ARMA-GJR-GARCH Model for Determining Value-at-Risk and Back testing of Some Stock Returns

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ABSTRACT

Investment is a certain amount of money or other resources that is done in the hope of obtaining benefits in the future. Stocks are a common investment that is much in demand by investors. In investment activities, there is the most important component, namely volatility. Such volatility is identical to the conditional standard deviation of stock price return. The important thing in investing in addition to return is risk. Value at Risk (VaR) is a statistically estimated method of risk assessment of the maximum losses that may occur on a capital market instrument at a certain confidence level. To evaluate the quality of VaR estimates, models should always be back tested with appropriate methods. Back testing is a statistical procedure in which actual gains and losses are systematically compared to appropriate VaR estimates. The goal of this research is a time series model to determine Value at Risk and back testing. So the model that will be used is the Autoregressive Moving Average-Glosten Jagannathan Runkle-Generalized Autoregressive Conditional Heteroscedastic (ARMA-GJR-GARCH) model. ARMA is a combination of AR and MA models, while GJR-GARCH is a development of the GARCH model by incorporating leverage effects. The time series method used in this research is the ARMA-GJR-GARCH model on determining Value at Risk. The results obtained based on return data used against the two stocks analyzed were the ARMA (3.1)-GJR-GARCH (2.0) and ARMA (1.2)-GJR-GARCH (1.1) models. The value at risk return of the two stocks analyzed was 0.0225 and 0.0439, respectively. Back testing conducted against both stocks had a good performance, with QPS values of 0.1573 and 0.0364. Such research can be used by investors as a consideration in investing decision-making.

Keywords:

Return, volatility, leverage effect,Value at Risk, Back testing, ARMA-GJR-GARCH

1. Introduction

Investment is the placement of funds in the form of other assets during a certain period with certain expectations (Kalfin et al., 2019a; Kalfin et al., 2019b). In the object of investment, assets are generally divided into two, namely rill assets and financial assets. Real assets related to infrastructure and financial assets related to stocks (Dwipa, 2016). Investors choose to invest shares in a company based on the desire to get profits in the future that can be seen from the amount of stock returns. Investing in stocks will be faced with high risk because stock returns are volatile (Kalfin et al., 2020; Sukono er al., 2020). The return of the stock will change in a very fast span of time so that the value of the stock index will also change, this movement is known as the volatility of stock returns. High volatility produces risk that is similar if low volatility produces low risk. Therefore, it needs to be overcome by using mathematical models (Hasbullah et al., 2020).

Some researchers have used various time series models, one of which is the ARCH model introduced by Engle (1982). According to Dwipa (2016) financial returns have three characteristics. The first is the grouping of volatility, meaning that very large changes occur at certain times and small changes in other periods. The second is fat tailedness (excess kurtosis) means that financial returns display a taperingance greater than normal distributions. Third, there is a leverage effect, a situation in which bad news and good news conditions have an unsymmetrical effect on return volatility.

The Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model has a more flexible structure to accommodate the volatility nature in financial data, Bollerslev (1986). Tamilselvan and Vali (2016) used the GARCH model in researching Muscat's shares in the market by concluding that the GARCH model (1.1) is the best estimate of symmetrical data and there is no leverage effect on the data used. However, the GARCH model cannot be used on data that has a leverage effect. Ali (2013) used the EGARCH, GJR-GARCH, TGARCH, AVGARCH, NGARCH, IGARCH and APARCH models to determine the functional relationships of the pantogen indicator time series to activate reactions on the beach. However, the TGARCH model is marginally better than other models in capturing the response of pantogen variable indicators. Mittnik, Paolella and Rachev (2002) examined the stationary of the stable GARCH process. However, the process on the GARCH model cannot explain the asymmetric phenomenon, therefore the Glosten Jagannatan Runkle-Generalized Autoregressive Conditional Heteroscedasticity (GJR-GARCH) model will be used. The GJR-GARCH model can overcome asymmetrical effects (Lee, 2007).

