Bond Graphs for Manufacturing Control: Model's Improvement

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Abstract

Industry 4.0 paradigm calls for responsive scheduling and production control systems. Feedback loops incorporated to these systems allow decision making updates based on the real-time shop floor state. The bond graph approach enables the simulation of production control with feedback loops, but previous research with this theme is scarce. In the existing models, the buffers are modeled with infinite capacity, causing the effort variable to become zero and introducing a non-linear function that cramps the design of controllers. We propose modifications to an existing model in order to overcome this limitation, and simulate a 4-station shop floor as a proof of concept. With an optimal control approach, the rate of the material sources could be adjusted to keep the work in process at desired levels. The approach contributes to the literature with a model with more realistic assumption (limited buffers) and with the design of responsive production systems, addressing a research thread that is relevant to recent digitization trends.

Keywords
Bond graphs, dynamic modeling, production control, Industry 4.0 and closed-loop models.

1. Introduction

Competitive forces have been increasingly demanding flexibility and responsiveness from services and production systems. Recent trends of digitization aim to meet this demand. The paradigm of Industry 4.0 implies a higher level of automation and connectivity to enable high customization and flexibility whilst simultaneously sustaining high levels of productivity and operational efficiency. More flexible and reactive control systems are based on the cooperation and autonomous decision of agents, which is enabled by connectivity and data analyzing capabilities (Derigent et al. 2020). In the Cyber-Physical Systems (CPS), feedback loops coming from physical processes provide information for carrying out control actions. Thus, systems with these feedback loops, *i.e.* closed-loop systems, are an essential element of CPS. The vertical integration of the different elements within a factory is also a relevant feature of a flexible/reconfigurable manufacturing system (Wang et al. 2016). This greater flexibility requires the production scheduling and control systems to be dynamic (for examples see (Sagawa and Nagano 2013)). The production mix and rates have to react to disturbances and be based on the real-time shop floor state. In this paper, we propose a responsive closed-loop model of a shop floor based on bond graphs. Bond graph is a modeling technique based on the pictorial representation of power transmission between components. The state equations of the model can be derived from the graphs. Existing models based on bond graphs to represent shop floor (Sagawa and Nagano, 2015; Sagawa and Mušič 2019) are scarce. Moreover, some models are based on pseudographs and cannot be simulated; other models present simplifying assumptions. Taking (Ferney 2000)'s model as a reference, we remove one of the model's simplifications, defining production buffers with finite capacity (instead of with unlimited capacity). Then, we redevelop the model's equations accordingly and simulate an illustrative example. A Linear Quadratic Integral strategy is used to control the system, yielding a closed loop model. The results attested the viability of the proposed approach. This work contributes to the research field by overcoming one of the limitations of the existing models, turning them more realistic, and by addressing the mentioned gap regarding bond graphs models for shop floor representation. The simulation of these dynamic models may provide prescriptive directions on how to control relevant variables of the shop floor as a function of its real-time state, granting more responsive systems, aligned with the Industry 4.0's paradigm.

2. Literature Review: Bond Graphs for Modeling Production Systems

Bond graphs are a technique based on power flow diagrams to model dynamic systems independently of their physical domain (Das 2009). It relies on the generalized variables of effort ($\boldsymbol{e(t)}$) and flow ($\boldsymbol{f(t)}$), whose product results in power. The technique encompasses supply, storage, dissipation and transformation elements (Samantaray and Bouamama 2008). Most industrial processes have a multidisciplinary nature; the bond graph is a unifying modeling approach from which the state equations can be derived. Bond graph models to represent physical systems are numerous. For production systems' representation, previous research is scarce and proposed conceptual or peseudograph models, preventing the simulation of these models. (Ferney 2000)'s model was one of one of the firsts to respect the formalism of the approach, allowing simulations and analyses of the system's dynamic response**.**

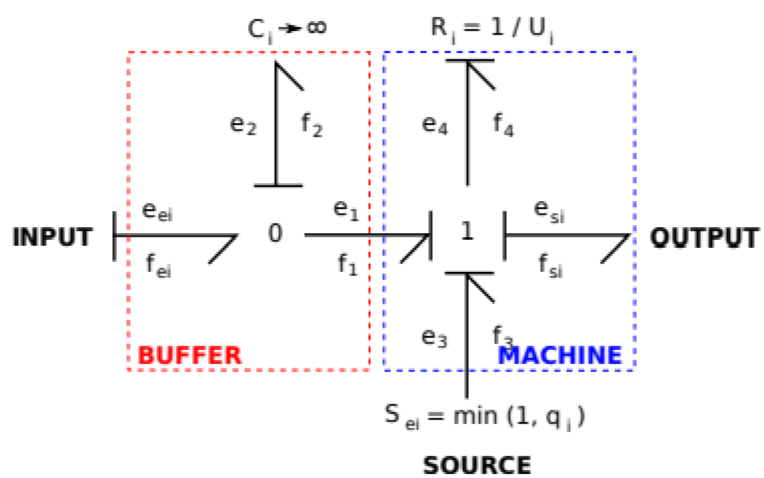

Figure 1. Bond graph representation of a production station *i*.

