

$$I_{imvk'c} \leq CM_{vc}(1 - o_{imvk''c}), \forall v \in V, \forall (i,m) \in N_p, \forall (k',k'') \in P_v, \forall c \in C_v, k' \neq k'' \tag{15}$$

$$\sum_{k\in P_v} o_{imvkc} \leq 1, \forall v \in V, \forall (i,m) \in N \cup \{o(v),1\}, \forall c \in C_v \tag{16}$$

$$t_{im} - t_{i(m-1)} \geq 0, \forall (i,m) \in N, m \neq 1 \tag{17}$$

$$t_{im} \leq TH, \qquad \forall (i,m) \in N \cup \{o(v),1\} \tag{18}$$

$$x_{imjnv}\left[t_{im} + \sum_{k\in P_v} \sum_{c\in C_v} TQ_{ik} q_{imvkc}\right] + TT_{ijv} - t_{jn} \leq 0, \forall v \in V, \forall (i,m,j,n) \in A_v, i \neq j \tag{19}$$

$$q_{imvkc} \leq s_{imk} + J_{jk} R_{ik}\left[\sum_{k\in P_v} \sum_{c\in C_v} TQ_{ik} q_{imvkc}\right], \forall v \in V, \forall (i,m) \in N_p, \forall k \in P_v, \forall c \in C_v \tag{20}$$

$$s_{imk} = IS_{ik} + J_{jk} R_{ik} t_{i1}, \forall (i,k) \in H \times K_i \tag{21}$$

$$s_{i(m-1)k} - \sum_{k\in P_v} \sum_{c\in C_v} TQ_{ik} q_{i(m-1)vkc} + J_{jk} R_{ik}\left(t_{im} - t_{i(m-1)}\right) - s_{imk} = 0, \forall (i,m,k) \in N \times K_i, m \neq 1 \tag{22}$$

$$SMN_{ik} \leq s_{imk} \leq SMX_{ik}, \forall (i,m,k) \in N \times K_i \tag{23}$$

$$SMN_{ik} \leq s_{imk} - \sum_{k \in P_v} \sum_{c \in C_v} TQ_{ik} q_{imvkc} + J_{jk} R_{ik} (TH - t_{im}) \leq SMX_{ik}, \forall (i,m,k) \in N \times K_i, m = MX_i \quad (24)$$

$$x_{imjnv} \in \{0,1\} \quad ,\forall v \in V, \forall (i,m,j,n) \in A_v \quad (25)$$

$$y_{im} \in \{0,1\} \quad ,\forall (i,m) \in N \cup \{o(v),1\} \quad (26)$$

$$z_{imv} \in \{0,1\} \quad ,\forall v \in V, \forall (i,m) \in N \cup \{o(v),1\} \quad (27)$$

$$o_{imvkc} \in \{0,1\} \quad ,\forall v \in V, \forall (i,m) \in N \cup \{o(v),1\}, \forall (k,c) \in P_v \times C_v \quad (28)$$

$$s_{imk} \geq 0 \quad ,\forall v \in V, \forall (i,m) \in N \quad (29)$$

$$t_{im} \geq 0, \quad \forall (i,m) \in N \cup \{o(v),1\} \quad (30)$$

$$I_{imvkc}, q_{imvkc} \geq 0 \quad ,\forall v \in V, \forall (i,m) \in N \cup \{o(v),1\}, \forall (k,c) \in P_v \times C_v \quad (31)$$

