Analysis of Students Critical Thinking Skills on The Limit of Algebraic Functions

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Abstract

This study aims to describe the critical thinking skills of students at SMA N 1 Karanganom Klaten in the 2020/2021 academic year. The subjects used in this study were 3 students in class XI. The technique used was purposive sampling. The research uses descriptive qualitative approach. Data collection was carried out through tests and interviews. The test given to students is in the form of description questions on the limit of algebraic function material online due to the covid-19 pandemic with indicators of interpretation, analysis, evaluation, and inference. Interpretation is to understand the problem that is shown by writing what is known or what is being asked about the questions correctly. Analysis is identifying the relationship between statements, questions and concepts shown by making a mathematical model appropriately. Evaluation is using the right strategy in solving problems, complete and correct in calculations. Inference is making conclusions correctly. The results of the tests that have been done by students showed that the students' critical thinking skills on the limit of algebraic functions were still low. Therefore, interviews were conducted with students to strengthen the data.

Keywords
Limit of Algebraic Functions, Covid-19 Pandemic, Ability to Think Critically

1. Introduction

Education is an effort that is done through learning, guidance and serves to prepare the source of power of man who is good, the days to come Anwar (2014). According to Hasibuan & Prastowo (2019) education in the 21st century is more emphasized on students' ability to think critically, being able to integrate knowledge. That has been learned in everyday life, is familiar with technology, information, and has communication skills and the ability to collaborate. This is in line with Pratiwi et al., (2019) opinion on Human Resources who are required to have critical thinking skills, creative thinking and problem solving skills. According to Rachmadullah et al., (2020), this capability will be indispensable in producing human resources that can compete with other developed countries.

One of the knowledge fields that can be applied to resolve the problems in life everyday is mathematics. With their math students are trained to reason (Fajri, 2017; Kusumawardani et al., 2018; Minami, 2010; Mikrayanti, 2016). Not only that, students also need to deepen their critical thinking skills (Basri et al., 2019; Sk & Halder, 2020). The
reason for Capacity of thinking critically is very necessary in the face of challenges MEA (Sulistiani & Masrukan, 2016; Kurniawati, 2020; Salim Nahdi, 2019; Živković, 2016; Wulandari et al., 2017; Elisanti et al., 2018).

According to (Dwyer, Hogan, 2015) the ability to think critically is the ability to think systematically, be able to carry out interpretation, collaboration, and be able to solve problems. In addition, Larsson (2017) explained that critical thinking is an attempt to check the truth based on evidence, logic, and awareness. (Yuwono et al., 2019) also explains that the ability to think critically is used to resolve the problem. Solving problems requires higher-level thinking skills. Application can be done by applying previously acquired knowledge to reach possible answers (Ansari et al., 2020). In addition, critical thinking skills can help students have a critical view and make decisions in accordance with the expected goals (Tosuncuoglu, 2018). Students who are accustomed to critical thinking will be accustomed to analyzing, synthesizing, and making decisions on a logical basis on any information they receive (Karakoc, 2016).

One of the materials that are considered difficult by most major students is the limit of a function algebraic. In accordance with the opinion (Sudirman, 2013), the difficulty of students in the limit material is not understanding the value of the function, the difficulty of factoring, and being accustomed to using a calculator. Besides that, the students can master theorems limit, students are also required to master the material prerequisites other.

According Lestari (Andrian & Rusman, 2019) students have not been able to actualize ability which is owned as at the time of learning in school, teachers active in explaining the material while students tend to be passive, namely by listening alone. Mathematics learning tends to be teacher-centered and not student-centered (Mudjiran, 2020). So that students tend not to maximize their critical thinking skills (Shofwan et al. 2021). This is in accordance with (Yuliati & Lestari, 2018) opinion which states that students' abilities in the affective domains of analysis, evaluation and creation are still low. It is seen from the data the Program for International Student Assessment (PISA) (OECD, 2019), show that for mathematics Indonesia occupied rank 74 of the 79 countries. It is caused due to students not yet accustomed to working on issues that HOTS. It is line with the results of interviews of teachers of mathematics in high school N 1 Karanganom Klaten who say that students experiencing difficulty when working on a matter which is slightly different to the matter that exemplified the teacher, can be seen from the many mistakes the students when processing a matter either of steps workmanship and answers.

Based on research (Nugraha et al., 2017; Dewi et al., 2016; Ismail, 2016; Sulistiono & Yuni Sri Rahayu, 2014) explained about the critical thinking skills of students at the junior high school level. Besides that (Jusmaya & Efyanto, 2018; Santi et al., 2018; Kirana & Kusairi, 2019; Ghofur & Raharjo, 2018; Anugraheni, 2019) outlines that the ability to think critically students too low.

