Optimization of the Signal Noise Ratio index using Simultaneous Perturbation Stochastic Approximation Algorithm

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Abstract

This paper proposes a linear search Simultaneous Perturbation Stochastic Approximation (SPSA) algorithm to maximize the signal-to-noise ratio (SNR) index in the least number of iterations and determine which succession measure converges and maximizes the signal-to-noise ratio (SNR) in the least number of iterations. The analysis and validation are performed with experimental simulation and validated with four case studies collected from the literature. The case studies were evaluated in ten experiments with different combinations of the succession measures a_k and c_k. The results show that the proposed SPSA is an iterative, efficient, and easy to use method to maximize the quality indexes of production processes, which is feasible to implement within the six sigma methodology. Also, the results show that experiments 3 and 5 converge to the best results in the four case studies analyzed.

Keywords
Signal-To-Noise Ratio; Simulation; Optimization; Simultaneous Perturbation Stochastic Approximation.

1. Introduction

The Simultaneous Perturbation Stochastic Approximation (SPSA) algorithm is an iterative optimization method for multivariate stochastic systems (James C Spall, 1992), of the Kiefer and Wolfowitz type, Wolfowitz is designed for stochastic problems (Lu & Zhou, 2019; Radenković, Stanković, & Stanković, 2018) and has the advantage of not requiring extensive knowledge of the process being analyzed (Miranda & K Del Castillo, 2011), it can be used when the relationship between response variables and their controllable factors is unknown.

This algorithm has the following characteristics: it is efficient in high-dimensional problems as it generates a solution in a small number of measurements of the objective function (Olguín Tiznado, García Martínez, Camargo Wilson, & López Barreras, 2011). The gradient approximation uses only two values for the objective function per iteration, regardless of the dimension of the problem, to reduce the objective function and the measurement frequency for the estimation of the gradient (Ding, Xia, Wang, Li, & Ou, 2015; Miranda & K Del Castillo, 2011).

One of the statistical applications of the SPSA algorithm in the industry is evaluating the potential and performance that processes have to work within the specification limits (Pearn, Shiau, Tai, & Li, 2011). The scalar values that determine this ability of the processes are known as process capability indexes (Kashif, Aslam, Jun, Al-Marshadi, & Rao, 2017), which have become widely used tools to evaluate the performance of processes in recent years (Goyal & Grover, 2012). These indices quantify the relationship between actual process performance and product specification limits (Ebadi & Shahriari, 2013), which is why this concept is beneficial for manufacturers when evaluating the ability of their production processes.

One of the most used indices in process optimization is the Signal to Noise Ratio SNR index. (Y. Lagunes Rangel, 2011). Taguchi uses this index to measure the variation of the desired value in a quality characteristic. These characteristics are classified according to the case study in "the least is the best," "the most is the best," and "the nominal is the best" (Tu, Yih, Chou, & Chou, 2020). In this research work, we take an adaptation for this index that proposes an alternative way of measuring the SNR function from the process capability index C_{pk} (Amiri, Bashiri, & Mogouie, 2011), as shown in equation (1):

$$SNR_{max} = -10Log\sum_{i=1}^{m}\left(\frac{1/C_{pki}^2}{n}\right) \tag{1}$$

Where: SNR represents the signal-to-noise ratio index, C_{pki} represents the real process capability index, n represents the number of replicates evaluated that for this research work we will take the index reported by (García et al., 2019). In this article, four numerical examples selected from the literature are presented to carry out the experimental simulation analysis as shown below: First, the dual second-order regression model reported by Olguín Tiznado et al. (2011) is presented, which measures the percentage of final moisture in the dehydrated garlic process with its target value of five percent of final moisture.

