

# Simulation of Texture Geometries to Optimize Tribological Properties.

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## Abstract

Surface texturing is a process that enhances the tribological properties of different materials. This topic has been widely studied because it is a tool that saves resources such as materials, money, and time. To obtain the most useful set of parameters to get the smallest Coefficient of Friction (COF), several simulations in the program COMSOL were made, varying depth, width, texture density and oil film thickness. Design of experiments with the methodology of Box Behnken was applied through Minitab to reduce the number of simulations that had to be performed (from 81 to 27 runs), and to identify the most significant parameter that provided a lower COF. Minitab showed that every input parameter (oil film thickness, width, depth, % textured area) has an important role for reducing COF, being the most important the lubricant film thickness. The results showed us improvements depending on the lubrication regime ranging from 60% and 95% where the greatest decreases in COF were observed in boundary lubrication conditions. This demonstrates the importance of texturing for reducing the COF in Tribological tests and the potential it has for manufacturing processes, as well as the use of simulation programs to help reduce the consumption of laboratory resources. A design guide was made including Minitab and COMSOL for different manufacturing processes considering boundary, mixed and hydrodynamic lubrication.

## Keywords:

Surface texturing, Laser surface texturing, Simulation, Design of experiments, Tribology.

## 1. Introduction

The surface texturing is a very effective technique used to improve tribological properties of mechanical components where micro/nano cavities are created on a pattern such as circles, channels, S-shape, chevron, crosshatch, etc. The result of this process is a significant improvement in load capacity, wear resistance, COF, and the useful life of tribological mechanical components (Chen, 2018) (Etsion, 2005). There are different types of

texturing such as mechanical texturing, electrochemical micro machining, lithography, micro- or nano casting, laser surface texturing, among other types of texturing, which involves multiple steps for generation of textured surfaces (Patel, 2018). Laser surface texturing is one of the best methods for surface texturing because the laser is fast, friendly with the environment and provides control of the shape and size of the cavities. Zhang (Etsion, 2005) made a groove textured surface and found that COF was 12.7% lower than the non-textured surface under oil lubrication. (Zhang, 2018) Chouquet performed surface texturing on a DLC/steel lubricated sliding contact, and compared to a non-textured surface, the COF was reduced by 63% (Chouquet, 2010). The importance of this investigation is how to use different parameters to optimize the dimensions in laser texturing, this type of combinations are mainly done experimentally in which there is a lot of time consumption and lab resources, with the use of simulations this can be done faster and cheaper, furthermore the next steps and investigations must be focused in simulations.

Table 1. Summary of literature related to surface texturing for different applications.

Authors/year	Shape	Dimensions	Depth	Tests	Effect on COF
Dan L, Xuefeng Y., Chngyang L. et al. Tribology international. (Xuefeng, 2020)	Chevron texturing	0.7 x 0.1 x 0.3 mm	32 $\mu$ m	Pin on disk	Reduced by 52%
Chouquet C et al. Materials chemistry and physics. (Chouquet, 2010)	Dimple texturing	7 $\mu$ m	0.3 $\mu$ m	Ball on disk	Reduced by 63%
Zhang D et al. Wear. (Zhang, 2019)	Channels	1 mm	400 $\mu$ m	Pin on disk	Reduced by 20%
Chen P et al. Applied surface science. (Chen, 2016)	Triangles	1000 $\mu$ m	400 $\mu$ m	Pin on disk	Reduced by 27%
Rosenkranz A et al. Tribology International. (Rosenkranz, 2019)	Crosshatch	10 x 10 x 7 mm	39 $\mu$ m	Test rig	Reduced by 44%
Kovalchenko, A. Wear (Kovalchenko, 2011)	Dimple texturing	78 $\mu$ m	5.5 $\mu$ m	Pin on disk	Reduced by 44%
Ezhilmaran, V. et al Journal of tribology (Ezhilmaran, 2021)	Dimple texturing	100 +- 6 $\mu$ m	No specified	Reciprocating tribometer	Reduced by 53%
Wu, Z. et al. Surface & coatings technology. (Wu, 2017)	Dimple texturing	70 $\mu$ m	50 $\mu$ m	Ball on disk	Reduced by 40%
Li K. et al. Tribology International. (Li, 2014)	Dimple texturing	1100 $\mu$ m	10 $\mu$ m	Pin on disk	Reduced by 50%
Li J. et al. Tribology International (Li, 2010)	Dimple texturing	150 $\mu$ m	40 $\mu$ m	Ball on disk	Reduced by 50%

Recently some authors have been concerned about future surface texture applications, Gachot had observed an important need in this field, simulating the effects of surface texturing under different regimes and predicting the optimum texture parameters to overcome the time consuming and expensive trial and error approach. (Gachot, 2017) Kumar V, mentions that LST faces many challenges, the most important one is the determination of optimum texture design and laser characteristics. They concluded that LST trials to get an optimal design takes a lot of time for evaluation, so computational modelling is a very suited technique to study the parametric behavior before actual experimental studies. (Kumar, 2020)

## 2. Experimental Methods

### 2.1. Methodology

The tribological test T-05 block on ring, evaluates the behavior of wear in materials at different conditions, this allows a confident selection in both materials for specific tribological applications (Leroux, 2016). This test as shown in Figure 1a. uses a conformal block with a Load, and a ring with a rotational velocity in which there are

variations such as in pressure, velocities, loads and time. With these parameters the COF and the wear of the block is determined. In this procedure the lubricant is added and with different tests a graphic of the change in the COF can be determined. (Maldonado, 2021)

