Profile of Student's Mathematical Representation based on Teacher's Accepted-Promoted-action

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Abstract
Mathematics learning certainly involves interaction between teacher and students. The interaction can be teacher gives promoted-actions and students will give respons. When the teacher's promoted-actions can stimulate students' mathematical ideas, students will communicate their thinking verbally or non verbally. The way students communicate their ideas is called mathematical representation. Students' representation becomes important thing to evaluate learning because mathematical representation indicates students' understanding of mathematical concepts. This research aimed to describe student's mathematical representation on accepting teacher's promoted-action. This research was a qualitative research with two students in high and middle mathematical ability (HA-MA) as subjects. Data was collected through online observation and interview. The validity of the data was checked by using time triangulation. The results showed that HA-subject's representation was appear when teacher stimulates students to answer verbally then subject pronounce mathematical object frequently and correctly, and when teacher gives an exercise then subject write the mathematical expression correctly. MA-subject's representation was appear when teacher stimulates students to answer verbally then subject pronounce mathematical object correctly in several times, and when teacher gives an exercise then subject write mathematical expression.

Keywords
Mathematical Representation, Promoted Action, Mathematical Communication.

1. Introduction
The pandemic COVID-19 cause a change the learning method from face-to-face into online learning. Nevertheless, the learning process certainly still involve interaction between teacher and students. The interaction in online learning occur trough various applications commonly used by the teachers. Such as whatsapp group, google classroom, youtube, and zoom meeting. All of these application allow the use of video, so that students can see the teacher's expression during explaining the lesson.

Teacher's facial expression is the one of soft skill attributes in teaching mathematics (Hidayati, 2015), so that it become a thing needed to consider in determining what application should be used. On the other hand teacher also needs to assess students' affective. For this purpose, teacher can choose the application that enable the teacher to observe students' activities during online learning easily. A mathematics teacher in MA Madrasatul Quran Tebuireng Jombang prefer to use zoom meeting application for online learning. The reason is it allow teacher and students to interact like face-to-face learning.

The interaction between teacher and students in mathematics learning occurs from beginning to the end of the lesson. In this process, teacher gives some instructions to students that will stimulate students' responses. For example teacher starts learning by saying greetings, asks students to open the book and prepare their stationery, asks students to copy the materials, asks students to solve the problem, invite students to ask question, invite students to make conclusion, and etc. All the teacher activities that make students take action or behave to acquire new skill is called promoted-action(Iffah, 2015).

Based on students' response, promoted-action is categorized into accepted, rejected, and pseudo promoted-action. Promoted-action is accepted when the students conduct a particular instruction given by the teacher, rejected when the students do not conduct particular instruction given by the teacher, and pseudo when the

students apparently accepted the promote action but it was actually not(Iffah, 2017). The accepted promoted action (A-PA) that appears is to provide assistance to formulate concepts and provide opportunities to ask students questions; provide questions related to prerequisite materials to explore students' initial knowledge; ask students to identify parts of the material; deliver material in a structured manner using media; ask students to record material and respond to the questions given(Iffah, 2016).

When the teacher's promoted-actions can stimulate students' mathematical ideas, students will communicate their thinking verbally or non verbally. Verbal mathematical communication is defined as a process of delivering ideas / thoughts mathematics in oral form in the form of utterances (Hidayati, 2016). It covers everything students express about mathematical objects. While non verbal mathematical communication is defined as a process of conveying mathematical ideas in writing (Hidayati, 2015). In this process, students can interpret mathematical ideas in various ways. Such as charts, tables, geometrical shape, mathematical expression, and words. The way students communicate their ideas is called mathematical representation.

The term representation refers both to process and product that are observable externally as well as to those that occur "internally" in the minds of people doing mathematics(NCTM, 2000). The ability of the mathematical representation is the ability to express mathematical ideas (problem statement, solution, definition, etc.) into one of the forms: Drawing, diagram, charts, or tables; mathematical notation, numeric/symbol algebra; and the written text/words, as an interpretation of the mind(Nizarudin, 2014).