Second, dual models of second-order regression are presented, calculated based on the results reported by (Gedi, Bakar, & Mariod, 2015), which measure the performance of the sardine oil extraction process (YIELD = yμ2), and the relationships of Eicosapentaenoic Acid (EPA = yμ3), and DocosaHexaenoic Acid (DHA = yμ4). Therefore, the Yield measures the percentage of the total extraction yield and the ratio of the EicosaPentaenoic and DocosaHexaenoic acids. The models are presented below for the mean and for the standard deviation in each of the evaluated responses:

This document proposes a combination of classical statistical tools (such as DOE) with a linear search algorithm (such as SPSA algorithm) to optimize production processes. The objective is to improve the results reported by the authors of the case studies by at least 10%. Also, it is proposed to define the combination of succession measures a_k and c_k (parameters of the SPSA algorithm) that allow converging to a better solution in a smaller number of iterations.

The rest of this document is organized into five sections as follows: after the introduction and objectives, the second section presents the literature review, section three presents the method proposed, section four shows the results and discussions of the experimental analysis of the four study cases evaluated with the method proposed, section five presents the conclusions of this investigation.

2. Literature Review

Response surface methodology is commonly used to design the experiments, and it minimizes the number of experiments for a specific number of factors and their levels. It has many advantages over Taguchi method of design SNR index(Chelladurai et al., 2021). From this methodology, we derive the Taguchi signal-noise ratio, which is widely used for the optimization of a wide variety of processes (Costa, 2017).

On the other hand, within the SPSA parameters we have the succession measures a_k and c_k, which largely affect the convergence of the algorithm. The values typically used are $a_k = {}^{a}/_{k}$ and $c_k = {}^{c}/_{(k+1)^{\gamma}}$ (Altaf, Heemink, Verlaan, & Hoteit, 2011). However, these succession measures a_k y c_k are proposed parameters developed based on previous studies, but they may vary since there are not a couple of these succession measures that are robust for any type of application or study (James C Spall, 1998).

Therefore, in this article, five succession measures of a_k and two succession measures of c_k are evaluated to analyze the behavior of the proposed SPSA algorithm in such a way that they converge and maximize the SNR quality indices in the least number of iterations. The a_k and c_k succession measures found in the literature are shown in Table 1.

Table 1 Different measures of a_k and c_k succession found in the literature

Succession measures	References	Frequency
$a_{k1} = a/(A+k)^{\alpha}$	(Pouladi et al., 2020; Tahir, Moazzam, & Ali, 2018)	2
$a_{k2} = a/(A+k+1)^{\alpha}$	(Jesmani, Jafarpour, Bellout, & Foss, 2020; Markou, Papathanasopoulou, & Antoniou, 2019; Pouladi et al., 2020)	3
$a_{k3} = a/k$	(Bartkutė & Sakalauskas, 2007; Hernandez & Spall, 2019)	2
$a_{k4} = a/k^{2/3}$	1(García Martínez, 2000)	1
$a_{k5} = a/k^{\alpha}$	(Bangerth, Klie, Matossian, Parashar, & Wheeler, 2005; Sadegh & Spall, 1998)	2
$c_{k1} = c/k^{\gamma}$	(McClary, Syrotiuk, & Kulahci, 2010; Pouladi et al., 2020; Reardon, Lloyd, & Perel, 2010)	3
$c_{k2} = c/(k+1)^{\gamma}$	(Jesmani et al., 2020; Markou et al., 2019; Pouladi et al., 2020)	3

Where α is the initial value of the step size of the gradient factor, c is the initial value of the step size in the approximation of the gradient factor, A is the stability factor, k is the iteration number evaluated, α = is the reduction rate of the step size of the gradient factor, where $\gamma =$ the reduction rate of the step size in the approximation of the gradient factor (Finck & Beyer, 2012).

The purpose of testing these ten experiments is to confirm which of them is most effective in optimizing response variables such as the SNR index, with the objective of finding the best value of the response variable in the fewest number of iterations. In order to confirm that the combination of classical statistical tools such as the response surface methodology with linear search algorithms such as SPSA works for the optimization of this type of response variables.