The objective function (1) minimizes the total traveling cost. Constraint (2) guarantees that each ship might travel from or remain at its initial position. A ship either leaves or finishes its route at node (*i,m*) can be confirmed by Eqs. (3) and (4). Constraint (5) confirms that each node call is visited at most once. The flow precedence constraints are given by Eq. (6). Moreover, constraint (7) restricts that no route between consumption ports. Eqs. (8) relate ship-port compatibility constraints. Before and after a node call (*i,m*), the quantity on board is given by constraint (9). Eq. (10) ensures that the ship's compartments onboard must be empty if the ship back to the production port. The quantity on board the ship at the beginning of the time horizon will be the same as the initial quantity of product loaded in the compartment, verified by Eq. (11). Eqs. (12) and (13) determine the total amount of cement charged or discharged on the ships that should not exceed the ship's capacity. Eq. (14) is related to loading or unloading activities. Constraint (15) makes sure that each ship can be loaded a product in its compartment if it only has the same product. Constraint (16) controls that only one ship can perform a loading or unloading setup.

Scheduling time constraints are described by Eqs. (17)-(19). Constraint (17) avoids the violation of the time precedence. It ensures that a ship must complete its service before the next ship starts its service in the same port. The limitation of ship arrival time is given by constraint (18) and Eq. (19) calculates the time of the route. Eqs. (20)-(24) represent the inventory constraints. Constraint (20) ensures that the total amount of cement k loaded is within available product space in the storage. The product's storage level at the time of the first arrival is given by Eq. (21). Tracking the storage level of the previous and current visits of ships will be determined by Eq. (22). Constraints (23) and (24) ensure that the inventory at each port for each product should not exceed its limit. Eqs. (25)-(28) are the binary value for the ship routing and loading or unloading constraints. Eqs. (29) and (30) declare the inventory level and arrival time variable, respectively. Lastly, Eq. (31) provides continuous value for loading or unloading constraints.

3.2 Genetic Algorithm for the MIRP

Genetic algorithm (GA) is an optimization approach inspired by evolutionary processes happening in the natural system (Holland, 1975). GA intends to find and arrange the good features of different individuals within the population to produce better individuals. GA begins with setting the parameters, such as the population size of each individual or chromosome, the crossover probability, and the mutation probability. To get the best chromosome, the evolution within the population should be created in each generation. We evaluate the fitness value of each individual in each generation. Usually, the fitness value equals the value of the objective function in the problem being solved. This initial solution is stochastically selected based on the fitness value, where fitter solutions are typically more likely to be selected. Then, crossover and mutation are carried out to reach the desired solution. The crossover is a process in which members of the current population are mated randomly based on a spinning roulette wheel characterized by fitness for population size times. A pair of offspring is then generated, combining elements from two parents, which hopefully have improved fitness value. Before accepting the new generation, the feasibility must be evaluated. If both children are feasible, the children replace their parents. Otherwise, the existing feasible solution must be kept. This process will be repeated until two feasible children are obtained. A mutation is a sporadic random modification in the sequence of an individual. The mutation result may be selected if it is feasible. An elitism operation proposed by Santosa and Ai (2017) can be used to maintain the best solution in each generation.

GA has been applied to solve MIRP before. Therefore, we develop our proposed GA with specific solution representation to solve MIRP. This method consists of two phases: constructing a feasible initial solution by a construction algorithm and then improving by GA. This section explains our solution representation scheme first, then discusses how we generate the initial solution. Finally, we present the proposed GA procedures.

3.2.1 Solution Representation

A solution representation comprises *v* ships, *p* consumption ports, and quantity on board *q* in each ship's compartment for each ship route. A ship's assignment is the combination of ship routes in which each ship route represents a voyage of the ship from a production port for loading product(s) to a consumption port for unloading product(s). Each ship will be given an initial position at a port or point at sea and possibly load onboard with a known destination at the beginning of the time horizon. A new assignment will be added one by one, from left to right, into the current assignment until all demands are fulfilled. Each assignment will change the condition of variables defined as a state, such as a ship's position, compartment levels, contents, and port storage levels. In our method, the length of the chromosome is determined by the number of assignments. The number of assignments depends on the planning horizon's length and ensures that all consumption port's needs are fulfilled during the planning horizon. The longer the length of the planning horizon, the larger the size of the solution space. Fig. 1 illustrates an example of solution representation. The blue-shade column donates the visited port. As seen in Fig. 1, the first blue column in the first assignment is defined as that Port 3 will be visited by Ship 1 and Ship 2 go to Port 2. Meanwhile, the beige-shade column represents the quantity of cement loaded to the compartment and delivered to a specific consumption port by a particular type of vehicle.