The result of previous research showed that students with high, middle, and low mathematical ability have different representation(Rahayu, 2019). Students with high and middle mathematical ability present the correct representation more than students with low mathematical ability during solve problem. It indicates that students with the high and middle mathematical ability have greater possibility to accept teacher's promoted-action than students with low mathematical ability.

Mathematical representation plays a vital role in mathematical activities. Mathematical representation ability as an important component of mathematical literacy has become the educational aim in many countries(Zhe, 2012). As like in the mathematics learning objective in Indonesian curriculum it is stated that the one of indicators to understand mathematical concepts is present concepts in multiple-representation, students should be able to communicate ideas, reasoning and proof by using sentence, symbol, table, diagram, or other tools to make the problem clear(2014).

Students' representation becomes important thing to evaluate learning because mathematical representation indicates students' understanding of mathematical concepts. But in daily teaching, teachers should recognize that cultivating students' mathematical representational ability is not only the goal of the curriculum standard, but an important aspect of improving students' mathematical literacy(Zhe, 2012).

From the above, a research to describe high and middle mathematical ability students' mathematical representation on accepting teacher's promoted-action during online learning via zoom meeting is needed. The research will be useful to evaluate students' representation during studying maths and help the teacher determine what should teacher do to increase students' representation skills and minimize the students' incorrect representation.

1.1 Research Aim

This research aimed to describe:

1. High Mathematical Ability-Student's mathematical representation on accepting teacher's promoted-action.
2. Middle Mathematical Ability-Student's mathematical representation on accepting teacher's promoted-action.

2. Methods

This research was a qualitative research with descriptive method. The subjects were two students of class X MA Madrasatul Quran Tebuireng, Jombang with high and middle mathematical ability (HA-MA). Students' ability was classified based on mathematics final score in junior high school and high school enterance test score. Students will categorized into high mathematical ability if the both scores are greater than 85, middle mathematical ability if the both scores are in interval 70 until 85, and low mathematical ability if the both scores are less than 70. The data were collected through online observation via zoom meeting to observe students' verbal representation, and online interview to get the data of nonverbal representation and also confirmed subjects' representation. Researcher used observation sheet during data collection process. The validity of the data was checked by using time triangulation. The techniques of data analysis included data reduction, data display and conclusion drawing.

3. Results and Discussion

3.1 High Mathematical Ability-Student's Mathematical Representation

HA-subject's representation was appear when the teacher stimulates students to answer verbally then subject pronounce mathematical object frequently and correctly.

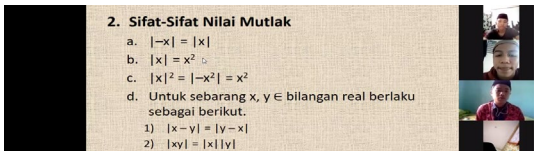

Figure 1. Teacher's slide presentation

Figure 1 shows the material presented by the teacher during online learning. Teacher has asked the students to prepare stationery first before explain the material, and then allow students to copy the material they need. Teacher's explanation is as follows:

T : "The properties of absolute value. Firstly, absolute value of negative x will be equivalent or is equal to absolute value of x. Secondly, absolute value of x is equal to square root of x squared. Thirdly, the absolute value of x squared is equal to the absolute value of minus x squared and is equal to x squared. For example the absolute value of three squared, what is the absolute value of three?" (in this moment, HA-Subjects follows the teacher to pronounced the mathematical symbols. But when the teacher said "negative", HA-Subject said "minus")

HA-S : Three

T : Then, what is three squared?

HA-S : Nine

T : it will be equivalent with the absolute value of negative three-squared. It means that we should find the value of three squared first. Three squared is nine, then the absolute value of negative nine is nine, and is equal to three squared. For each x and y element of real numbers, the absolute value of x minus y is equal to the absolute value of y minus x, the absolute value of xy is equal to the absolute value of x multipled by absolute value of y".