The risk measurements in this research used Value at Risk and were tested by back testing. According to Bakhtiar et al. (2020) Value at Risk is one of the most popular tools used by investors in risk measurement. Value at Risk is used as a measuring tool that can assess the worst losses in investing at a given time and level of trust. Some risk measurement level studies use Value at Risk. Sukono et al. (2019) examined the ARIMA-GARCH model conducted to estimate and shortfall expectations of several stocks in the Indonesian capital market. Based on the analysis, acquired the preferred shares. Bank Mandiri shares have the lowest level of risk and Mustika Ratu shares have the highest level of risk, with the value-at-risk of stocks generally smaller than the expected shortfall value. Bucevska (2012) conducted a relative test of selected GARCH type models in terms of the ability to estimate volatility and extended empirical research on VaR estimates in financial markets. Nilsson (2017) found the best APARCH model for estimating volatility, while to estimate VaR the best model is APARCH, GJR-GARCH or EGARCH depending on which VaR level is used.

The research used the Autoregressive Moving Average-Glosten Jagannatan Runkle-Generalized Autoregressive Conditional Heteroscedasticity (ARMA-GJR-GARCH) model to determine the Value at Risk and Back testing value of stock returns. The goal of the research was to determine the best ARMA-GJR-GARCH model to estimate value at risk and use the Back testing test on stock return data.

2. Methods

2.1 Return

According to Ruppert (2011) return is the return on the results obtained due to making investments. In general, the formula of return is as follows:

$$r_t = \ln\left(\frac{S(t_i)}{S(t_{i-1})}\right) \tag{1}$$

where r_t is the return of the stock at the t-time, $S(t_i)$ is a stock price in the t_i and $S(t_{i-1})$ is the stock price in the t_{i-1} period.

2.2 Stasionarity

The stasionary test is the underlying assumption in statistical procedures used in time series analysis. According to Tsay (2005) data percentage test can use the Augmented Dickey Fuller test (ADF). Augmented Dickey Fuller test (ADF) is a stationary test in average, where the ADF test statistics are as follows:

$$ADF = \frac{\hat{\delta}}{SE(\hat{\delta})} \tag{2}$$

where $SE(\hat{\delta})$ is a standard error for $\hat{\delta}$. The decision making is if the value $ADF < \alpha$ then refused H_0 in other words, stationary data. If the vaue $ADF > \alpha$ then accepted H_0 in other words, the data is not stationary.

2.3 ARMA Model

The purpose of the ARMA model is to discuss the the average model in the time series. Autoregressive Moving Average (ARMA) model can be expressed in the following equation:

$$r_t = \omega + \sum_{i=1}^{p} \phi_i r_{t-i} + a_t - \sum_{j=1}^{q} \theta_j a_{t-j} \tag{3}$$

with r_t is the value of return at the time to t, a_t is a process of the white noise or error at the time t (Sukono, *et al.,* 2017).

ARMA modeling process. In general, the ARMA modeling process is: (i) Identify the model by determining the values p and q with the autocorrelation function (ACF) and partial autocorrelation function (PACF) of the correlogram plot. (ii) Parameter estimation can use the smallest square method or maximum likelihood. (iii) Diagnostic test with white noise and non-correlation test against residual using Box-Pierce or Ljung-Box. (iv) Forecasting, if the model is suitable can be used for predictions made recursively.

2.4 ARCH Model

The ARCH model is used to estimate volatility introduced by Engle (1982). This model is used when the variance error in the model follows an autoregressive (AR) form. To model a time series using the ARCH process (p).

$$a_t = \sigma_t v_t, \quad \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \cdots + \alpha_m a_{t-m}^2 \tag{4}$$

where v_t is the order of *independent and identically distributed* (iid), a_t is a residual return from the average model, σ_t^2 is a residual variance squared at time to t and $\alpha_1 a_{t-1}^2$ is a component of ARCH.

ARCH effect test. The most widely used test to detect the ARCH effect is Lagrange Multiplier (LM). Based on equations (4) for the ARCH (p) effect test based on the null hypothesis $H_0: \alpha_1 = \alpha_2 = \cdots = \alpha_p = 0$ there is no ARCH effect and $H_1: \exists \alpha_i \neq 0$, $i = 1,2,\ldots,p$ there is an ARCH effect. The test statistics used are as follows.

$$LM = nR^2 \tag{5}$$

with n is a lot of data R^2 is the coefficient of determination in the previous regression model.

