Performance of the Markowitz Model Investment Portfolio in Some Mining and Energy Sector Stocks

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ABSTRACT
Investment for mining and energy exploration in Indonesia needs to be a priority and continue to be encouraged to maintain the level of reserves as raw materials for future industrial development, including downstream. This study aims to measure the performance of investment portfolios in several stocks in the Mining and Energy Sector. The portfolio optimization method is carried out using the Markowitz model. To measure the performance of the investment portfolio is done by using the Sharpe Ratio. Based on the results of the analysis, it is obtained that the combination and proportion of capital allocation in several stocks in the formation of an investment portfolio that has better performance, so that it can be used as a consideration for investors who want to invest in several shares of the Mining and Energy Sector analyzed.

Keywords:
Stocks in the mining and energy sector, investment portfolios, Markowitz model, optimization method, Sharpe ratio.
1. Introduction

The mining and energy sector is one of the development sectors and is a strategic industry that has an important role for Indonesia. Where the mining and energy sector is a sector that absorbs labor from Indonesia which is quite large. With a large number of workers and potential new expansion projects, the jobs offered by the mining industry are very promising. Pandiangan et al., (2021). In addition, the mining and energy sectors are also very important supporting elements in the process of economic growth and one of the main sectors in generating foreign exchange for Indonesia. The availability of mining products in Indonesia does not need to import energy resources, even most of the production can be exported. One of the uses of energy resources is to generate electricity which is indispensable for the development of other sectors (Chow et al, 2003).

On the other hand, the mining and energy sectors are quite ogled by investors in making investments. This is due to the increase in oil prices on the world market and the need for energy which continues to increase every year (Espinoza and Rojo, 2017). This situation causes mining and energy sector stocks to have large capitalization values to drive the JCI and the mining sector stock index, thereby causing demand for mining and energy sector stocks to have very high demand (Pandiangan et al., 2021) Investments in the mining and energy sectors have great benefits but also have a large risk of loss. Therefore, in making investments, investors need to pay attention to what will be invested, how much amount they want to invest, and the level of risk that is ready to be borne by the investor. investors to achieve their investment goals (Kalfin et al., 2019a; Kalfin et al., 2020). One alternative that investors can do is to form a portfolio which is a combination of two or more stocks (Hasbullah et al., 2020). Portfolio investment contains an element of uncertainty risk, for that it is necessary to first conduct a scientific analysis so that they can choose investments that are truly safe or provide minimum risk and provide optimal returns (Sukono et al., 2020; Kalfin et al., 2019b).

Research related to portfolio optimization in investing has been carried out by many previous researchers. For example, research by Pandiangan et al., (2021), determines the optimal investment portfolio. In addition, in this research, risk analysis is measured using Value-at-Risk (VaR), so that optimization modeling is carried out using a quadratic investment portfolio approach with a Mean-VaR model with risk-free assets. The data used are share prices in the mining and energy sectors obtained from the JCI. In addition, the results of the analysis show that the greater the level of risk aversion, the smaller the VaR value followed by the smaller the average value of the portfolio. In the research of Elfiswandi et al., (2020) analyzing the level of stock returns of mining and energy sector companies by paying attention to the influence of macroeconomic factors and energy consumption. The shares used are companies in the mining and energy sectors listed on the Indonesia Stock Exchange (IDX) for the 2014-2018 periods. In addition, Antono et al., (2019) conducted an analysis of the factors that affect the stock prices of mining and energy companies. Factors that affect stock prices used are world oil prices, inflation and exchange rates. The data used were obtained from the IDX, Energy Information Administration (EIA), and Bank Indonesia as many as 35 companies. The results showed that the Price to Earnings Ratio (PER) and world oil prices had a positive and significant effect on stock prices. In addition, inflation has a negative and significant effect on stock prices while the exchange rate has no significant effect on stock prices.

Based on the description of the problems mentioned above, this study is interested in determining the performance of the Markowitz model investment portfolio in several stocks in the mining and energy sectors. This is based on the fact that portfolios that provide a higher rate of return do not always perform better than other portfolios. This is due to the importance of considering risk factors so that there is a need for standard portfolio performance measurements. From the goal, investors are also expected to get the maximum possible return with minimum risk. The method that will be used is the Markowitz model. The implications of this research can provide assistance to investors to consider the selected asset when making an investment.