2. Literature Review

2.1. Critical Thinking

According to (Hartati et al., 2019) Critical Thinking Ability is a basic mathematical ability that is essential and needed by students in learning mathematics. The ability to think critically is one of the capabilities in the ability to think level higher. According to Gokhale (Sidik et al., 2018) critical thinking ability is a thinking process that involves analyzing, synthesizing, and evaluating concepts (Fakhruddin & Shofwan, 2019). By thinking critically, students can distinguish, sort to be grouped and look for their relationship to the information obtained. Then students process information systematically to state the truth of their opinions. Critical thinking skills are needed to deal with everyday problems and face an ever-changing future, not easy to accept without knowing the reasons, and being able to account for their opinions with logical reasons.

2.2. Indicators Of Critical Thinking

This is indicated by indicators of critical thinking skills, including analysis and evaluation which is an indicator of the ability to think at high in the realm of cognitive. According to Facione (Karim & Normaya, 2015) indicators of critical thinking in this study are 1) interpretation, 2) analysis, 3) evaluation, and 4) inference. Where every student has the ability think critically are different, both from the category of low, medium and high.
3. Methods

The method of research that used using the approach of descriptive qualitative. The subjects in this study were three students taken from class XI MIPA in SMA N 1 Karanganom Klaten. The purpose of selecting subject by purposive sampling. Foreigners for each category both low, medium, and high were taken by 1 student. At the implementation stage of data collection critical thinking skills through tests and interviews. The instrument in this study was a test of critical thinking skills. In this study, the data validity uses triangulation method. The data analysis technique in this study was on during the process in the field along with data collection, reducing involves summarizing, selecting the main points, and focusing on the problem to be analyzed. With data reduction, the picture of students' critical thinking skills is getting clearer. Followed by the presentation of data in the description of the description, chart, or the relationship between categories for easy explanation and planned description here and then conclusions. The final result will be described about the critical thinking skills of students of class XI MIPA 5 SMA N 1 Karanganom Klaten.

4. Result and Discussion

According to Facione (Karim & Normaya, 2015) indicators of critical thinking in this study are interpretation, analysis, evaluation, and inference. These indicators can be seen in Table 1.

Table 1. Indicators of Critical Thinking Ability

<table>
<thead>
<tr>
<th>No</th>
<th>General indicators</th>
<th>Indicator</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Interpretation</td>
<td>Understand the problems indicated by writing known or asked questions appropriately.</td>
</tr>
<tr>
<td>2</td>
<td>Analysis</td>
<td>Identify the relationships between the statements, questions, and concepts given in the problem shown by making a mathematical model appropriately and giving an appropriate explanation.</td>
</tr>
<tr>
<td>3</td>
<td>Evaluation</td>
<td>Using the right strategy in solving questions, complete and correct in doing calculations.</td>
</tr>
<tr>
<td>4</td>
<td>Inference</td>
<td>Make the right conclusions.</td>
</tr>
</tbody>
</table>

The materials are used in a matter of a pretest that is the limit of the function algebraic. The amount of matter that is used there are 3, in which each question is used to measure the ability to think critically students. Subjects who used as many as three students were each grouped into categories of low, medium and high. Taking the subject based on the highest score from each category. Problem test were used, namely:

1. The value of \( \lim_{x \to -2} \frac{\sqrt[3]{7-x} - 3}{\sqrt[3]{x+6} - 2} \) is ...

2. The velocity of the object at any time is determined by the formula \( V(t) = 0.2t^2 - 0.4t \). The change in velocity for \( t \) approaching 5 is formulated by \( \lim_{t \to 5} \frac{V(t) - V(5)}{t - 5} \), the value of the change in velocity of the object is ...

3. Given the function \( f(x) = \begin{cases} x^2 + 4x - 5, & \text{if } x \neq 1 \\ 2, & \text{if } x = 1 \end{cases} \)

   Investigate whether the function \( f(x) \) is continuous at \( x = 1 \)?

Figure 1, in question 1, the error subject 1 lies in multiplying the rational root of the fraction. Subject 1 does lim \( x \to -2 \frac{\sqrt[3]{7-x} - 3}{\sqrt[3]{x+6} - 2} \) multiplying the numerator and denominator by \( \frac{\sqrt[3]{x+6} + 2}{\sqrt[3]{x+6} + 2} \). However, subject 1 forgets that numerator also need to be rationalized because in the developer there is a root value. The form \( \lim_{x \to -2} \frac{\sqrt[3]{7-x} - 3}{\sqrt[3]{x+6} - 2} \) when multiplied by the same root becomes \( \lim_{x \to -2} \frac{\sqrt[3]{7-x} - 3}{\sqrt[3]{x+6} - 2} \times \frac{\sqrt[3]{x+6} + 2}{\sqrt[3]{x+6} + 2} \). The final result in the answer subject 1 is an infinite value, in this case it is wrong.
Figure 2, in question 2, subject 1 has substituted \( t = 5 \). After that, the value of \( V(t) = 3 \) is obtained dan subject 1 substitutes the value of \( V(t) = 3 \) to the limit value. Subject 1 was incorrect in substituting the value \( V(t) \). The understanding of subject 1 is not quite right, because the value of \( V(t) = 0.2t^2 - 0.4t \) is already known in the problem, so that it resulted in the student's wrong answer.