In this model, the production flow $f(t)$ represents the evolution of the material flow in a given section of the system and is expressed in material units per second. The effort variable $e(t)$ defines the active or passive characteristic of an element. A buffer, for instance, is a passive element whose storage rate is imposed by the machine upstream and the machine downstream. The effort in this element is zero. Ferney (2000) defines a production station *i* as composed by a buffer C_i (capacitor, storage element), a machine R_i (dissipating element) and a source of effort S_{ei} (supply element). The bond graph model is shown in Figure 1. The production flow $f_4(t)$ is proportional to the effort variable $e_4(t)$ yielding:

$$f_4(t) = U_i e_4(t) = \frac{1}{R_i} e_4(t). \tag{1}$$

where U_i is the production rate of the machine *i* expressed in processing frequency (per second). The well known linear relationship between the electrical current f_e (flow) and the electrical voltage e_e (effort), $e_e = e_e/R_e$, holds, where R_e is the electrical resistance.

The rate of material stored in a buffer i, $f_2(t)$, is the difference between the input and the output flows, where a type 0-junction represents the flow conservation. The stored production volume is represented by the displacement variable $q_i(t)$, where $q_{i0}(t)$ is the initial volume stored (Eq. 3). This volume is related to the effort variable by the constitutive equation of a capacitor (Eq. 4).

$$f_2(t) = f_{ei}(t) - f_1(t) \tag{2}$$

$$q_i(t) = q_{i0}(t) + \int_{t_0}^{t} f_2(\tau) d\tau \tag{3}$$

$$e_2(t) = \frac{1}{C_i} q_i(t). \tag{4}$$

In Ferney (2000), the capacity of the buffers is considered unlimited, *i.e.* $C_i \rightarrow \infty$. Thus, the output effort of the buffer tends to zero, and a source of effort (S_{ei}) is added to establish a coupling interface between machine and buffer:

$$S_{ei}(t) = e_3(t) = \min(1, q_i). \tag{5}$$

The variable $e_i(t)$ assumes 0 when there is no material coming from the station downstream (machine starvation); the current machine is then activated when there is any amount of material $q_i(t)$ in the previous buffer ($q_i(t) < 1$); when $q_i(t)$ is at least one unit, the machine will process the input flow with its total processing frequency U_i.
The properties of the 0- and 1-junctions in the bond graph technique yield:

$$\begin{gathered} e_{ei} = e_1 = e_2 \text{ (0-junction)}, \\ f_1 = f_3 = f_{si} = f_4 \text{ (1-junction), and} \\ e_4 = e_1 + e_3 - e_{si} \text{ (1-junction).} \end{gathered} \tag{6}$$

Using Eqs. 1-5 and 6, the output of a station is:

$$f_{si}(t) = U_i(\frac{q_i}{C_i} + \min(1, q_i) - e_{si}). \tag{7}$$

The quotient q_i/C_i tends to zero since $C_i \rightarrow \infty$. Station i is connected to a downstream station k and the output effort from station i (e_{si}) is equal to the input effort from station k (e_{ek}), *i.e.* $e_{si} = e_{ek} = q_k/C_k$. Since the capacity of the buffer k also unlimited, $C_k \rightarrow \infty$, $e_{si} = 0$, yielding:

$$f_{si}(t) = U_i \min(1, q_i). \tag{8}$$

The combination of Eq. 5 and 8 yields the basic state equation, expressing the rate of material storage in station i:

$$\dot{q}_i = f_{ei} - U_i \min\left(1, q_i\right). \tag{9}$$

The flow of finished products is unconstrained and represented by wells, *i.e.* buffers of infinite capacity. Sources of imposed flow supply raw material to the system (Figure 2(b)). The flow is a function of the frequency $U_i$multiplied by one material unit. In controlled sources, the control acts on this frequency U_i. The flow distribution among production stations is performed by converging or diverging junctions, which can be represented in bond graphs by 0-junctions or transformers, respectively. For more details, please see Ferney (2000).