3. Methods

The materials for the analysis and assessment of the experimental simulation of this research were a laptop with an AMD E-450 processor at 1.65 GHz and 4 GB of RAM; Minitab 17® software to perform statistical analysis of the data; and MATLAB R2014® for the simulation of the SPSA algorithm proposed in the optimization of the SNR indices.

The methodology proposed in this research consists of six stages, which are presented in summary form for SNR index below in Table 2.

Table 2 Summary of the SPSA algorithm Iterative Process for Signal to Noise Ratio $\widehat{SNR}$

1	$\widehat{\boldsymbol{y}_{\boldsymbol{\mu}}}, \widehat{\boldsymbol{y}_{\boldsymbol{\sigma}}}$	// Second-order regression models for mean and variance are defined.
2	Δ_k^+, Δ_k^-	// The perturbation gradient is defined
3	A, α, c, γ, a	// Non-negativity values are stated
4	USL and LSL	// The specification limits are defined
5	a_k y c_k	// Succession measures are defined
6	For k=1:100	
7	$\widehat{C_{pk}}(\text{ULS, LSL}, \widehat{\boldsymbol{y}_{\boldsymbol{\mu}}}, \widehat{\boldsymbol{y}_{\boldsymbol{\sigma}}})$	// response variable is evaluated
	$\widehat{SNR}(\widehat{C_{pk}})$	
9	$X_k^+ = X_k + c_k(\Delta_k^\pm)$	// the positive increase in the controllable factors is realized
10	$\widehat{y_\mu^+}, \widehat{y_\sigma^+}$	
11	$\widehat{C_{pk}}^+$ (USL, LSL, $\widehat{y_\mu^+}$, $\widehat{y_\sigma^+}$)	// the response variable is evaluated with the positive increase
12	$\widehat{SNR}^+(\widehat{C_{pk}}^+)$	
13	$X_k^- = X_k + c_k(\Delta_k^\pm)$	// the negative increase in the controllable factors is realized
14	$\widehat{y_\mu^-}, \widehat{y_\sigma^-}$	
15	$\widehat{C_{pk}}^-$ (USL, LSL, $\widehat{y_\mu^-}$, $\widehat{y_\sigma^-}$)	// the response variable is evaluated with the negative increase
16	$\widehat{SNR}^-(\widehat{C_{pk}}^-)$	
17	$\emptyset_k^+ = \dfrac{\widehat{SNR}^+ + \widehat{SNR}^-}{2c_k\Delta^-}$	
18	$\hat{X}_{k+1}^+ = X_k - a_k\emptyset_k^+$	// the new values for the controllable factors are evaluated
19	$\emptyset_k^- = \dfrac{\widehat{SNR}^+ - \widehat{SNR}^-}{2c_k\Delta^-}$	
20	$\hat{X}_{k+1}^- = X_k - a_k\emptyset_k^-$	
21	End	

In stage one, a second-order dual regression model is obtained for the mean (μ) and the standard deviation (σ) using an experimental design from a vector of controllable factors (X_k) and response variables (y_i)as recommended by Garcia et al. (García et al., 2019).

These models represent four case studies: they are named: model 1 ($\widehat{y_{\mu1}}$)), which is a function of the controllable factors of temperature (X_1) and time (X_2) (Olguín Tiznado et al., 2011), and models 2 ($\widehat{y_{\mu2}}$), 3 ($\widehat{y_{\mu3}}$) and 4 ($\widehat{y_{\mu4}}$), which are a function of the controllable factors of pressure (X_1) y and temperature (X_2) (Gedi et al., 2015).

Stage two, implementation of the proposed SSA is divided into sub-stages: First, it defines the disturbance vector, the values Δ_k^+ y Δ_k^- for Model 1 are Δ_k: $\Delta_k^+ = 3°C$ y $\Delta_k^- = -0.3$ hours, and for models 2, 3 y 4 the values are $\Delta_k^+ =$ 141.4 Bars; $\Delta_k^- = -21.5°C$.