2.5 GARCH Model

Bollerslev (1986) develop the ARCH model into the GARCH model (p,q) where q is the ARCH order and p is the GARCH order. In general, the GARCH model is as follows:

$$a_t = \sigma_t v_t, \qquad \sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i a_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{6}$$

where ϵ_t is the order of *independent and identically distributed* (iid), σ_t^2 is a residual variance a the t, ω is a constant component, α_i is the parameter to i of the ARCH, a_{t-i}^2 is the square of the residual in time (t-i), β_j is the parameter to j of GARCH, σ_{t-j}^2 is a variance of residuals at time (t-j) . Equations (3) indicates that conditional variance is a past shock seen from residual square (p) and pas residual variance (q) (Olowe dan Ayodeji, 2010).

Volatility model process. In general, the volatility model process is: (i) Estimated ARMA model with time series model. (ii) use residuals from the ARMA model to test the ARCH effect. (iii) If there is an ARCH effect, the volatility model estimate, and the combined estimates from the ARMA model and the volatility model. (iv) Perform diagnostic tests to observe the suitability of the model. (v) If the model has matched, use it to predict based on recursive predictions.

2.6 Asymmetry

According to Bakhtiar, et al (2020) asymmetrical test is a property that shows an imbalance of certain conditions or objects. In time series asymmetrical properties are called leverage effect or high volatility. To determine the nature of asymmetries is with skewness and kurtosis. Skewness is an imbalance of degrees in distribution. Asymmetrical test can be done using cross correlation between residual lags (ε_t) with residual squares (ε_t^2).

2.7 GJR-GARCH Model

Glosten Jagannathan dan Runkle (1993) introduced the asymmetric GARCH model, the GJR-GARCH model. The advantage of the GJR-GARCH model is that it can measure volatility due to the different effects of bad news and good news. The difference between the GJR-GARCH model and the GARCH model is that the GJR-GARCH model has parts that represent asymmetrical properties. The GJR-GARCH model is as follows:

$$a_t = \sigma_t \epsilon_t, \qquad \sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i a_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 + \gamma_i I_{t-i} a_{t-i}^2 \tag{7}$$

and

$$I_{t-i} = \begin{cases} 1, a_{t-i} < 0 \\ 0, a_{t-i} \geq 0 \end{cases} \tag{8}$$

with α_i is the parameter of ARCH, β_j is the parameter to j of GARCH and γ_i is the parameter to i laverage effect, I_{t-i} is a dummy variable which means a functional index that is worth zero when a_{t-i} positif and worth one when a_{t-i} negative. If parameter $\gamma_i > 0$ then negative error does not work which means that the influence of bad news will be greater than the influence of good news (Dritsaki, 2017).

GJR-GARCH model Process: (i) Estimated GARCH model with time series model. (ii) Use residuals from the GARCH model to test the ARCH effect. (iii) Perform diagnostic tests to observe the suitability of the model. (iv) Asymmetric effect test. (v) If there is an asymmetric effect, it can be used to predict based on recursive predictions.

2.8 Value at Risk

According to Dwipa (2016) VaR is defined as the maximum potential loss in a given period with a certain level of confidence under normal (market) circumstances. VaR at the trust level $(1-\alpha)$ and time interval t can be formulated as follows:

$$VaR = \inf\{r_t | F_l(r_t) \geq \alpha\} \tag{9}$$

where F_l is a distribution function of return r_t. Then VaR for the next period with a level of trust α can be formulated as follows:

$$VaR = \mu + \sigma_t F^{-1}(\alpha) \tag{10}$$

with μ is the mean, σ^2 is the variance and σ is a standard deviation.

2.9 Back testing

Back test is a method used to measure the performace of VaR that has been estimated. If r_t declaring a profit or loss at time t and VaR_t is a VaR prediction at time t. In 1998 Lopez introduced the following size-adjusted frequency approach:

$$C_t = \begin{cases} 1 + (r_t - VaR_t)^2, r_t > VaR_t \\ 0, \quad r_t \leq VaR_t \end{cases} \tag{11}$$

Statistics used to test VaR risk performance are using quadratic probability score (QPS). The QPS equation is as follows:

$$QPS = \left(\frac{2}{n}\right)\sum_{i=1}^{n}(C_t - p)^2 \tag{12}$$

where n that's a lot of data, p is a probability value. The QPS value is between the $[0,2]$ range with 0 being the minimum value that occurs when $r_t \leq VaR_t$ and 2 is the maximum value that occurs when $r_t > VaR_t$. VaR performance is said to be good when small QPS approach 0 (Sukono dkk., 2019).