A. Illustrative example

The production system shown in Figure 3 is modeled by Ferney (2000). It contains four production stations (Figure 1), a well (Figure 2(a)), and two sources of flow (Figure 2(b)). The mathematical model is based on the state equation for an individual station (Eq. 9), adding 2 sources and a converging 0-junction:

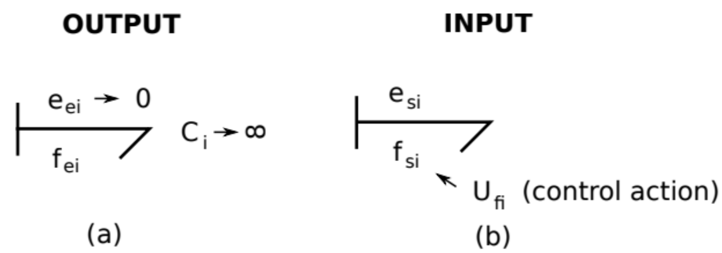

Figure 2. Bond graph representation of a (a) well and (b) input control (controlled source of material flow)

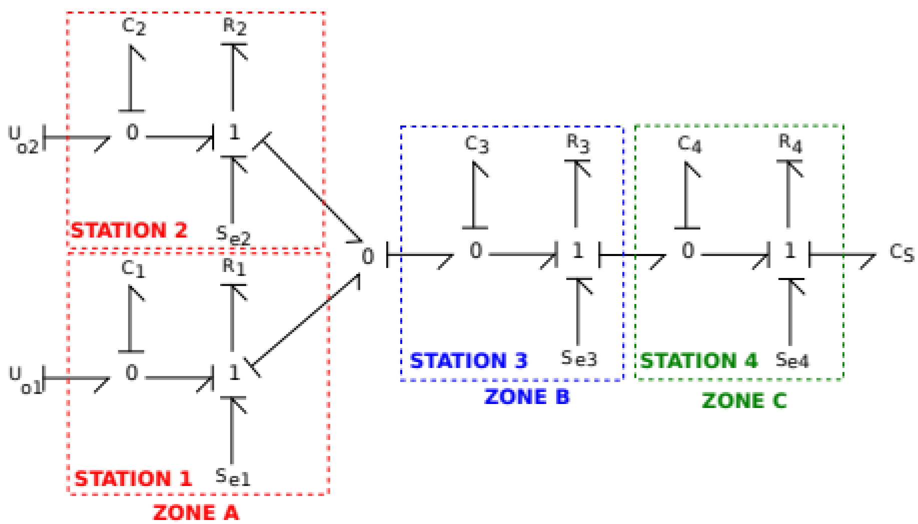

Figure 3: Example with four production stations, a well and two control inputs (existing model)

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ 1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} U_1 \min(1, q_1) \\ U_2 \min(1, q_2) \\ U_2 \min(1, q_3) \\ U_2 \min(1, q_4) \\ U_{01} \\ U_{02} \end{bmatrix} \tag{10}$$

3. Proposal: Ferney's Model Modification

In Ferney (2000), by setting infinite capacities to the buffers, the effort becomes null (see Eq. 4), eliminating the power transmission, which is precisely the bond graphs' strongest point. The buffers are connected to other elements by means of a 0-junction, so the effort in these elements also becomes null. As a result, the machine (element R) becomes disconnected from the buffer, and a source effort ($S_{ei} = \min(1, q_i)$ has to be added to couple machine and buffer. This introduces a nonlinearity in the model and makes the design of controllers more difficult. Thus, we propose a modification in the production station representation to preserve the effort variable and to avoid using the function minimum.

The same interpretation for the variables proposed in Ferney (2000) applies to the proposed model, but the capacity of the buffers now assumes a finite value. Thus, the previously null effort variables also assume finite values (Eq. 4). This allows removing the source of effort introduced to activate the machine, as shown in Figure 4. Eqs. 1-4 and 6 give the output of a station:

$$f_{si}(t) = U_i(\frac{q_i}{C_i} + U_{ei} - e_{si}). \quad (11)$$

A. Illustrative example

To redevelop the equations according to the new station model, the same production system presented in Ferney (2000) was considered. Figure 5 shows the new bond graph model. The input material flow is given by the flow sources' with processing frequency U_{01} e U_{02}:

$$f_{e1} = U_{01} \text{ and } f_{e2} = U_{02} \quad (12)$$

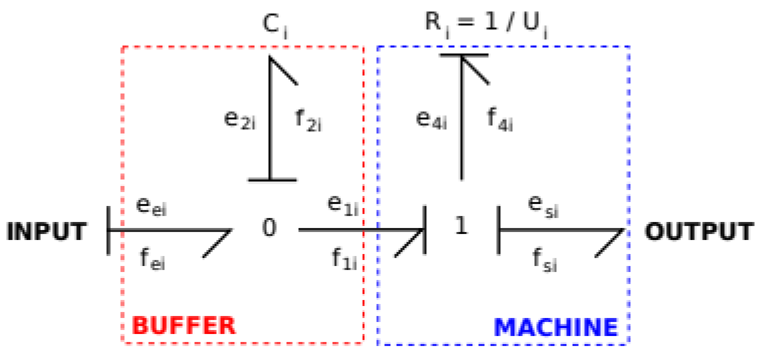

Figure 4. Proposed bond graph representation of a production station *i*

The source of flow U_{01} is connected to the buffer by a 0-junction, so that the effort exerted on the two elements is equal. Machine 1 (R1) is connected to the outlet of buffer 1 by a 1-junction, which is connected to the 0-junction downstream the station. Thus:

$$e_{e1} = e_{11} = f_{e1} = e_{21}(\text{0-junction}), \text{ and} \quad (13)$$
$$e_{41} = e_{11} - f_{e1} = e_{s1}(\text{1-junction}).$$

The constitutive equation of element C applied to station 1 yields:

$$e_{11} = e_{21} = \frac{q_i}{C_i}. \quad (14)$$

Looking at station 3, where the flows come together at a 0-junction, it can be deduced that:

$$e_{s1} = e_{s2} = e_{e3} \text{ (0-junction), and} \quad (15)$$
$$e_{e3} = e_{23} = e_{13} \text{ (0-junction).}$$

The effort $e_{23}$is directly linked ot the buffer of station 3:

$$e_{s1} = e_{13} = \frac{q_3}{C_3} \quad (16)$$

The output flow station 1 is the same as the flow in R1 (property of a 1-junction), that is:

$$f_{s1} = f_{41} = Ue_{41} = U_1(e_{11} - e_{s1}) \quad (17)$$

Replacing 14 and 16 into 17 leads to:

$$f_{s1} = U_1(\frac{q_1}{C_1} - \frac{q_3}{C_3}). \quad (18)$$

The equations for station 2 are analogous to those for station 1, yielding:

$$f_{s2} = U_2(\frac{q_2}{C_2} - \frac{q_3}{C_3}). \tag{19}$$

The input flow of station 3 is ruled by the converging junction, while the output is ruled by the dissipation element:

$$f_{e3} = f_{s1} = f_{s2} \text{ (0-junction), and} \tag{20}$$

$$f_{s3} = f_{34} = U_3 e_{43} \text{ (1-junction and dissipation element)}$$

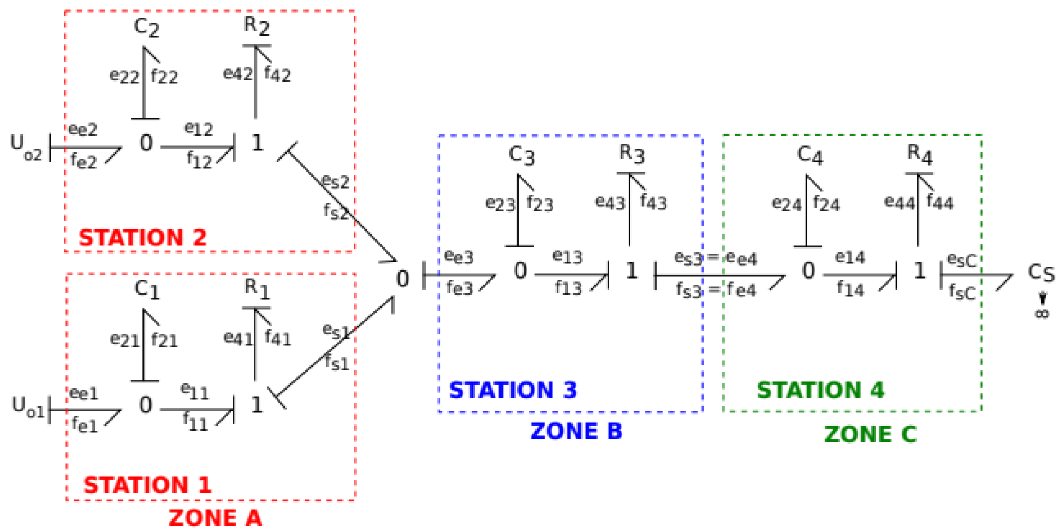

Figure 5: Example with four production stations, a well and two control inputs (proposed model).