Second, the non-negativity coefficients a, c, A, α, and γ. A useful guide to choosing A is to define it as a value smaller than the maximum number of iterations allowed or expected (Altaf et al., 2011). In this research, we chose the values A= 100, α= 0.16, c= 1, a= 1 and γ = 0.101.

Third, Lower (LSL) and upper Specification Limits (USL) are defined. The values of the specification limits for the case studies are USL1 = 6% and LSL1 = 4% humidity; USL2 = 10 and LSL2 = 5%, USL3 = 10 and LSL3 =4%, USL4 = 21 y LSL4 = 16%. After this step, a counter k=1:500 iterations is determined.

Fourth, the succession measures a_k and c_k; are chosen; the choice of these values is critical to the performance of the SPSA algorithm.

To evaluate which combination of the a_k and c_k succession measures is the best to maximize the SNR in the least number of iterations, ten experiments were performed that resulted from the combination of the five a_k succession measures and the two c_k succession measures. Table 3 shows these experiments.

Table 3 Succession measure experiments in the SPSA algorithm.

Experiment	a_k	c_k
1	$a_k = {}^{a}/_{(A+k)^{\alpha}}$	$c_k = {}^{c}/_{k^{\gamma}}$
2	$a_k = {}^{a}/_{(A+k+1)^{\alpha}}$	
3	$a_k = {}^{a}/_{k}$	
4	$a_k = {}^{a}/_{k^{2/3}}$	
5	$a_k = {}^{a}/_{k^{\alpha}}$	
6	$a_k = {}^{a}/_{(A+k)^{\alpha}}$	$c_k = {}^{c}/_{(k+1)^{\gamma}}$
7	$a_k = {}^{a}/_{(A+K+1)^{\alpha}}$	
8	$a_k = {}^{a}/_{k}$	
9	$a_k = {}^{a}/_{k^{2/3}}$	
10	$a_k = {}^{a}/_{k^{\alpha}}$	

In stage three, the functions to maximize DR and SNR are evaluated. The initial values are used for model 1: 1: $X_1 = 65°\text{C};\ X_2 = 4h{:}\,30\ min$, and for models 2, 3, and 4: $X_1 = 300\ Bars;\ X_2 = 55°\text{C}$. Next, the values corresponding to the independent variables (X_1, X_2) of the second-order regression models are substituted by study case $\widehat{y_{\mu 1}}$,$\widehat{y_{\sigma 1}}$; $\widehat{y_{\mu 2}}$,$\widehat{y_{\sigma 2}}$; $\widehat{y_{\mu 3}}$,$\widehat{y_{\sigma 3}}$ and $\widehat{y_{\mu 4}}$,$\widehat{y_{\sigma 4}}$.

On the other hand, to calculate the $\widehat{SNR}$ response variable, an adaptation of the conventional formula of the signal-to-noise ratio index is used, based on the adequacy of equation (1) of the $\widehat{C_{pk}}$ index (Amiri et al., 2011) as shown in the equation (2). In such a way that the estimated values of $\widehat{C_{pki}}$ for each of the models evaluated in this work are $\widehat{C_{pk1}} = -0.321$, $\widehat{C_{pk2}} = 3.888$, $\widehat{C_{pk3}} = 11.555$ y $\widehat{C_{pk4}} = 4.653$. Next, equation (4) is used, obtaining the following estimated values for the SNR: $\widehat{SNR}_1 = -9.869$, $\widehat{SNR}_2 = 11.794$, $\widehat{SNR}_3 = 21.255$ y $\widehat{SNR}_4 = 13.354$.

$$\widehat{C_{pk}} = Min\left\{\frac{USL - \widehat{y_\mu}}{3\widehat{y_\sigma}}; \frac{\widehat{y_\mu} - LSL}{3\widehat{y_\sigma}}\right\} \qquad (2)$$