3	2	1500	0	0	0	5200	0	0	0	1st Assignment
3	3	1500	0	0	0	1771	0	0	0	2nd Assignment

Figure 1. An example of solution representation

3.2.2 Initial Solution

We create the initial solution using a construction algorithm. This algorithm is to find a feasible and good solution as input in the GA stage. The algorithm starts with a known initial plan. The route specifies the ships start either empty, with the initial time and position given by a port call, or starts with loaded ships. The algorithm then repeats the following steps until all storage of port is fulfilled during the planning horizon.

Step 1. Set the number of assignments, denoted by $a \in A$. In our algorithm, the number of assignments is based on the largest total demand during the planning horizon at a particular consumption port divided by the minimum ship's capacity. However, not all consumption ports must be served as many as the number of assignments. Once the total demand at a particular consumption port is fulfilled, this port does not need to be served in the next assignment. Hence, we determine the number of assignments.

Step 2. For each assignment, select the critical silo and ship selection. At the beginning of each assignment, the coverage day (*CD*) is calculated. This parameter aims to specify which port and its storage need to be served. The less *CD* value of the consumption port, the more urgent it needs to be served. This term has the same meaning with the term that described by Savelsbergh and Song (2007), Al-Khayyal and Hwang (2007). The quantity of product that loaded to satisfy the demand of the selected port is denoted by QPH_{aik}. The largest total capacity of the ship will prioritize serving the consumption port with the largest *QPH* value.

Step 3. We assign the quantity of the specific type of cement to the assigned compartment based on the selected port's product demand. The larger compartment will be assigned to the larger demand. The quantity on board in each compartment is randomly generated but set to be either equals to the quantity of product k to satisfy the demand of consumption port *i* until the end of the planning horizon, denoted by QPH_{aik}, or the compartment capacities the ships.

Step 4. Update the compartment level and the new storage level. If the new storage level does not violate the rules, record the shipment; otherwise, the neighborhood move is employed in the current assignment, a swap move, to make the shipment feasible. The last step is calculating time and total cost for all assignments.

3.2.3 The GA heuristic

In this stage, we evaluate *N* chromosomes generated from the previous solution based on the fitness function. Then, we copy the best chromosome three times if *N* value is odd or four times if *N* value is even to create a new population in each generation. This operation is called the selection phase. A new set of chromosomes are then generated through the crossover and mutation process. Genetic operators such as selection, crossover, and mutation are employed to create a better solution and replace them with those existing in the initial population to obtain a near-optimum solution. Each chromosome in the population is evaluated in GA framework by using a fitness function. If the storage is occurred shortages due to the longer arrival time, it will occur the penalty cost. However, when the consumption port's storage level has not covered its demand until the end of the planning horizon, the completion cost will have occurred. The fitness function is highly influenced by the values of three parameters: w_1, w_2, w_3. We set w_2 and w_3 significantly high compared to w_1. Thus, it will avoid shortages and can satisfy all demands during the planning horizon.

There are two different crossover points in each assignment of the selected chromosome that used in this study. These points are randomly generated along the chromosome's length and exchanged among parents to create offspring. The larger the fitness value of the chromosomes, the larger probability of being selected as parents. Then, a random value *r* is generated. We use two conditions so that the offspring is performed if *r* value is less than *Pcross* and if the offspring satisfies the feasibility. Otherwise, both parents are selected as offspring without crossing them. The crossover results are considered feasible if the quantity on board does not exceed the compartment's capacity and the appropriate type of cement is transferred with the product at the consumption port.