The combination of Eqs. 18, 19, and 20 leads to:

$$f_{e3} = U_1\left(\frac{q_1}{C_1} - \frac{q_3}{C_3}\right) + U_2(\frac{q_2}{C_2} - \frac{q_3}{C_3}) \tag{21}$$

Analogous to Eqs. 19 and 18,

$$f_{s3} = U_3(\frac{q_3}{C_3} - \frac{q_4}{C_4}). \tag{22}$$

Due to the 1-junction of station 4, $e_{14} = e_{44} - e_{s4}$. The final output of the model is represented by a well with unlimited capacity, $C_s \to \infty$. Thus, $q_s/C_s \to 0$, and:

$$f_{e4} = f_{s3} = U_3(\frac{q_3}{C_3} - \frac{q_4}{C_4}), \tag{23}$$

$$f_{s4} = U_4\left(\frac{q_4}{C_4} - \frac{q_s}{C_s}\right) = U_4(\frac{q_4}{C_4}) \,. \tag{24}$$

The rates of material storage $\dot{q}_i$ are based on Eqs. 2 and 3. Their combination with Eqs. 18, 19, 21 and 22-24 yields the state model, which can be written in two forms. In the first one, the processing frequencies of the machines are factorized:

$$\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\\ \dot{q}_4\end{bmatrix}=\begin{bmatrix}-\left(\frac{q_1}{C_1}-\frac{q_3}{C_3}\right) & 0 & 0 & 0 & 1 & 0\\ 0 & -\left(\frac{q_2}{C_2}-\frac{q_3}{C_3}\right) & 0 & 0 & 0 & 1\\ \left(\frac{q_1}{C_1}-\frac{q_3}{C_3}\right) & \left(\frac{q_2}{C_2}-\frac{q_3}{C_3}\right) & -\left(\frac{q_3}{C_3}-\frac{q_4}{C_4}\right) & 0 & 0 & 0\\ 0 & 0 & \left(\frac{q_3}{C_3}-\frac{q_4}{C_4}\right) & -\left(\frac{q_4}{C_4}-\frac{q_s}{C_s}\right) & 0 & 0\end{bmatrix}\begin{bmatrix}U_1\\ U_2\\ U_3\\ U_4\\ U_{01}\\ U_{02}\end{bmatrix} \quad (25).$$

This first form is somewhat analogous to Eq. 10. However, in the (Ferney, 2000)'s model, the characteristic matrix does not depend on the variable q_i . Let the state vector and the input vector be, respectively, $\boldsymbol{x^*}^T = [q_1 \quad q_2 \quad q_3 \quad q_4]^T$ and $\boldsymbol{u^*}^T = [U_1 \quad U_2 \quad U_3 \quad U_4 \quad U_{01} \quad U_{02}]^T$. Then:

$$\dot{X}^* = \boldsymbol{A}^*(\boldsymbol{x}^*)\boldsymbol{u}^* \quad (26).$$

where the matrix $\boldsymbol{A}^*$ depends on the state variables $\boldsymbol{x}^*$. This form is not canonical and is not linear due to the product between input ($\boldsymbol{u}^*$) and state ($\boldsymbol{x}^*$). Hence, the design of a controller for this system is not a trivial task. In the second form, the volumes of material in the buffers are factorized:

$$\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\\ \dot{q}_3\\ \dot{q}_4\end{bmatrix}=\begin{bmatrix}\frac{-U_1}{C_1} & 0 & \frac{U_1}{C_3} & 0\\ 0 & \frac{-U_2}{C_2} & \frac{U_2}{C_3} & 0\\ \frac{U_1}{C_1} & \frac{U_2}{C_2} & \frac{-(U_1+U_2+U_3)}{C_3} & \frac{U_3}{C_4}\\ 0 & 0 & \frac{U_3}{C_3} & \frac{-(U_3+U_4)}{C_4}\end{bmatrix}\begin{bmatrix}q_1\\ q_2\\ q_3\\ q_4\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\\ 0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}U_{01}\\ U_{02}\end{bmatrix} \quad (27)$$

Defining the state vector as $\boldsymbol{x^*}^T = [q_1 \quad q_2 \quad q_3 \quad q_4]^T$, the input vector as $\boldsymbol{u}^T = [U_{01} \quad U_{02}]^T$ and the system's output **y** based on matrices **C** and **D** leads to the canonical state model:

$$\dot{\boldsymbol{x}}^* = \boldsymbol{A}\boldsymbol{x}^* + \boldsymbol{B}\boldsymbol{u} \quad (28)$$
$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x}^* + \boldsymbol{D}\boldsymbol{u} \quad (29)$$

This form is advantageous since the controller design is simpler. On the other hand, to assume the standard form, the processing frequencies of the machines have to be interpreted as fixed parameters and only the processing frequencies of the sources of material (U_{01} and U_{02}) are used as control variables.