Next, the values for the vector of controllable factors X_k^+, are calculated, which is based on the initial values of X_k defined in stage 3 for each study case, the succession Measure c_k and the disturbance vector $\Delta_k^{\pm}$ defined in stage 2 with the values defined in stage 2, and 3 $X_k^+ = X_k + c_k(\Delta_k^{\pm})$, and the new controllable values for model 1 are obtained: $X_1^+ = 68°\text{C}$ y $X_2^+ = 4h\ 12min$; and for models 2, 3, and 4 $X_1^+ = 441.4\ Bars$ y $X_2^+ = 33.5°\text{C}$. These new values are substituted in the second-order regression models to calculate the values of $\widehat{y_\mu^+}$ y $\widehat{y_\sigma^+}$.

The estimated values are $\widehat{y_{\mu 1}^+} = 4.417$, $\widehat{y_{\sigma 1}^+} = -0.902$; $\widehat{y_{\mu 2}^+} = 5.535$, $\widehat{y_{\sigma 2}^+} = 0.237$; $\widehat{y_{\mu 3}^+} = 7.012$, $\widehat{y_{\sigma 3}^+} = 0.449$; $\widehat{y_{\mu 4}^+} = 20.814$, $\widehat{y_{\sigma 4}^+} = 0.367$, and with these values, we proceed to calculate the response variable $\widehat{SNR}^+$ as shown in equations (4).

The values to calculate the $\widehat{C_{pk}^+}$ are obtained with equation (3), giving the following results per model: $\widehat{C_{pk1}^+} = -0.585$, $\widehat{C_{pk2}^+} = 0.237$, $\widehat{C_{pk3}^+} = 2.218$ and $\widehat{C_{pk4}^+} = 0.168$, these results are used to calculate the $\widehat{SNR}^+$.

$$\widehat{C_{pk}^{+}} = Min\left\{\frac{USL - \widehat{y_{\mu}^{+}}}{3\widehat{y_{\sigma}^{+}}}; \frac{\widehat{y_{\mu}^{+}} - LSL}{3\widehat{y_{\sigma}^{+}}}\right\} \tag{3}$$

$$\widehat{SNR_{max}^{+}} = -10Log\sum_{i=1}^{m}\left(\frac{1/\widehat{(c_{pk\iota}^{+})^{2}}}{n}\right) \tag{4}$$

The estimated values for the $\widehat{SNR^{+}}$ are: $\widehat{SNR_1^{+}} = -4.656$, $\widehat{SNR_2^{+}} = -12.505$, $\widehat{SNR_3^{+}} = 6.919$ y $\widehat{SNR_4^{+}} = -15.493$. In addition, values for the controllable factors X_k^- are calculated which, like X_k^+, re a function of the initial values of X_k defined in stage 3 for each study case; the succession measure c_k and the disturbance vector $\Delta_k^{\pm}$ defined in stage 2, with the values defined in stage 2 and 3, the values of $X_k^- = X_k - c_k(\Delta_k^{\pm})$, are calculated, so that, the new controllable values for model 1 are $X_1^- = 62°C$ y $X_2^- = 4h\ y\ 48\ min$; and for models 2, 3, and 4 $X_1^- = 158.6\ Bars$ y $X_2^- = 76.5°C$. These new values are substituted in the second-order regression models to calculate the values of $\widehat{y_{\mu}^{-}}$ y $\widehat{y_{\sigma}^{-}}$ (Olguín Tiznado et al., 2011). and it is obtained $\widehat{y_{\mu 1}^{-}} = 5.616$, $\widehat{y_{\sigma 1}^{-}} = -0.883$; $\widehat{y_{\mu 2}^{-}} = 3.532$, $\widehat{y_{\sigma 2}^{-}} = 1.123$; $\widehat{y_{\mu 3}^{-}} = 10.183$, $\widehat{y_{\sigma 3}^{-}} = 0.28$; $\widehat{y_{\mu 4}^{-}} = 15.054$ $\widehat{y_{\sigma 4}^{-}} = 0.235$, then, we proceed to calculate the values for the response variable $\widehat{SNR^{-}}$ as shown in the equations (6).