3. Data Collection

The data used is data return from PT. Unilever Indonesia Tbk (UNVR) and PT Indofood CBP Sukses Makmur Tbk (ICBP). The amount of data used as much as 746 data over an interval of years. Data collection starts from August 27, 2018 to August 24, 2021. This data can be downloaded through the website https://finance.yahoo.com/. the usefulness of data is to determine the model of ARMA-GJR-GARCH to estimate the Value at Risk and Back testing.

4. Results and Discussions

4.1 Data Return

The data used is data return from PT. Unilever Indonesia Tbk (UNVR) and PT Indofood CBP Sukses Makmur Tbk (ICBP). The data analyzed is the closing price of the stock. The data is daily data starting from August 27, 2018 to August 24, 2021, the amount of data used as much as 746 data. For example Figure 1 is the closing price of the stock and Figure 2 is the return of UNVR shares.

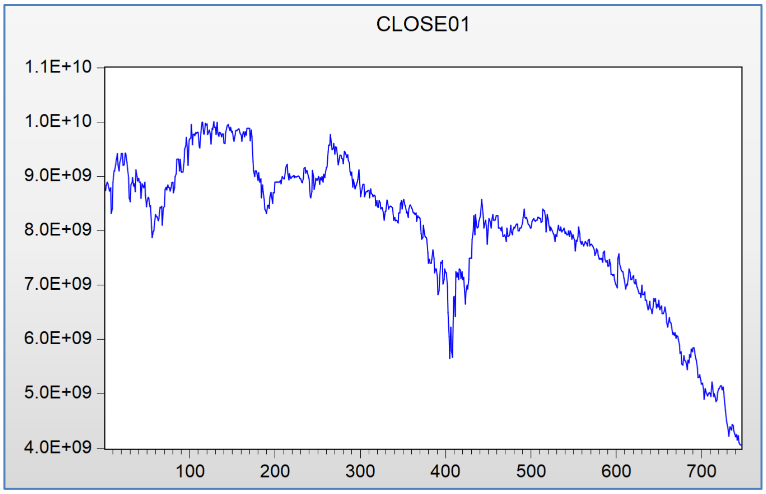


Figure 1. UNVR Share Price

In Figure 1, it can see the fluctuating stock price movement up and down. But the stock price movement on the data to 400 is seen to fall and on data to 700 and above the UNVR stock price dropped. The stock price in the initial data has increased and the share price of UNVR is decreasing. From the closing data of the stock price will be calculated the value of the return of the stock using the equation (1). So that, the return of shares can be seen in Figure 2 below. The same calculation will be donr on ICBP shares.

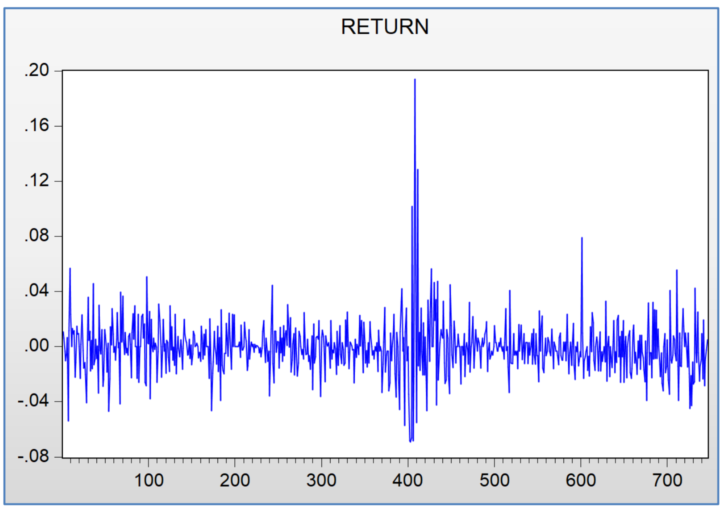


Figure 2. UNVR Stock Return

Seen in the Figure 2 is the data return of UNVR shares. The data return above looks fluctuating or up and down and forms clusters where there are clusters that are relatively high and clusters that are relatively low. After obtaining the stock return data will continue stationary testing of data return to section 4.2.