4. An optimal control approach for wip control

The reduction of shop floor lead times (i.e., the total time to manufacture an item, including the waiting times) is a relevant performance objective, especially when customers prioritize delivery performance in their buying decisions. To achieve that, the work in process (WIP) has to be kept under control. In this work, we use an optimal control approach to adjust the processing rate of the material supply sources ($\boldsymbol{u} = [U_{01} \quad U_{02}]^T$ in Eq. 27), aiming to reduce the difference between the desired and actual WIP in the buffers (*i.e.* $e = r - x$ in Figure 6). Machines rates $[U_1 \quad U_2 \quad U_3 \quad U_4]$ and buffer capacities $[C_1 \quad C_2 \quad C_3 \quad C_4]$ are considered parameters (contants). Therefore, the state-space model in the standard/canonical form (Eqs. 28 and 29) can be used.
As the objective is to keep the work in process in the buffers at desired values, it is necessary to add an integral term so that the system converges to the reference value (see Figure 6). This can be done by augmenting the states with extra states x_I yielding the following state vector:

$$x = [x_I \quad x^*]^T \quad (30)$$

In steady-state $\boldsymbol{x}^* = 0$, these time derivatives of extra states should be equal to the desired values $\boldsymbol{r}$, yielding:

$$\dot{x}_I = Cx^* + r \tag{31}$$

The state of the plant (Eq. 28) should be augmented with the extra state x_I :

$$\dot{x} = \begin{bmatrix} \dot{x}_I \\ \dot{x}^* \end{bmatrix} = \begin{bmatrix} 0 & \boldsymbol{C} \\ 0 & \boldsymbol{A} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_I \\ \boldsymbol{x}^* \end{bmatrix} + \begin{bmatrix} 0 \\ \boldsymbol{B} \end{bmatrix} \boldsymbol{u} + \begin{bmatrix} \mathbf{I} \\ 0 \end{bmatrix} \mathbf{r} \tag{32}$$

In control systems, the system output is used to derive a control action. In the full state feedback control, the state variables are fed back instead of the output. The relation between the control action u and the augmented states is:

$$\boldsymbol{u} = -[\boldsymbol{K}_I \quad \boldsymbol{K}_R][\boldsymbol{x}_I \quad x] = -\boldsymbol{K}\boldsymbol{x}. \tag{33}$$

The gain matrix **K** can be adjusted according to the desired closed-loop characteristics through various synthesis methods. One of the simplest methods is the pole allocation strategy (Sagawa and Mušič, 2019). Furthermore, according to (Franklin *et al.*, 2009), the Linear-Quadratic Regulator (LQR) is widely used by designers for the control of linear systems due to its effectiveness.

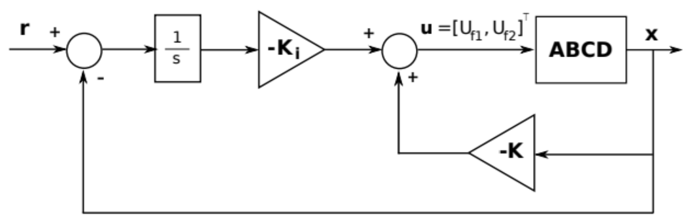

Figure 6. Determining $U_{01}$and U_{02} using optimal control approach.

Given a state-space representation (Eqs. 27 and 28) with a non-zero initial state, $\boldsymbol{x}(0)$, the LQR problem (Skogestad and Postlethwaite 2007) consists in finding the input signal $\boldsymbol{u}(t)$ that will cause the system to return to the zero state ($\boldsymbol{x}(0)$) in a limited amount of time and minimizing the performance index:

$$\mathbf{J} = \int_0^\infty (\boldsymbol{x}^{\boldsymbol{T}} * \mathbf{Qx} + \boldsymbol{u}^{\boldsymbol{T}} * \mathbf{Ru}\ )\mathrm{dt}, \tag{34}$$

where matrix **Q** is a positive (or semi-definite positive) or real symmetric hermit matrix and **R** is a positive or real symmetric defined hermit matrix. This optimization problem can be solved using the Ricatti equation. Since it is applied to the augmented (integral) system (Figure 6), this strategy is named Linear Quadratic Integral (LQI) control.

5. Simulation and preliminary results

Initially, the open-loop system (*i.e.*, without a controller) was simulated. Arbitrary values (as in Ferney 2000) were defined for the processing frequencies of the supply sources, that is, the inputs $\boldsymbol{u} = [U_{01} \quad U_{02}]^T = [4/3 \quad 8/3]^T$. The parameters of the system, *i.e.* machines' processing rates $[U_1 \quad U_2 \quad U_3 \quad U_4]$ and buffer capacities $[C_1 \quad C_2 \quad C_3 \quad C_4]$ were set according to Table 1. The level of WIP in the buffers increases monotonically (Figure 7). This shows the need of implementing a control strategy (closed-loop system).