The values to calculate the $\widehat{C_{pk}^{-}}$ are calculated with equation (5), getting the following results: $\widehat{C_{pk1}^{-}} = -0.610$, $\widehat{C_{pk2}^{-}} = -0.433$, $\widehat{C_{pk3}^{-}} = -0.218$ and $\widehat{C_{pk4}^{-}} = -1.377$ (García et al., 2019), then, these results are used to calculate the $\widehat{SNR^{-}}$.

$$\widehat{C_{pk}^{-}} = Min\left\{\frac{USL - \widehat{y_{\mu}^{-}}}{3\widehat{y_{\sigma}^{-}}}; \frac{\widehat{y_{\mu}^{-}} - LSL}{3\widehat{y_{\sigma}^{-}}}\right\} \tag{5}$$

$$\widehat{SNR_{max}^{-}} = -10Log\sum_{i=1}^{m}\left(\frac{1/\widehat{(c_{pk\iota}^{-})^{2}}}{n}\right) \tag{6}$$

The estimated values for the $\widehat{SNR^{-}}$ for each study model are: $\widehat{SNR_1^{-}} = -4.293$, $\widehat{SNR_2^{-}} = -7.27$, $\widehat{SNR_3^{-}} = -13.23$ and $\widehat{SNR_4^{-}} = 2.778$.

In stage four, the approximation of the gradient $\emptyset_k^{+}$ and $\emptyset_k^{-}$ is determined and the simultaneous disturbance gradient is generated for $\widehat{SNR}$ with equations (7).

$$\widehat{\emptyset_{kSNR}^{+}} = \frac{\widehat{SNR^{+}} - \widehat{SNR^{-}}}{2c_k\Delta^{+}} \qquad \widehat{\emptyset_{kSNR}^{-}} = \frac{\widehat{SNR^{+}} - \widehat{SNR^{-}}}{2c_k\Delta^{-}} \tag{7}$$

The values of $\widehat{\emptyset_k^{+}}$ and $\widehat{\emptyset_k^{-}}$calculated for the response variable $\widehat{SNR}$ are shown in Table 4.

Table 4 Value of variable $\widehat{SNR}$ calculated for each model

Model	$\widehat{\emptyset_{kSNR}^{+}}$	$\widehat{\emptyset_{kSNR}^{-}}$
1	-0.0455	0.455
2	0.018	0.121
3	0.071	-0.468
10	-0.064	0.424

In stage five, the values of the controllable factors $\widehat{X_k}$, are updated, estimating the new values for the iterative process $\widehat{X_{k+1}^{+}}$ and $\widehat{X_{k+1}^{-}}$; this is achieved using the standard equations (8) and (9) of the Stochastic Algorithm as follows: The results calculated for the $\widehat{X_2^{+}}$ and $\widehat{X_2^{-}}$values are shown in Table 5.

$$\widehat{X_{k+1}^{+}} = X_k - a_k\emptyset_k^{+} \tag{8}$$

$$\widehat{X_{k+1}^{-}} = X_k - a_k\emptyset_k^{-} \tag{9}$$

Table 2 Values of iterations n 2 $\widehat{X_2^+}$ and $\widehat{X_2^-}$ for variable $\widehat{SNR}$ calculated for each model.

Model	$\widehat{X_2^+}$	$\widehat{X_2^-}$
1	65.045	4.045
2	300.018	54.878
3	299.928	55.468
10	300.064	54.575

Stage six indicates returning and updating the value of k by k+1 to repeat this iterative process until reaching the maximum iteration value defined for this investigation, k=500.