2. Material and Methodology

2.1 Material

The data used in this study is in the form of stock data in the mining and energy sectors, in the form of daily data for three years from April 5, 2016 – April 5, 2019. The data used is obtained from the Indonesia Stock Exchange (IDX), which is accessed through the website www.yahoofinance.com. At the data analysis stage, the software used in the form of MS Excel and Matlab in helping research data processing. The stock data used consists of 10 selected mining and energy sector stocks, which include stock prices: ADRO, ANTM, BSSR, DKFT, ELSA, PGAS, PSAB, RUIS, SMRU, and TINS. Of the 10 selected stocks, the best 5 stocks will be selected based on the level of the stock ratio.
2.2 Mean Vector, Unit Vector, and Variance-Covariance Matrix

In the process of calculating portfolio returns, several things that need to be considered are the proportion of each stock and the return of the stock. Suppose a portfolio has $N$ assets with each asset's average being $\mu_1, \mu_2, \ldots, \mu_N$ and the return of each asset is $r_1, r_2, \ldots, r_N$ with the proportion or weight of each asset being $w_1, w_2, \ldots, w_N$. So, it can be expressed through a vector as follows (Sukono et al., 2019):

$$
\mathbf{r} = \begin{pmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{pmatrix}, \quad \mathbf{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_N \end{pmatrix}, \quad \mathbf{w} = \begin{pmatrix} w_1 \\ w_2 \\ \vdots \\ w_N \end{pmatrix}, \quad \text{and} \quad \mathbf{e} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}
$$

with,

- $\mathbf{r}$: vector Return Vector
- $\mathbf{\mu}$: mean Vector
- $\mathbf{w}$: weight vector
- $\mathbf{e}$: Unit Vector

Furthermore, $\Sigma$ and $\mathbf{I}$ represent the covariance matrix and the identity matrix, respectively, which are expressed as:

$$
\Sigma = \begin{pmatrix}
\sigma_{11} & \sigma_{12} & \cdots & \sigma_{1N} \\
\sigma_{21} & \sigma_{22} & \cdots & \sigma_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
\sigma_{N1} & \sigma_{N2} & \cdots & \sigma_{NN}
\end{pmatrix}
$$

and

$$
\mathbf{I} = \begin{pmatrix} 1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1
\end{pmatrix}
$$

where $\sigma_{ij}$ represents the covariance between stocks $i$ and $j$ where $\sigma_i = \sqrt{\sigma_{ii}}$ $(i = 1, 2, \ldots, N)$ is called the standard deviation.

2.3 Portfolio Optimization using Markowitz Model.

In the Markowitz approach, the selection of portfolios by investors is based on their preference for the expected return and risk of each portfolio. In portfolio theory, there are concepts of efficient portfolio and optimal portfolio. An efficient portfolio is a portfolio that provides maximum return for investors with a certain level of risk of return, or a portfolio that offers the lowest risk with a certain level of return. While the optimal portfolio is a portfolio that is chosen by investors from the many choices in an efficient portfolio. According to Tandelilin (2010), the optimal portfolio is a portfolio that is chosen by an investor from the many choices available in an efficient portfolio collection. Optimal portfolio can also be interpreted as something unique to investment in risky assets.

Furthermore, in determining the expected return of the portfolio in vector form, it can be stated as follows:

$$
\mu_p = \mathbb{E}\{r_p\} = \mathbf{\mu}^T \mathbf{w} = \mathbf{w}^T \mathbf{\mu}
$$

(1)

and portfolio risk can be expressed by variance in vector form, can be expressed as follows:

$$
\sigma_p^2 = \text{Var}\{r_p\} = \mathbf{w}^T \Sigma \mathbf{w}
$$

(2)

Mean-Variance Optimization, efficient portfolio is defined as follows:

**Definition 1:** A portfolio $p^*$ (Mean-Variance) is efficient if there is no portfolio $p$ with $\mu_p \geq \mu_{p^*}$ and $\sigma_p^2 < \sigma_{p^*}^2$ (Panjer et al., 1998).