In this study, we use two procedures of mutation operation in each assignment for the selected chromosome. First, we perform swapping mutation on certain genes that represent the routing. The number of ships determines the number of genes. Second, we modify some genes representing the cement quantity on board by randomly generated within its boundary. The mutation operation will perform if a random value *r* is less than *Pmut* value. The result of mutation will introduce a new offspring. Then, each offspring is evaluated in terms of the fitness function, as mentioned before. The termination will occur if one of the following conditions is met: (i) reaches the maximum iteration, denoted as *maxit,* or (ii) all *CD* values of the consumption port are fulfilled, and the current best solution, TC_n, is not improved. Otherwise, the iteration is updated and performed. Finally, the solution of MIRP-UC is derived from *BestX,* and the total cost is updated based on *BestX.*

4. Data Collection

The proposed GA is tested on an instance adopted from real-world problems to understand its performance. The dataset has (6,5,2,2) configuration of the number of ships, ports, products, and compartments, and then run for 15 and 30 days as planning horizon, called model 1 and model 2, respectively. We present the MIRP that involves the delivery of multiple products, *K1* and *K2*, which cannot be mixed and uses heterogeneous types of ships in terms of capacity, time, and specific ship characteristics for delivery. Meaning that only particular ships can visit a particular consumption port due to the ship-port compatibility constraints. Ships also differ in the number of compartments, *C1* and *C2* that are not dedicated to specific products. Each ship's sails start and end at the production port. A ship can only visit at most one consumption port in each trip. It is also assumed that each consumption port allows several ships to perform their unloading process simultaneously but cannot load or unload different products at the same time. For the unloading process, waiting time is permitted until there is enough space to unload more products.

Each ship will either be given an initial position (at a port or point at sea) or load onboard with a known destination at the beginning of the planning period. In this problem, we assume that the producer owns the production port and controls the inventory storages at the source and destination. Therefore, the inventory costs do not come into play. Moreover, we ignore the fixed cost of the ship due to the number of ships and its capacities are fixed and sufficient to fulfill all demands during the planning horizon. We design a set of combinations of the planning horizon's length, ranging from short-term to medium-term planning horizon. The depiction of problem instances is illustrated in Table 2 to Table 4.

Table 2. Data for types of port, consumption rate, initial level, and cost

Port	Types of Port		Consumption Rate (ton)		Initial Level (ton)		Minimum Level (ton)		Minimum Level (ton)		Loading/Unloading Time (days/ton)		Cost (Thousand IDR/ton)
	1	2	1	2	1	2	1	2	1	2	1	2	
0	1	1	0	0	0	0	0	0	2000000	2000000	0.0002	0.0002	240
1	-1	-1	719	279	2624	1018	2624	1018	12000	0	0.0002	0.0002	183
2	-1	-1	811	0	1549	0	1549	0	12000	0	0.0001	0	105
3	-1	-1	527	0	632	0	632	0	12000	0	0.0003	0	250
4	-1	-1	204	0	4140	0	1000	0	6000	0	0.0003	0	255
5	-1	-1	302	0	3083	0	1293	0	10000	0	0.0003	0	255

Table 3. Data for types of port, consumption rate, initial level, and cost (cont.)

Port	Types of Port		Consumption Rate (ton)		Initial Level (ton)		Minimum Level (ton)		Minimum Level (ton)		Loading/Unloading Time (days/ton)		Cost (Thousand IDR/ton)
	1	2	1	2	1	2	1	2	1	2	1	2	
6	-1	-1	462	0	7446	0	2231	0	10000	0	0.0004	0	240

Table 4. Ships data

Ship	Compartment Capacity (ton)		Initial Level (ton)		Initial Position	
	1	2	1	2	Destination Port	Remaining Time (days)
1	1500	0	0	0	-	0
2	5200	0	5200	0	P3	0.6
3	4000	3500	0	0	-	3.3
4	3400	2500	0	0	-	0
5	4000	0	0	0	-	0