The proposed production control strategy using the LQI approach was simulated with the parameters shown in Table 1. The buffer levels $\boldsymbol{q} = [q_1 \quad q_2 \quad q_3 \quad q_4]$ should reach the reference values, $\boldsymbol{r} = [r_1 \quad r_2 \quad r_3 \quad r_4]$ in Table 1. Using this approach, the processing frequencies of the sources, the inputs $\boldsymbol{u} = [U_{01} \quad U_{02}]$, are derived from the closed-loop scheme

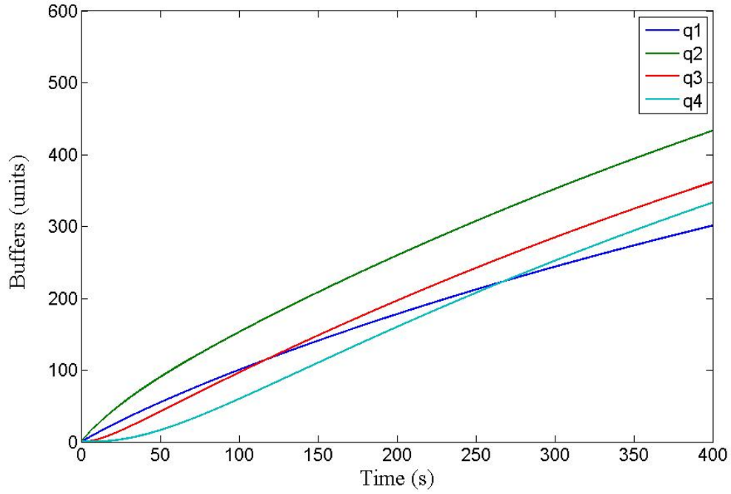

Figure 7. Open-loop numerical evaluation: q values for $\boldsymbol{u} = [U_{01} \quad U_{02}]^T = [4/3 \quad 8/3]^T$

Table 1. Nominal Values

Buffer	Machine	Reference
C_1 100	$U_1 = 1/R_1$ 4/3	r_1 90
C_2 100	$U_2 = 1/R_2$ 8/3	r_2 90
C_3 150	$U_3 = 1/R_3$ 4	r_3 90
C_4 300	$U_4 = 1/R_4$ 4	r_4 90

shown in Figure 6. Figures 8 and 9 depict the input values, $\boldsymbol{u} = [U_{01} \quad U_{02}]$, and the volumes of material in the buffers, $\boldsymbol{q} = [q_1 \quad q_2 \quad q_3 \quad q_4]$.

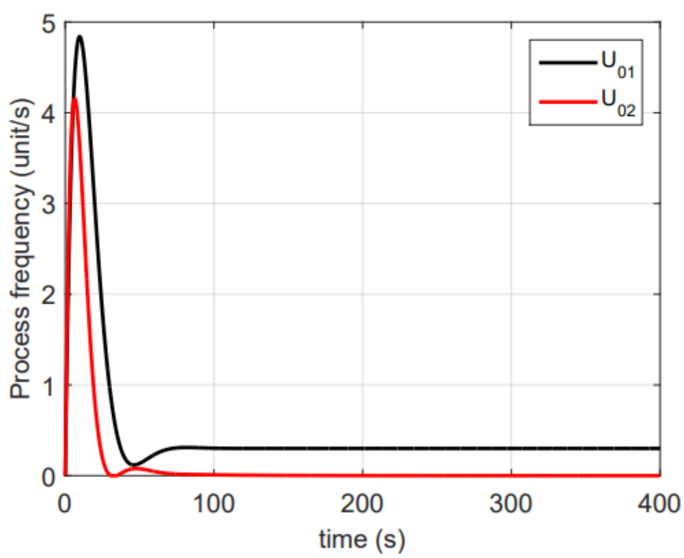

Figure 8. U_{01} and U_{02} values via LQR approach

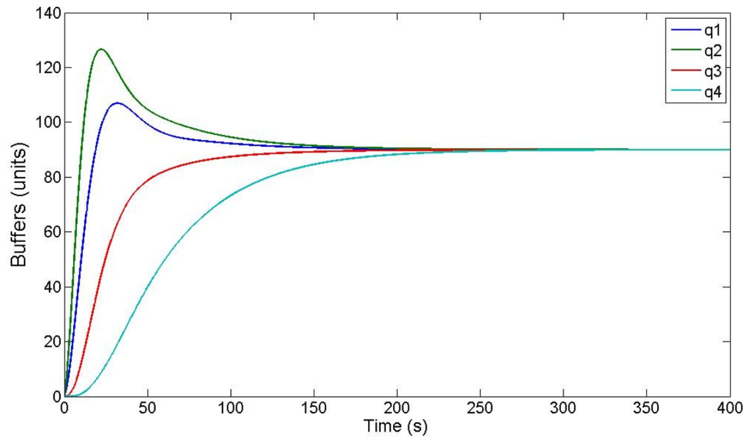

Figure 9. Buffer levels q via LQR approach

A saturation block was used to impose a lower bound to the inputs, avoiding negative values. The weighting matrices **R** and **Q** are chosen according to the Bryson's rule:

$$\boldsymbol{R} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } \boldsymbol{Q} = \begin{bmatrix} \boldsymbol{Q_{xi}} & 0 \\ 0 & \boldsymbol{Q_x} \end{bmatrix} \tag{35}$$

Where $\boldsymbol{Q_x} = diag\{1/r_1^2 \quad 1/r_2^2 \quad 1/r_3^2 \quad 1/r_4^2\}$ and $\boldsymbol{Q_{xi}} = 0.01\boldsymbol{Q_x}$ (see Table 1).