4. Results and Discussion

4.1 Numerical Results

Below are the results of the ten experiments carried out in the tables listed from 6 to 9. Where, in the first column is defined the number of the experiment of the combination of the succession measures that was raised in table 1, in columns two and three are indicated the optimal values for the controllable factors (X_k^*), in the fourth column is illustrated the maximum value found in the variable of $\widehat{SNR}$ evaluated at 500 iterations, and in the fifth column the number of iteration in which was found the maximum value for the response variable to optimize was found.

One of the objectives of this paper is to improve the results reported by the authors of each case study. In order to verify that this combination of techniques improves the results obtained with classical techniques. It is for that reason that in the sixth column, we show the percentage value (PV) of the gain in the response variable that we calculated with the efficiency between the maximum values obtained by the author $\widehat{SNR}$ and this simulation process ($\widehat{SNR}^*$), which is calculated with equation 10.

$$pv = \frac{\widehat{SNR}^* - \widehat{SNR}}{\widehat{SNR}} \tag{10}$$

In the seventh column, we show a weighted value (Wv) that we calculate with the quotient of the efficiency between the maximum values obtained by the author $\widehat{SNR}$ and this simulation process ($\widehat{SNR}^*$), and the number of iterations (ki), which is calculated with the equation 11.

$$Wv = \left(\frac{\widehat{SNR}^* - \widehat{SNR}}{\widehat{SNR}}\right) \Big/ k_i \tag{11}$$

Table 6 shows the results of the maximum values for the $\widehat{SNR}$ variable in model number 1. Considering the objectives of this research (the highest value in the least number of iterations), the best results are shown in experiments 3 and 9 with percentage values of 0.3660 and 2.002; and weighted values of 0.0407 and 0.0128, respectively.

Table 3 Results of the SNR variable for model 1 in the simulation process from 1 to 500 iterations

Experiment	SNR max	Iteration	Percentage Value	Weighted Value
1	-8.9213	500	-1.8066	-0.0036
2	-8.9318	500	-1.8076	-0.0036
3	15.1075	9	0.3660	0.0407
4	-5.6304	6	-1.5091	-0.2515
5	33.6639	449	2.0438	0.0046
6	-8.9271	500	-1.8072	-0.0036
7	-8.9375	500	-1.8081	-0.0036
8	37.8824	259	2.4252	0.0094
9	33.2026	157	2.0021	0.0128
10	32.3160	250	1.9219	0.0077

Table 7 shows the results of the maximum values of the response variable $\widehat{SNR}$ for model 2, where the maximum value achieved is 27.1620 in iteration 1. With a percentage value and weighted value of -0.3004 for all experiments. It is important to note that this value was the same in the 10 experiments analyzed in this research.

Table 4 Results of the SNR variable for model 2 in the simulation process from 1 to 500 iterations

Experiment	SNR max	Iteration	Percentage Value	Weighted Value
1-10	27.1620	1	-0.3004	-0.3004

Table 8 shows the results of the maximum values of the ($\widehat{SNR}$) variable for model 3, and these values do not present a significant difference. However, the weighted value shows that the best experiments are 4, 5, 9, and 10 with percentage values of 1.7156, 1.7180, 1.7157, 1.7181, and weighted values of 0.0093, 0.116, 0.0094, and 0.0118 respectively.

Table 5 Results of the SNR variable for model 3 in the simulation process from 1 to 500 iterations

Experiment	SNR max	Iteration	Percentage Value	Weighted Value
1	60.8754	500	1.7049	0.0034
2	60.8752	500	1.7049	0.0034
3	60.9415	500	1.7079	0.0034
4	61.1154	185	1.7156	0.0093
5	61.1700	148	1.7180	0.0116
6	60.8755	500	1.7049	0.0034
7	60.8754	500	1.7049	0.0034
8	60.9429	500	1.7079	0.0034
9	61.1169	183	1.7157	0.0094
10	61.1712	146	1.7181	0.0118

Table 9 shows the results of the maximum values of the response variable $\widehat{SNR}$ of model 4; these values are between 30.7588 and 30.9313. The difference between these values is not significant, although in terms of the number of iterations, the difference is very significant since their minimum values of iterations are between 48 and 58. The weighted value shows that the best experiments are 6 and 7, with values of 0.1514 and percentage value of 7.2664 for both experiments.