Table 5. Traveling time

Port	Time Travel (Day)						
	0	1	2	3	4	5	6
0	0	3.45	1.71	1.1	3.61	4.18	4.63
1	2.8	0					
2	1.4		0				
3	0.87			0			
4	1.76				0		
5	3.39					0	
6	2.63						0

5. Results and Discussion

We implemented our proposed GA in Matlab 2015 by using an Intel® Core i7-6700 CPU at 3.4 Ghz and 8 GB of RAM under Windows 10 Professional. CPLEX is utilized to solve the MILP. We also compare our GA solution result with the MILP result to evaluate our proposed GA's performance. The parameter settings may influence the performance of our proposed GA. Thus, we conduct a preliminary experiment to determine the best parameter setting. The parameter values tested are as follows:

1. Number of chromosomes (*N*) = 100, 300, 500.
2. Maximum iteration (*maxit*) = 1000, 3000, 5000.
3. Crossover probability (*Pcross*) = 0.2, 0.4, 0.6.
4. Mutation probability (*Pmut*) = 0.2, 0.4, 0.6.

Table 5. Comparison result of the MILP and the result of proposed GA

No	Model	Planning Horizon	Proposed MILP		Proposed GA		Gap (%)
			Total Cost (Million IDR)	CPU (s)	Total Cost (Million IDR)	CPU (s)	
1	1	15	4.388	71280.00	4.392	314	0.09
2	2	30	-*	>> 1.728E+6	15.023	58,559	-

(*) A feasible solution was not found before the time limit of 48 hours

5.1 Numerical Results

The parameter setting $N = 300$, $maxit = 3000$, $Pcross = 0.4$, $Pmut = 0.4$ more likely to generate the best results among all possible combinations. Therefore, this combination of parameters is used in this study. The effectiveness of our proposed GA by solving two models and then comparing the results with those solved by CPLEX is summarized in Table 5. The CPLEX solutions obtained within the predetermined 48 hours limit are optimal.

5.2 Graphical Results

The results of sensitivity analysis are presented in Figs. 2-5. Blue and orange lines in the figures depict the objective function values and computational time, respectively. Figures 2 and 3 show that the parameter number of chromosomes and maximum iteration seem not to influence the objective function by changing its value significantly. This result occurred because of increasing the set parameter, resulting in a lower objective function that takes a long time for being solved, but it then increases after *N* and *maxit* value reaches 300 and 3000, respectively. Therefore, we can conclude that the promising values of *N* are 300 and 500, while for parameter *maxit,* the promising values are 3000 and 5000. The crossover probability gives an opposite effect on the running time and objective function. When a higher value of the crossover probability is tested in our proposed GA, the smaller average of the total cost is obtained, but the computational time tends to be longer when the higher value of *Pcross* is used. Although a higher value setting for *Pcross* may give a good solution result, we need to consider the computational time. We consider decreasing the total cost and increasing the computation time to determine the setting parameters (Yu and Lin , 2015). Therefore, increasing the crossover probability to 0.6 from 0.4 seems to not significantly improve the objective function while taking much more computing time. Parameter mutation probability also seems to have an opposite effect on the objective function and the computational time. Still, it has not significantly decreased the total cost until *Pmut* is 0.4 and increases computational time. Therefore, the preferable value of *Pmut* is 0.4.