To obtain an efficient portfolio, it is customary to use the maximizing objective function,

$$
2\tau \mu_p - \sigma_p^2, \tau \geq 0
$$

(3)

where the risk tolerance of investors is expressed by the parameter $\tau$. This means that investors with risk tolerance ($\tau \geq 0$) must solve portfolio problems:

Maximize $\{2\tau \mu_p - \sigma_p^2\}$

$w \in \mathbb{R}^N$

With the provision of $\sum_{i=1}^N w_i = 1$

or

Maximize $\{2\tau \mathbf{\mu}^T \mathbf{w} - \mathbf{w}^T \Sigma \mathbf{w}\}$

$w \in \mathbb{R}^N$

With the provision of $\mathbf{e}^T \mathbf{w} = 1$

(4)

with $\mathbf{e}^T = (1, 1, \ldots, 1) \in \mathbb{R}^N$. It should be noted that the solution (4), for all $\tau \in [0, \infty)$ forms the complete set of efficient portfolios.
2.4 Rasio Sharpe

Sharpe (1966) attempted to formulate a measure to assess portfolio performance. The calculation uses the concept of the Capital Market Line or better known as the Reward-to-Variability-Ratio (RVAR). RVAR is obtained by comparing the average excess rate of portfolio return from the average risk-free interest rate (called portfolio risk premium), with portfolio risk. In this case the portfolio risk is the total risk and is expressed by the standard deviation.

In the form of a mathematical equation, RVAR can be expressed as follows:

\[
RVAR = \frac{(AR_{pt} - AR_{ft})}{\sigma_p}
\]  

(5)

with:

- \(AR_{pt}\): Average portfolio return rate over t period
- \(AR_{ft}\): Average risk-free interest rate over t period
- \(\sigma_p\): Portfolio standard deviation
- \(AR_{pt} - AR_{ft}\): Portfolio risk premium (Sharpe performance index)

The above equation shows the magnitude of the risk premium per unit standard deviation of the portfolio (total risk). That is, RVAR tries to measure the amount of additional risk premium, if the total portfolio risk increases by one unit. Thus, the higher the Sharpe Index of a portfolio, the better the portfolio's performance (Tandelilin, 2010).

3. Results and Discussion

3.1 Stock Price Data

The data used in this study is in the form of stock data in the mining and energy sectors, in the form of daily data for three years from April 5, 2016 – April 5, 2019. Data from the 10 nominated stocks, each share price fluctuated up/strengthened or decreased/weakened during research period. The chart of stock price data that has been nominated as shown in Figure 1 is as follows:

![Stock Price Chart](image)

**Figure 1.** Nominated Stock Price Chart

Based on the stock price chart in Figure 1, several characteristics of the stock prices that were nominated can be explained. Stock prices increase or decrease around a certain time. Where in certain periods sometimes rises
3.2 Estimated Expectation and Stock Return Variance

Based on stock price data, stock returns are then determined, this is needed to obtain the expected value and variance of stock data returns. To get the expected value and variance of return data is easier to do with the help of excel software. The results of the expectation and variance of stock returns for the 10 nominated stocks along with the ratio between expectations and variance of returns are summarized in Table 1.

<table>
<thead>
<tr>
<th>Stock Name</th>
<th>Mean ((\mu))</th>
<th>Variance ((\sigma^2))</th>
<th>Ratio((\frac{\mu}{\sigma^2}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADRO</td>
<td>0.000829947</td>
<td>0.000731517</td>
<td>1.134554716</td>
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<td>ANTM</td>
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<td>0.765463305</td>
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<td>-7.22637E-15</td>
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<td>0.000999263</td>
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<tr>
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<td>TINS</td>
<td>0.000842023</td>
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<td>1.046646854</td>
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</table>

The next stage is to form an investment portfolio consisting of 5 nominated stocks. Therefore, from the 10 stocks in Table 1, five randomly selected nominees were selected. The following stocks are included in the investment portfolio selected at random, presented in Table 2.