During the learning process, students answer the teacher's question in many times. HA-Subject communicates the correct representation verbally. HA-Subject can identify that all numbers less than zero is called negative number and determine the absolute value of a number.

T: : What is the number value if the number less than zero?

HA-S : Negative (value)

T : if x is a positive number, what is the absolute value of x?

HA-S : x

T : How about the absolute value of negative five-x?

HA-S : five-x

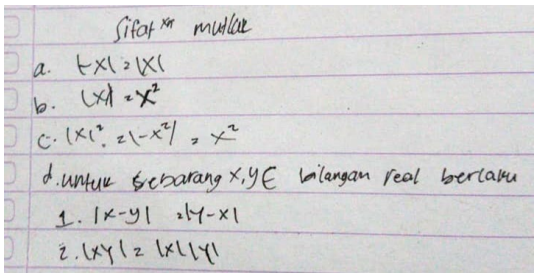

Figure 2. Copy of material by HA-Subject

Figure 2 shows that HA-Subject accepted teacher's promoted-action to copy the material. In this case, HA-Subject copies as like as presented by the teacher. Based on the interview, HA-subject pronounces the mathematical notation he has written, some of the pronounciations are correct but there was incorrect pronounciation also.

HA-S: "The properties of absolute value. Firstly, absolute value of minus x is equal to absolute value of x. Secondly, square root of absolute value of x is equal to square root of x. Thirdly, square root of the absolute value of x is equal to the absolute value of square root of minus x. For each x, y real number, the absolute value of x minus y is equal to the absolute value of y minus x, the absolute value of xy is equal to the absolute value of x and absolute value of y".

The correct pronounciations include absolute value, equality operator, quadratic, substraction, and variables. While the incorrect pronounciation includes the negative value. HA-Subject pronounced negative value by "minus". This problem has been discovered by Hidayati, the subject of his study was wrong in saying the reduction (substraction) operation with the notation "negatives" that should be pronounced "reduced", but pronounced "negative"(Hidayati, 2016). It is also relevant to the finding of Chirume's study that most students fail to interpret or understand the meaning of mathematical symbols due to the way by which they are taught to read, pronounce and use them(Chirume, 2012). HA-Subject stated that "minus" and "negative" have the same meaning, or in another words, minus is the synonym of negative but it will be better to pronounce it by "negative" because it is the formal language. While substraction and minus have different meanings. HA-Subject can identified which is the negative symbol and which is the substraction operator symbol. HA-Subject differentiates the negative symbol and substraction operator based on the symbols'characteristic. The negative symbol usually side by side with the operation of arithmetic and sometimes separated by bracket. So it can be said that the internal representation about mathematical facts of HA-Subject is accordance to the concepts, but the external representation that communicated orally is not appropriate.

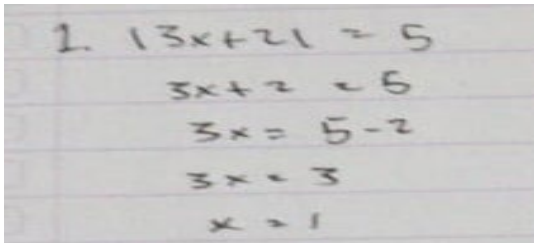

Figure 3. HA-Subject's Answer

Figure 3 shows that HA-Subject accepted teacher's promoted-action to answer the question. Based on the interview, HA-Subject explained that $|3x + 2|$ equal to $3x + 2$ refers to the properties of absolute value that $|x| = x$ and said that there is no properties $|x| = -x$.

R : can you explain how did you solve this problem?

HA-S : $|3x + 2|$ equal to $3x + 2$ because the concept is $|x| = x$, then (number) two moves to another side so that it be negative, $3x = 5 - 2$, $3x = 3$, x is equal to $3/3$, so $x = 1$.