Table 6 Results of the SNR variable for model 4 in the simulation process from 1 to 500 iterations

Experiment	SNR max	Iteration	Percentage Value	Weighted Value
1	30.7592	49	7.2665	0.1483
2	30.7592	49	7.2665	0.1483
3	30.8670	54	7.2955	0.1351
4	30.9313	58	7.3127	0.1261
5	30.7836	50	7.2730	0.1455
6	30.7588	48	7.2664	0.1514
7	30.7588	48	7.2664	0.1514
8	30.8571	53	7.2928	0.1376
9	30.9171	56	7.3089	0.1305
10	30.7813	49	7.2724	0.1484

4.2 Graphical Results

Figure 1 below shows the weighted values from the ten experiments analyzed to optimize the response variable $\widehat{SNR}$ evaluated in the four models proposed in this work. Where it can be seen that experiments 3 and 5 are those reaching the higher values for the four models.

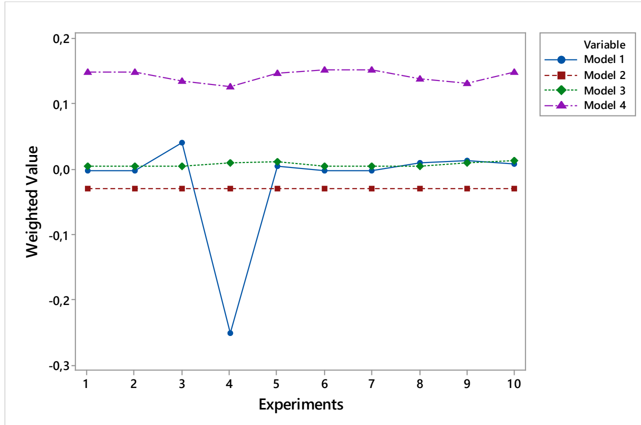

Figure 1 Weighted values for each experiment evaluated in models 1, 2, 3, and 4 at 500 iterations.

The results obtained in this research show that the difference between the results of each experiment is relatively small. Only in model 1 a significant change is observed between the results of ten experiments, which does not happen in the results obtained in the evaluations of models 2, 3 and 4 in which the maximum values obtained are practically the same in all 10 experiments.

5. Conclusion

This research proposes an SPSA algorithm to maximize the signal-to-noise ratio ($\widehat{SNR}$) in the least number of iterations, and it is also sought to determine which of the succession measures cited in the literature of this algorithm converge to maximize these quality indices that evaluate. The results show that the proposed SPSA algorithm maximizes the ($\widehat{SNR}$) for the four case studies in this research.

Another objective of this work is to improve by at least 10% the results reported by the authors of each case study analyzed. It is important to highlight that this objective was not achieved. However, there were improvements, greater than 2% for model 1, greater than 1% in model 3, and greater than 7% in model 4, only in model 2 there was no percentage improvement.

Furthermore, the succession measures that converge on the maximum values of the variable ($\widehat{SNR}$) that its combinations of experiments of the succession measures for the proposed SPSA algorithm in the four case studies are 3 and 5.

Finally, the results obtained show that the proposed SPSA algorithm is an easily implemented, efficient, and iterative method, which can be considered as an experimental tool for the improvement of quality indices that evaluate the aptitude of the processes within the Six Sigma methodology.

Future research should propose a Stochastic Simultaneous Perturbation Stochastic Approximation (SPSA) algorithm capable of working with discrete and mixed controllable factors, and that is robust to the process quality indices to be optimized with the succession measures evaluated in this research.

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