<table>
<thead>
<tr>
<th>Nama Saham</th>
<th>Mean ((\mu))</th>
<th>Variance ((\sigma^2))</th>
<th>Ratio((\frac{\mu}{\sigma^2}))</th>
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</thead>
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</tbody>
</table>

3.3 Creating Mean Vectors, Unit Vectors, and Variance-Covariance Matrices

In this section, we intend to construct the mean vector, unit vector, and covariance matrix as well as the inverse of the covariance matrix of returns from random stocks. Using the estimated mean value in Table 2, it is arranged in the form of a mean transpose vector \(\mathbf{\mu}^T = (0.000227414 \ 0.000829947 \ 0.000118866 \ 0.000842023 \ 0.001011078)\), then formed the unit transpose vector \(\mathbf{e}^T = (1 \ 1 \ 1 \ 1 \ 1)\). While the covariance matrix \(\Sigma\) as follows:
The Markowitz Model Investment Portfolio Optimization Process

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>RUIS</th>
<th>ADRO</th>
<th>ELSA</th>
<th>TINS</th>
<th>SMRU</th>
<th>$w^T \epsilon$</th>
<th>$\mu_p$</th>
<th>$\sigma_p^2$</th>
<th>$\mu_p / \sigma_p^2$</th>
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</thead>
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<td>0.2587</td>
<td>0.2397</td>
<td>0.000764421</td>
<td>0.000254736</td>
<td>3.00810090596810</td>
<td></td>
</tr>
</tbody>
</table>

Looking at the results in Table 3, taking the risk tolerance value is only for a value of $0 \leq \tau \leq 0.25$. This is because the risk tolerance value $\tau > 0.25$ produces a negative weight. Considering Table 3, it can be seen that with a risk tolerance of $\tau = 0.00$, a portfolio composition that produces a minimum portfolio return expectation of 0.00053248 with a minimum variance of 0.000194431 is obtained.

How changes in the average value and variance of portfolio returns along with the increase in risk tolerance value can be seen in Table 3. In this case, the maximum value of risk tolerance is $\tau = 0.25$, where the resulting portfolio weight composition with the highest expected portfolio return is $0.0007555$ and variance is $0.000250186$. Looking at the results in Table 3, it appears that every increase in the value of risk tolerance causes an increase in the expected return of the portfolio, and is also accompanied by an increase in portfolio variance.

A series of efficient portfolios is on the efficient frontier. The efficient frontier is the efficient surface on which lies portfolios whose returns are commensurate with the risks. Based on the results of the calculation process in Table 3, it is found that efficient portfolios are located along the line with a risk tolerance of $0 \leq \tau \leq 0.25$, where the highest
Portfolio return expectation is 0.0007555 and the minimum portfolio return is 0.00053248 as can be seen in Figure 2.

![Figure 2. Efficient Frontier Portfolio Markowitz B](image)

After obtaining a series of efficient portfolios, the next step is to determine the optimum portfolio composition. Every investor wants a portfolio investment that can generate large returns but is accompanied by a small level of risk. If it is assumed that investor preferences are only based on the average return and risk of the portfolio, the optimal portfolio selection can be determined based on the composition of the efficient portfolio, and produces the largest ratio between expected return and portfolio variance. The graph of the ratio between the average return and the variance of the portfolio looks as shown in Figure 3.

![Figure 3. Optimum portfolio of the Markowitz model](image)
Based on Figure 3, it can be seen that the ratio between the average and the largest portfolio return variance is 3.107739053221320 or obtained when the risk tolerance is $\tau = 0.16$. The ratio between the mean and variance of portfolio returns increased at the risk tolerance interval of $0 \leq \tau \leq 0.16$ and decreased at the risk tolerance interval of $0.16 < \tau \leq 0.25$. In Figure 3 it can also be seen that the optimal portfolio composed of 5 randomly nominated stocks is a portfolio with the composition of the weight vector as follows:

$$
\mathbf{w}^T = (0.194709686422055, 0.310118677009046, 0.004241209935416, 0.254558667888638, 0.236371758744846),
$$
respectively for RUIS, ADRO, ELSA, TINS, and SMRU stocks. This optimal portfolio composition produces an average return of 0.000675213 and a variance or risk level of 0.000217268.

4. Conclusion

Based on the results of the analysis, it can be concluded that the level of efficiency in the Markowitz Model investment portfolio chosen from the nominee stocks randomly produces the highest expected portfolio return of 0.0007555 and the minimum portfolio return of 0.00053248. While the results of the optimum portfolio composition resulted in a weight vector $\mathbf{w}^T = (0.249477285575, 0.264318283593, 0.062500431273, 0.217198530099, 0.206505469457)$, where the optimum portfolio composition obtained a portfolio return of 0.000675213 with a portfolio variance of 0.000217268.

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References


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