HA-Subject's external representation about arithmetic operation is correct but the internal representation is inappropriate. The subject gave the reason that number two is move to another segment. Actually, there is an addition process in both sides, $3x + 2 = 5$ then $3x + 2 + (-2) = 5 + (-2)$, so that $3x = 3$. The result of a research by discussing phenomenology conducted by Laja(2020) shows that the rules of moving segments are considered as a

result of the creativity of teachers in teaching mathematics, but another teachers do not accept this method as an alternative to solve linear equations because it can "destroy" the the essence of mathematics structure(Laja, 2020).

About the representation of mathematical concepts, the HA-Subject stated that the absolute value is a number whose value is absolute or cannot be changed. The subject also interpret the absolute value by giving an example in real life.
HA-S : "...for example God is one and only, namely Allah and that is absolute and cannot be changed".
R : Is there possibility that the absolute value be negative?
HA-S : Maybe, I have not learned about that.
It indicates that HA-Subject involved contextual representation and the indicator is describes situations of mathematical ideas in real life or in imagination (NCTM, 2014). The example given by HA-Subject is terminologically correct that absolute means certain. But the subject's internal representation about absolute value is mathematically inappropriate because the absolute value is a non-negative number while the subject was not sure that the absolute value always be positive.

3.2 Middle Mathematical Ability-Student's Mathematical Representation

MA-subject's representation was appear when teacher stimulates students to answer verbally then subject pronounce mathematical object correctly in several times, and when teacher gave an exercise then subject write the mathematical expression.
T : what is the absolute value of two minus three equals to?
MA-S : absolute value of two multipled by absolute value of three.
T : the absolute value of two is..?
MA-S : two
T : the absolute value of three is..?
MA-S : three

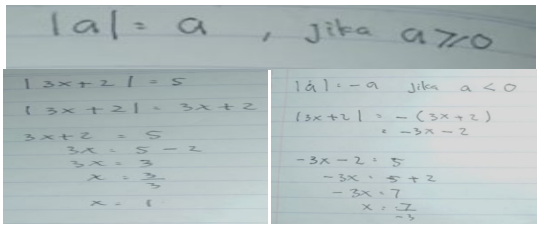

Figure 4. MA-Subject's Answer

Figur 4 shows that MA-Subject accepted teacher's promoted-action to answer the question. MA-Subject wrote two solutions for one problem given by the teacher. MA-Subject wrote the suitable properties first, then applied the properties to determine the value. Based on the subject's external representation, it was observed that there is two possibilities because the value of variable itself was not known yet. First, if the value of a is greater than or equal to zero, then absolute value of a equals to a; second, if a is less than zero, then absolute value of a equals to $-a$. And in this problem $3x + 2$ be a. So that $a = 3x + 2$ and $-a = -(3x + 2) = -3x - 2$. But actually, based on the interview, the subject stated that the first solution is the correct answer because absolute value is always be positive number, and he wrote these two solutions because at first he did not know yet which is the appropriate solutions. The subject can not relate the properties $|a| = -a$ if $a < 0$. The subject thought that $-a$ when $a < 0$ is the negative number. Even though the subject can represent that $a < 0$ is a number on the left side of number line. The result of interview showed that MA-Subject's internal representation was incorrect. This finding is accordance with Ciltas' result of study (2011) that has been detected the students' difficulties in forming a correct solution and could not fully internalized the concept of absolute value (Ciltas and Tatar, 2011).