4.2 Stationary Test

The stationer test uses Dickey Fuller with a probability value 5%. The formula used for this stationary test is to use the equation (2). Stationary testing on this research used Eviews 10 software, so it was obtained as follows:

Table 1. Stationary test of stock return

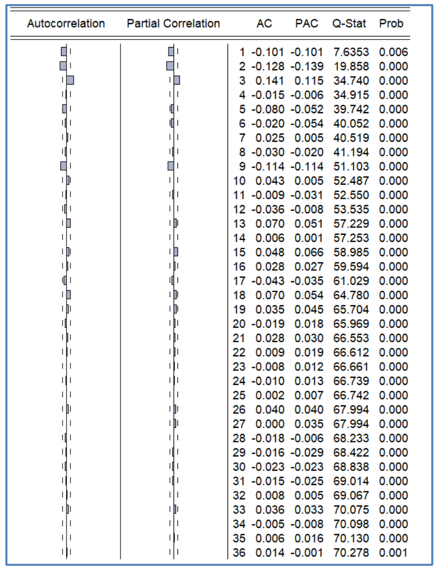
No	Stock Name	ADF Value	*Critical Value*	Probability	Stasioneritas
1.	UNVR	-15.7219	-2.8652	0.0000	Stasioner
2.	ICBP	-28.6429	-2.8652	0.0000	Stasioner

In Table 1 it can be explained that the value of ADF in UNVR shares is -15.72190 and ADF in ICBP shares is -286429, the critical value of both shares with a probability of 5% is -2.8652 and probability values in UNVR and

ICBP shares are 0.0000. The probability value obtained is less than the probability value used, which is 5%, meaning that the data return of UNVR and ICBP shares has been stationary. If the return data has been stationary, it will proceed to identify the ARMA model and diagnostic tests.

4.3 ARMA Models and Diagnostic Test

Stationary stock return data will be continued by identifying the ARMA model. ARMA models can be identified by looking at ACF and PACF on correlogram plots. Figure 3 is an example of the output of a correlogram plot on UNVR shares.



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.101	-0.101	7.6353	0.006
		2	-0.128	-0.139	19.858	0.000
		3	0.141	0.115	34.740	0.000
		4	-0.015	-0.006	34.915	0.000
		5	-0.080	-0.052	39.742	0.000
		6	-0.020	-0.054	40.052	0.000
		7	0.025	0.005	40.519	0.000
		8	-0.030	-0.020	41.194	0.000
		9	-0.114	-0.114	51.103	0.000
		10	0.043	0.005	52.487	0.000
		11	-0.009	-0.031	52.550	0.000
		12	-0.036	-0.008	53.535	0.000
		13	0.070	0.051	57.229	0.000
		14	0.006	0.001	57.253	0.000
		15	0.048	0.066	58.985	0.000
		16	0.028	0.027	59.594	0.000
		17	-0.043	-0.035	61.029	0.000
		18	0.070	0.054	64.780	0.000
		19	0.035	0.045	65.704	0.000
		20	-0.019	0.018	65.969	0.000
		21	0.028	0.030	66.553	0.000
		22	0.009	0.019	66.612	0.000
		23	-0.008	0.012	66.661	0.000
		24	-0.010	0.013	66.739	0.000
		25	0.002	0.007	66.742	0.000
		26	0.040	0.040	67.994	0.000
		27	0.000	0.035	67.994	0.000
		28	-0.018	-0.006	68.233	0.000
		29	-0.016	-0.029	68.422	0.000
		30	-0.023	-0.023	68.838	0.000
		31	-0.015	-0.025	69.014	0.000
		32	0.008	0.005	69.067	0.000
		33	0.036	0.033	70.075	0.000
		34	-0.005	-0.008	70.098	0.000
		35	0.006	0.016	70.130	0.000
		36	0.014	-0.001	70.278	0.001

Figure 3. Plot Correlogram UNVR Stock

From Figure 3 above, can identify the ARMA model (3,3), ARMA (3,2) dan ARMA (3,1) Stock of UNVR. So that from these 3 models can be obtained the best ARMA model, it is ARMA (3,1). The same thing will be done to ICBP shares so that the best model of ICBP dhares is ARMA (1,2).