The results showed that the processing frequencies of the sources could be successfully adjusted, leading the system to stability and leading the buffers $\boldsymbol{q}_i$ to the desired levels $\boldsymbol{r}_i$, with some overshoot (initial surplus of WIP) in buffers 1 and 2. These results are reached with the deactivation of one of the supply sources, which may be unfeasible in practice. Another operation point could likely be found with the optimization of the system's parameters (machine rates and buffer capacities). This could also contribute to reduce the overshoot.

To measure the quantitative improvements, the total shop floor throughput time (lead time) for the open-loop system and for the closed-loop system (i.e. with LQR) were calculated. To do so, the actual buffer levels were converted to time by dividing them by the output rate of the system, which is of 4 material units/s, in the studied case (Ferney, 2000). This rate is considered equivalent to the average demand rate. So, the throughput time related to each station was calculated by dividing the actual WIP by the demand rate, which is a procedure usually applied to the value stream maps of lean methodology. For the system of Figure 5, the total throughput time is the sum of the queue time at station 1 or 2 (which one is longer) and the queue times of stations 3 and 4. The results are shown in Table 2. For t =300s, the lead time of the closed-loop system was reduced in 70%.

Table 2. Quantitative improvement results (in terms of throughput time reduction)

		Buffer levels				Throughput times				reduction (%)
	time	q1	q2	q3	q4	max(t1,t2)	t3	t4	total	
open-loop system	100	101	153	97	60	38	24	15	78	-
	200	178	260	197	160	65	49	40	154	-
	300	244	352	285	252	88	71	63	222	-
closed-loop system	100	92	95	87	73	24	22	18	64	18%
	200	90	90	90	88	23	22	22	67	56%
	300	90	90	90	90	23	22	22	67	70%

6. Managerial Discussions

From the production control perspective, the model proposed here resembles a pull-system, since it limits the WIP in the system. This WIP limit brings relevant benefits, highlighted in the literature (Hopp and Spearman 2004), namely: (1) reduced manufacturing cycle time/lead time, based on Little's Law, *i.e.* a more responsive and flexible shop floor; (2) smoother production flow, since fewer fluctuations in WIP level lead to a more predictable output stream; (3) improved quality, because less WIP and smaller lots expose quality problems and allow earlier detection of errors; (4) reduced cost, since problems such as long set up times and uncoordinated part supplies for assembly show up in the form of blocking and starving of the machines and, once tackled, cost is reduced.

Other contributions are highlighted as follows:

• In many production systems, buffer capacities are restricted by economic factors, products' obsolescence or limitations in the storage area. The proposed modification in the model allows to incorporate this restriction;
• The elimination of non-linear functions (*e.g*. the function *min*) eases the design of controllers;
• The modularity of bond graphs enables to combine the production stations and junctions to depict different production configurations, and the corresponding state equations can be derived from the pictorial representation;
• The model can be used to study the dynamics of production control systems such as Workload Control, which relies on the limitation of order release*, i.e*. input control.

7. Conclusion

In this paper, we modify an existing bond graph model of shop floor dynamics by defining buffers with limited capacity and by eliminating the source of effort that coupled buffer and machine. As advantages, the proposed model supports a more realistic assumption, relevant in practice, and respects the power transmission principle of the bond graph approach.

The LQI controller implemented for WIP control was able to track the desired (non-zero) values for the buffers. This WIP control produces practical benefits, as already highlighted in the literature. The simulation of the model may provide directions to managers on how to respond to disturbances in the shop floor without an excessive cushion (since excessive WIP leads to long manufacturing times). The modularity of bond graphs allows modeling production systems with different configurations.

There is a gap concerning closed-loop models for production control; the studies are even more scarce when bond graph models are considered. This study addresses this gap and contributes to the Industry 4.0 context with a model of a responsive shop floor. As future research, optimization approaches could be used to better define the parameters of the model (machine processing rates and buffer capacities). Also, Model Predictive Controllers (MPC) can be proposed using the canonical state-space model or nonlinear approaches can be implemented using the nonlinear model, which has not been explored in this work.

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