MA-Subject's external representation in mathematics operation was correct. The subject performed the operation of addition, substraction, and division correctly. On the other hand, the subject stated that negative and minus have the different meanings but still related. Negative is any number less than zero, while the minus refers to the sign of negative numbers. Based on the subject's statement, for example, -3 is the negative number and the symbol "-" of -3 itself is minus. The subject's statement was not accordance to the theory that said there are three meanings of minus signs: firstly, indicates substraction (for example, $3 - 1$ could read as "three minus one"); secondly, it is part of the symbolic

representation for a negative number (for example −2 could read as "negative two"); thirdly, may be viewed as the "opposite of" (− − 4 could read as "the opposite of negative four")(Lisa, et.al., 2020).

4. Conclusion

Based on the online observation and interview conducted by the researchers, the results showed that HA-subject's representation was appear when teacher stimulates students to answer verbally then subject pronounce mathematical object frequently and correctly, and when teacher gives an exercise then subject write the mathematical expression correctly. HA-Subject the internal representation about mathematical facts of HA-Subject is accordance to the concepts, but the external representation that communicated orally is not appropriate, HA-Subject's external representation about arithmetic operation is correct but the internal representation is inappropriate, involved contextual representation that terminologically correct but mathematically inappropriate. MA-subject's representation was appear when teacher stimulates students to answer verbally then subject pronounce mathematical object correctly in several times, and when teacher gives an exercise then subject write mathematical expression. MA-Subject's external representation in mathematics operation was correct, stated that negative and minus have the different meanings but still related. The subject's statement was not accordance to the theory of minus signs.

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Biographies

Tria Rahayu is a student of Mathematics Education Masters Program. She has completed her bachelor's degree at STKIP PGRI Jombang in 2019, and now she is completing her thesis for master's degree at STKIP PGRI Jombang. She has completed her research that funded by Directorate General of Higher Education - educational minister of Indonesia. Her research are always related to Mathematical Representation.

Jauhara Dian Nurul Iffah Basic education history up to university is taken in Jombang regency, namely MI Perguruan Mua'llimat Cukir (graduated in 2000), MTsN Tambakberas (graduated in 2003), MA Perguruan Mu'allimat Cukir (graduated 2006). After graduating, she then continued her S1 Mathematics Education at STKIP PGRI Jombang (graduated in 2010), in the same year continued her S2 Mathematics Education at Universitas Negeri Surabaya with BPPDN On-Going (graduated in 2012). Furthermore, to increase insight and improve the quality of education, in 2013 continue S3 Mathematics Education with BPPDN scholarship at State University of Malang. Work history, in 2010 after graduation S1 has been a teacher at MTs Salafiyah Syafi'iyah Tebuireng Jombang. In the same year also accepted to be a lecturer in Mathematics Education Studies Program STKIP PGRI Jombang until today. During his time as a lecturer, he has served as research staff at the Center for Research and Community Service at STKIP PGRI Jombang.

Wiwin Sri Hidayati born on May 2, 1973, to a father of Soehoed Poerwoatmodjo and Mrs. Sukarlik. Born as the seventh child of 7 siblings. As a father's daughter who challenged in the field of education, her father also directed to pursue the field of education as well. A small year was spent in 1980 to 1986 to study at SDN Sukoiber 1 and SDN Pesanggrahan in Gudo District, Jombang Regency. After graduating from elementary school, from 1986 to 1989 the writer continued her studies at SMPN 1 Gudo and succeeded in SMA PGRI 1 Jombang from 1989 to 1992. During her undergraduate studies, the writer spent at STKIP PGRI Jombang in the mathematics education study program and graduated in 1996 After graduating from undergraduate, in 1998 until 2005 the writer became a lecturer at STKIP PGRI Jombang. In 2005 the writer was accepted as a lecturer in the DPk Kopertis East Java Region (now LLDikti East Java Region) until now. Over time, the author continued his studies in the mathematics education master program at Surabaya State University in 2005 and graduated in 2007 with the PostScholarship Program (BPPS) from the Directorate General of Higher Education. From 2009 to 2013, the author continued his doctoral program in the mathematics education study program at Surabaya State University with the Post-Scholarship Program (BPPS) from the Directorate General of Higher Education.