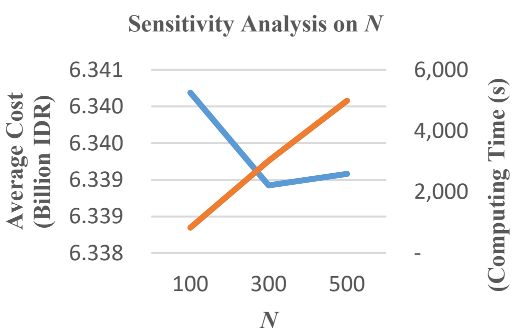

Figure 2. Sensitivity analysis on *N*

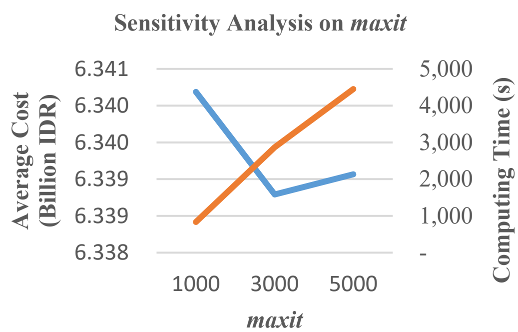

Figure 3. Sensitivity analysis on *maxit*

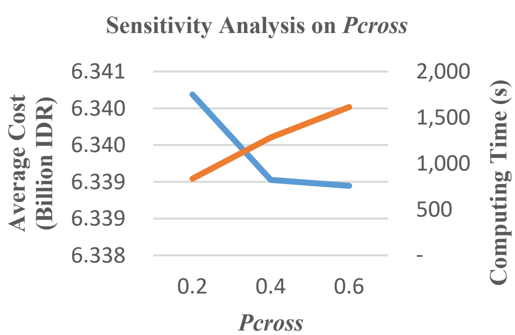

Figure 4. Sensitivity analysis on *Pcross*

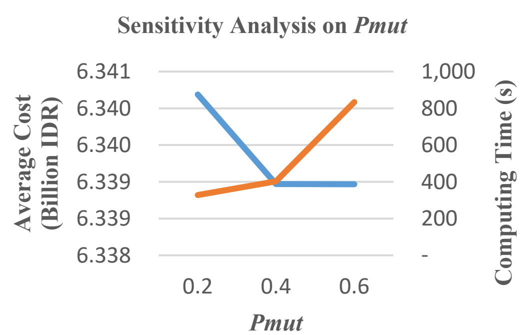

Figure 5. Sensitivity analysis on *Pmut*

Table 5. indicates that the optimal solutions could be acquired within set time limits. Unfortunately, the higher the number of ships, ports, and types of products, the longer the running time. It also applies to extending the planning horizon. The exact approach could not get a solution for model 2 regarding the planning horizon increased. Therefore, it was the reason why we develop and solve the problem with the metaheuristic approach.

The result of our proposed GA is close to the outcome of the CPLEX and, on average, obtains good performances with an average gap under 1% and moresuperior in safe the computational time. However, GA tends to have a longer computational time for small instances compared to CPLEX solution. Still, for medium instances, GA is more superior in terms of computational time. Moreover, it shows that a metaheuristic algorithm can efficiently solve medium problem instances with a longer planning horizon than the MILP approach. Therefore, the proposed GA to solve MIRP-UC is capable and more efficient in solving a bigger problem.

6. Conclusion

This study has formulated a mathematical model of the MIRP. This method can solve small problems and can be utilized to verify the performance of the heuristics approach. We conduct an efficient GA that applied and checked on the instance adapted from real-world problems and then we compare to the results solved by CPLEX. The proposed GA was reasonably good in solving the MIRP. Furthermore, CPLEX fails to solve medium-scale MIRP instances, whereas the proposed GA seems to perform better than CPLEX, as it can obtain the solutions within shorter times. The results show that GA is capable and competitive in obtaining the optimal solution. Therefore, the proposed model gives a better solution quality and superior computational time to solve big-scale problems.

Further research may consider other aspects such as environmental or social perspectives, such as considering the sustainable factor. Future studies may also consider other variant conditions, such as uncertainty, time-windowed problem, and dynamic routing structure, which recently widely studied in routing problems. Multiple routing structures can be implemented and it would be interesting to consider them under the MIRP framework so the company can select a more beneficial policy that increased their performances compared to the currently applied policy.

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