Figure 3, in question 3, subject 1 only substitutes the value \( x = 2 \) into the equation \( x^2 + 4x - 5 \). After that, the final result of the answer to subject 1 is 7. The location of subject 1 is error is that it cannot answer according to the question. The purpose of the command in the problem is that subject 1 are asked to prove whether the function \( f(x) = \begin{cases} x^2 + 4x - 5, & x \neq 1 \\ 2, & x = 1 \end{cases} \) continous at \( x = 1 \), but subject 1 do not understand the meaning.
4.1. Low Category Subject

In question 1, 2, and 3, subject 1 did not answer the questions correctly. Subject 1 did not understand the question, so do not write well-known and questioned about exactly (interpretation), does not create a mathematical model correctly (analysis), do not use strategy right in solving (evaluation), and does not make decisions with the right (inference). In problem number 1, the error lies in multiplying the rational root of the fraction. The form \( \lim_{x \to -2} \frac{\sqrt{7-x} - 3}{\sqrt{x+6} - 2} \) when multiplied by the same root becomes \( \lim_{x \to -2} \frac{\sqrt{7-x} - 3}{\sqrt{x+6} - 2} \times \frac{\sqrt{7-x} + 3}{\sqrt{7-x} + 3} \). Subject 1 should do this by multiplying the same root twice because the problem has two friend roots that are different from the usual problem. Subject 1 applies the concept of multiplying only one common root which is usually in the mention. Based on the results of the interview, it was found that subject 1 did not know that every algebra operation that was rooted had to be rationalized first, both the denominator and the numerator. So that subject 1 only rationalizes the existing roots in accordance with the questions that are usually given.

In question number 2, subject 1 has substituted \( t = 5 \). But subject 1 was incorrect in substituting the value \( V(t) \) so that it resulted in the student's wrong answer. Based on the results of the interview, it was found that subject 1 could not distinguish between \( V(t) \) and \( V(5) \).

In question number 3, subject 1 only substitutes the value \( x = 2 \) into the equation \( x^2 + 4x - 5 \). The location of subject 1's error is that it cannot answer according to the question, it is better if subject 1 does the limit operation by factoring and simplifying the results. After that, proceed to substitute \( x = 1 \). So that a value will be obtained that proves whether the function is continuous or not. Based on the results of the interview, it was found that subject 1 did not understand what is meant by continuous function.

Figure 4, Answer to subject 2

Figure 4, in question 1, subject 2 has answered the question correctly. The final result of subject 2 is \( \lim_{x \to -2} \frac{\sqrt{7-x} - 3}{\sqrt{x+6} - 2} = \lim_{x \to -2} \frac{\sqrt{7-x} + 3}{\sqrt{x+6} + 2} = \frac{2}{3} \). In the case, subject 2 already know how to solve the problem.
Figure 5, in question 2, subject 2 is correct in substituting the value \( V(t) = 0.2t^2 - 0.4t \) dan \( V(5) = 3 \). However, subject 3 error is wrong in substituting value for \( t = 3 \), \( \lim_{t \to 5} \frac{1}{5} (t + 5) = \lim_{t \to 5} \frac{1}{5} (3 + 5) \) which is \( t = 5 \). This causes the calculation to be inaccurate.

Figure 6, in question 3, subject 2 says only write the question back, namely \( f(x) = \begin{cases} x^2 + 4x - 5, & \text{if } x \neq 1 \\ 2, & \text{if } x = 1 \end{cases} \) and that the answer to question 3 is not continuous. Subject 2 do not know how to solve the problem and only answer modestly.

4.2. Medium Category Subject

In question 1, subject 2 has answered the question correctly. Subject 2 already understands the problem. In the process of working, it is not written what is known or what is asked about the problem correctly (interpretation), but can make mathematical models appropriately (analysis), use the right strategy in solving problems (evaluation), and make conclusions appropriately (inference).