Table 2. Best ARMA Models

No	Stock Name	ARMA Model	Parameter	Parameter Estimation	P-value
1.	UNVR	ARMA (3,1)	AR (3)	0.140621	0.0000
			MA (1)	-0.997077	0.0000
2.	ICBP	ARMA (1,2)	AR (1)	-0.998250	0.0000
			MA (2)	-0.998164	0.0000

Table 2 explains that the best models of the two stocks to be researched ar ARMA (3,1) and ARMA (1,2). For ARMA (3,1) with an estimated parameter value of AR(3) is 0.140621 and the estimated value of the MA (1) is -0.997077. As for ARMA (1,2) the estimated value of AR (1) parameters is -0.998250 and MA (2) that is -0.998164. The p-value of both models is 0.0000 less than the 5% probability value. From the estimated value of the parameters that have been obtained, a model will be made of that value. So that, the model can be seen in Table 3 by following the equation (3).

Table 3. ARMA Model

No	Stock Name	ARMA Model	Model Equations Stock
1.	UNVR	ARMA (3,1)	$z_t = 0.140621 z_{t-3} - 0.997077 a_{t-1} + a_t$
2.	ICBP	ARMA (1,2)	$z_t = -0.998250 z_{t-1} - 0.998164 a_{t-2} + a_t$

Once the ARMA models is obtained as in Table 3, diagnostic testing will be conducted on the ARMA model (3,1) and ARMA (1,2) to see that the models is well used. By performing diagnostic tests on the ARMA (3,1) and ARMA (1,2) models, it is obtained that the model has white noise. So that the ARMA (3,1) and ARMA (1,2) models can be continued to the next stage, it is the heteroscedasticity test.

4.4 Heteroscedasticity Test

Heteroscedasticity tests are performed to see if the model has a constant residual or not constant. This tests can use equation (5) with the hypothesis that if the value of p-value $< \alpha$ then there is an ARCH effect and if the value of p-value $> \alpha,$ there is no ARCH effect and cannot be continued for further model testing. In the ARMA models that have been obtained, residual tests for ARMA (3,1) and ARMA (1,2) models ar 0.0000 and 0.0001 which means ARMA (3,1) and ARMA (1,2) models have ARCH effect and can be continued to identify GARCH models.

4.5 GARCH Model and Diagnostic Test

The GARCH model is performed on UNVR and ICBP stock data that contains heteroscedasticity properties. The next process is the estimation of GARCH by looking at the ACF and PACF plots. The GARCH model can be seen in the following table.

Table 4. GARCH Model

No	Stock Name	GARCH Model	Model Equations Stock
1.	UNVR	GARCH (2,0)	$\sigma_t^2 = 0.000189 + 0.312884a_{t-1}^2 + 0.190794a_{t-2}^2 + u_t$
2.	ICBP	GARCH (1,1)	$\sigma_t^2 = 9.22 \times 10^5 + 0.221163a_{t-1}^2 + 0.490563\sigma_{t-2}^2 + u_t$

In Table 4 the GARCH model is obtained using the equation (7). So can see the constant value obtained from both GARCH (2,0) model which is 0.000189 and the value of each ARCH parameter are 0.312884 and 0.190794. For the GARCH (1,2) model the constant value is obtained 9.22×10^5, the value of the ARCH parameter is 0.221163 and the value of the GARCH parameter is 0.490563. From both models will then be diagnostic tests to see that the model is well used. By conducting diagnostic tests on GARCH (2,0) and GARCH (1,1) models, it can be obtained that the model has white noise. So that the GARCH (2,0) and GARCH (1,1) models can be continued to the next stage, namely the ARCH effect test.

4.6 ARCH Effect Test

The ARCH effect test is the same as the heteroscedasticity test to see if the model has a constant residual or not constant. In the GARCH (2,0) model by following the equation (5) obtained a value p-value of 0.2634. While in the GARCH (1,1) model obtained a p-value of 0.7805. So the GARCH (2,0) and GARCH (1,1) models have no ARCH effect on the model, meaning they do not contain heteroscedasticity. This proved that testing can be continued to the next stage, namely the asymmetric GARCH method.