In question 2, subject 2 did not answer the question correctly. When answering questions, subject 2 has made a mathematical model correctly (analysis), used the right strategy in solving the problem (evaluation), but did not write down what was known or what was asked about the question correctly (interpretation), and did not make the right conclusions (inference). The location of subject 2's error is wrong in substituting value for \( t = 3 \), which is \( t = 5 \). Based on the results of the interview, it was found that subject 2 was not careful in doing the questions.

In question 3, subject 2 did not answer the question correctly. Subject 2 says only that the answer to question 3 is not continuous. Based on the results of the interview, it was found that subject 2 ran out of time working on the questions and was just guessing in answering the questions.
Figure 7. Answers to the subject 3

Figure 7, in question 1, subject 3 is correct in rationalizing the roots of the numerator and denominator, namely \( \lim_{x \to -2} \frac{\sqrt[3]{7-x} - 3}{\sqrt[3]{x+6} + 2} \). However, it is miscalculated at \( x+6-4 = x-2 \) so that the numerator and denominator cannot be crossed out because the numerator \( x-2 \) and the denominator \( x+2 \). This causes the process to be less precise.

Figure 8. Answers to the subject 3

Figure 8, in question 2, subject 3 is substituting the value of \( V(t) \) so that we get \( \lim_{t \to 5} \frac{0.25t^2 - 0.4t - 3}{t-5(0.2t + 0.6)} \). Then the numerator is factored, we get \( \lim_{t \to 5} \frac{t-5(t)(0.2t + 0.6)}{t-5} \). The values of the \( t=5 \) numerator and denominator are simplified to obtain \( \lim_{t \to 5} 0.2t + 0.6 \). The error occurs when subject 3 error can be seen at writing \( \lim_{t \to 5} \). Subject 3 keeps writing the limit, while the limit value has been substituted.

Figure 9. Answers to the subject 3

Figure 9, in question 3, subject 3 has answered the question correctly. To work on the problem, subject 3 looks for the \( \lim_{x \neq 1} \) and looks for \( \lim_{x \to 2} \) if \( x = 1 \). For the value of \( \lim_{x \neq 1} \) the value is 0 and the value of \( \lim_{x \to 2} \) if \( x = 1 \) is obtained the value 2. The final result of the score \( \lim_{x \neq 1} \) dan \( \lim_{x \to 2} \) if \( x = 1 \) is different, it can be concluded that the function \( f(x) = \begin{cases} x^2 + 4x - 5, \text{ jika } x \neq 1 \\ 2, \text{ jika } x = 1 \end{cases} \) is not continuous at \( x = 1 \).
4.3. High Category Subject

In question number 1, subject 3 did not answer the question correctly. Subject 3 only wrote what was known and what was asked about the problem correctly (interpretation), but did not make a mathematical model appropriately (analysis), did not use the right strategy in solving problems (evaluation), and did not make correct conclusions (inference). Based on the results of the interview, it was found that subject 3 was in a hurry to work on the problem so that \(x + 6 - 4 = x - 2\). But the correct one was \(x + 2\). So that the denominator could not be crossed directly with the numerator.

In question number 2, subject 3 has answered the question correctly. At the time of answering questions, the subject 3 had written well-known and questioned about exactly (interpretation), is already making a mathematical model correctly (analysis) and using the appropriate strategy in solving (evaluation). But not making conclusions with accuracy (inference). The location of subject 3 error can be seen at writing \(\lim_{t \to 5}\). Subject 3 keeps writing the limit, while the limit value has been substituted. Based on the results of the interview, it was found that subject 3 did not understand when the limit was removed.

In question number 3, subject 3 has answered the question correctly. Subject 3 has written what is known and what is asked about the question correctly (interpretation), has made a mathematical model appropriately (analysis), can use the right strategy in solving problems (evaluation), and can make conclusions correctly (inference).

5. Conclusions

Based on the results and findings we concluded that the critical thinking skills students class XI MIPA 5 in SMA N 1 Karanganom Klaten are still low. S ISWA which have a poor performance then the critical thinking skills of students is not good. This can be seen from the questions not being answered correctly. This means that students do not understand the questions, so they do not write what is known or what is asked about the questions correctly (interpretation), do not make mathematical models appropriately (analysis), do not use the right strategy in solving problems (evaluation), and do not make correct conclusions (inference). Students who have moderate achievement have good critical thinking skills. In the process of working, it is not written what is known or what is asked about the problem correctly (interpretation), but can make mathematical models appropriately (analysis), use the right strategy in solving problems (evaluation), and make conclusions appropriately (inference). Meanwhile, students who have high achievement have good critical thinking skills. It can be seen from the writing that it is known or what the questions are asked correctly (interpretation), has made a mathematical model appropriately (analysis), can use the right strategy in solving problems (evaluation), and can make conclusions correctly (inference).

References

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