4.7 Asymmetric Test

This asymmetric test is also called cross correlation test which means multiplication between residual lag (u_t) and residual square (u_t^2). The multiplication is done to see if the GARCH model contained in Table 4 has asymmetric properties or not. For cross correlation cheking using Eviews 10 software. From both models, the results obtained using Eviews 10 software can be explained that there are some lags that go out of the line (barlet) which means the lag value differs significantly from zero. The value obtained is also not the same as zero, so the data used has asymmetric properties, so bad news and good news conditions have an asymmetric influence on volatility.

4.8 GJR-GARCH Model

The GJR-GARCH model can be done when it has known the asymmetric properties of the GARCH model found in the Table 4. The restoration of the GJR-GARCH model parameters from the selected model is GARCH (2,0) and GARCH (1,1). So that, the GJR-GARCH model is presented in the following table.

Table 10. GJR-GARCH Model Determination

No	Stock Name	GJR-GARCH Model	Parameter	Parameter Estimation	P-value
1.	UNVR	GJR-GARCH (2,0)	$\boldsymbol{\omega}$	0.000213	0.0000
			$\boldsymbol{\alpha_1}$	-0.007415	0.0000
			$\boldsymbol{\alpha_2}$	0.140679	0.0000
			$\boldsymbol{\gamma_1}$	0.522536	0.0000
2.	ICBP	GJR-GARCH (1,1)	$\boldsymbol{\omega}$	9.65×10^5	0.0000
			$\boldsymbol{\alpha_1}$	0.087935	0.0030
			$\boldsymbol{\beta_1}$	0.482334	0.0000
			$\boldsymbol{\gamma_1}$	0.233828	0.0003

According to Table 10, the asymmetric values produced from each model are not equal to zero. The GJR-GARCH coefficient value of Table 10 is greater than zero is 0.522536 and 0.233828 which means that there will be a crash so that the volatility of the return value for laverage effect will have a significant effect. From the results of both models the influence of bad news received will be greater in the volatility of return than in the influence of good news.

4.9 Value at Risk and Back testing

Before determining the value of value at risk, it will be predicted the average value and volatility of the stock's return one period ahead. Using the average model and volatility of stock returns for UNVR and ICBP, the value of the $\hat{\mu}_t = \hat{r}_t$ and $\hat{\sigma}_t^2 = \sigma_t^2$. These results can be seen in the following table.

Table 11. Mean, varian, VaR and QPS

No	Stock Name	$\hat{\boldsymbol{\mu}}_t$	$\hat{\boldsymbol{\sigma}}_t^2$	$\boldsymbol{VaR_t}$	QPS
1.	UNVR	0.0009	0.0002	0.0225	0.1573
2.	ICBP	0.0017	0.0008	0.0439	0.0364

The value at risk is determined by the result of the ARMA (3,1), ARMA (1,2) models for averages and GJR-GARCH (2,0), GJR-GARCH (1,1) for volatility (varian). Estimates of the $\hat{\mu}$ average and volatility (varians) σ^2 are found in Table 11. So obtained the standard deviation value for UNVR shares is 0.0142 and the standard deviation for ICBP shares is 0.0276. If the probability value is 5% then the normal distribution value $z_{0.05} = -1.65$ and the investment assumption is $S_0 = 1$ unit, then the value at risk is obtained by using the equation (10) and the result is in accordance with Table 11 in column VaR_t.

However, value at risk performance estimates should be evaluated using back testing. If the probability value used is 5% then by using the equation (11) and equation (12) the result of QPS is obtained according to Table 11. The QPS value obtained in Table 11 in the range of values [0,2] which means the performance of the value at risk is good. So that risk measurement based on the ARMA-GJR-GARCH model is well used in the stock return data analyzed.

5. Conclusions

In this research has been done to estimate value at risk using the time series model. The data analyzed is data on return of UNVR stock and ICBP stock. Of the two stocks analyzed were obtained the ARMA(3,1)-GJR-GARCH(2,0) and ARMA(1,2)-GJR-GARCH(1,1) models. The value at risk return of the two stocks analyzed was 0.0225 and 0.0439 respectively. Based on a relatively small QPS and in the value range [0,2] shows that risk measurement using value at risk in the stocks analyzed has a good performance. This research is useful for making decisions in investing, so it can help investors in stock selection.

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