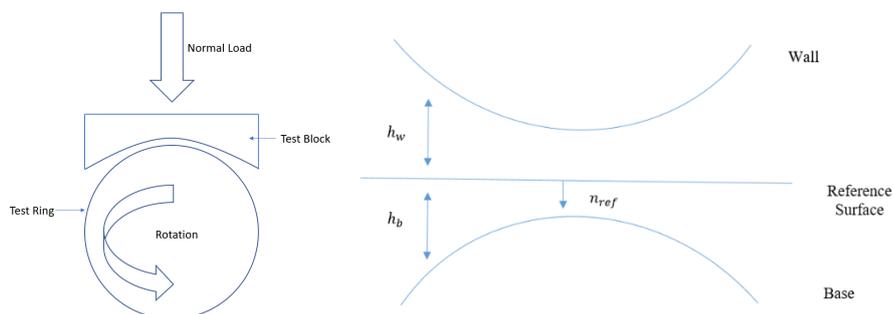


Figure 1.a) Block-on-ring test. b) Thin film flow COMSOL study.

For the study we used COMSOL Multiphysics, this is a powerful modeling and analysis tool, where one can simulate fluids on a determinate surface and obtain desired results such as COF, force, pressure, etc. The simulations on this research were made on COMSOL to reduce resources such as material, lubricant, time, and money. (Comsol, 2020) The most similar study of the block on ring test on COMSOL is called thin film flow, this study is used to solve the Reynolds equation or the modified equation in a narrow channel, by a surface within the geometry. It is used for lubrication, electrohydrodynamic, or gas damping situations which is why we applied this study for our research. (Comsol, 2018) In Figure 1b it can be seen how the study thin film flow applies a channel where the fluid passes through. In this case the wall has a velocity, and the base is fixed to simulate the test T-05 as shown in figure 1a. In Figure 1b the  $h_w$  is the distance from the reference to the upper wall in which the reference is the lubricant film thickness and the  $h_b$  is the distance from the base to the reference surface where the reference surface is the lubrication thickness. (Comsol, 2018)

## 2.2 Parameters and Lubricant

The parameters used in this study were found based on various studies in which they made the tests in the laboratory, so we decided to stick to reality and choose those materials and processes that were not far away so that we could check these simulations in COMSOL with an existing study. The variable parameters shown in Table 2, were the parameters that we were changing while doing the simulations, these parameters were the most appropriate because they were used in laboratory tests.

Table 2. Variable Parameters in the simulation

Variable Parameters		Author
Pressure	0.05 – 15 MPa	Suh, M. et.al. (2010)
Channel Width	20 – 70 $\mu\text{m}$	Maldonado, D. et.al. (2019)
Depth	20 – 70 $\mu\text{m}$	Maldonado, D. et.al. (2021)
Texturing Density	5 – 30 %	Dan, L. et.al. (2020)
Lubricant Film Thickness	1 – 6 $\mu\text{m}$	Chong, W. et.al. (2014)

Other parameters that we used were those observed in Table 3, these parameters are fixed as they were found in the study by Maldonado, et.al. (Maldonado, 2021), in this study the tests were carried out in the laboratory in the tribological test T-05 block on ring. In addition to the fact that the lubricant used in these tests was PAO4, we decided to use it since it is very common among lubricants and does not contain any additives, so we decided to use it in our studies (Maldonado, 2019).

Table 3. General Parameters in COMSOL.

General Parameters	
Disk Speed	250 RPM
Block Area	100.013 mm <sup>2</sup>
PAO4 Density	817.2162 $\frac{kg}{m^3}$
PAO4 Dynamic Viscosity	0.01373 Pa * s

### 2.3 Design of Experiments

Design of Experiments was used to reduce the number of runs to find the optimal COF, the method used was the Box Behnken design. This method groups together a group of simulations where for each simulation only certain factors change. The identity of the factors that vary in each group of runs changes from one group to another (Jones, 2013). With this design we create our model, the number of simulations decreases from 81 to 27. This model chooses 4 levels with 3 factors, we use Low, Medium, and High, and with this we obtain the 27 runs with their respective COF.

Table 4. Factors and Levels.

Level	Depth $\mu\text{m}$	Width $\mu\text{m}$	Texture Density %	Lubricant Film Thickness $\mu\text{m}$
Low	20	20	5	1
Medium	45	45	16	3
High	70	70	30	6

## 3. Results

### 3.1 Pressures Thin Film Flow 2D

As a first test, a test was carried out in the thin film flow study in 2D to demonstrate the distribution of the pressures, where it can be observed that in the flat figure of Fig. 4a the pressure is distributed throughout the entire figure, while in the texturing of Fig. 4b the pressure is concentrated in one area.

When researching the literature, we found that Rosenkranz et al. (Gachot, 2017) obtained similar results where the sample without texturing had the pressure distributed throughout the piece and in the textured samples the pressure was concentrated in a single area. He found that when the convergence ratio is high, the textured parts, especially the single pocket one produces a lower minimum film thickness, in this case, friction is reduced due reduced shear stress within the textured area. (Gachot, 2017)

After performing the pressure test in 2D, preliminary studies were carried out to find the appropriate use of COMSOL where we varied the pressures, the channel dimensions, and the texture density, where we found that the variables that affect the COF were the density of texturing, the thickness of lubrication and the geometry of texturing. What we observed in the preliminary tests was that the pressure does not decrease the lubrication thickness, so we had to vary the lubrication thickness manually and we noticed significant changes.

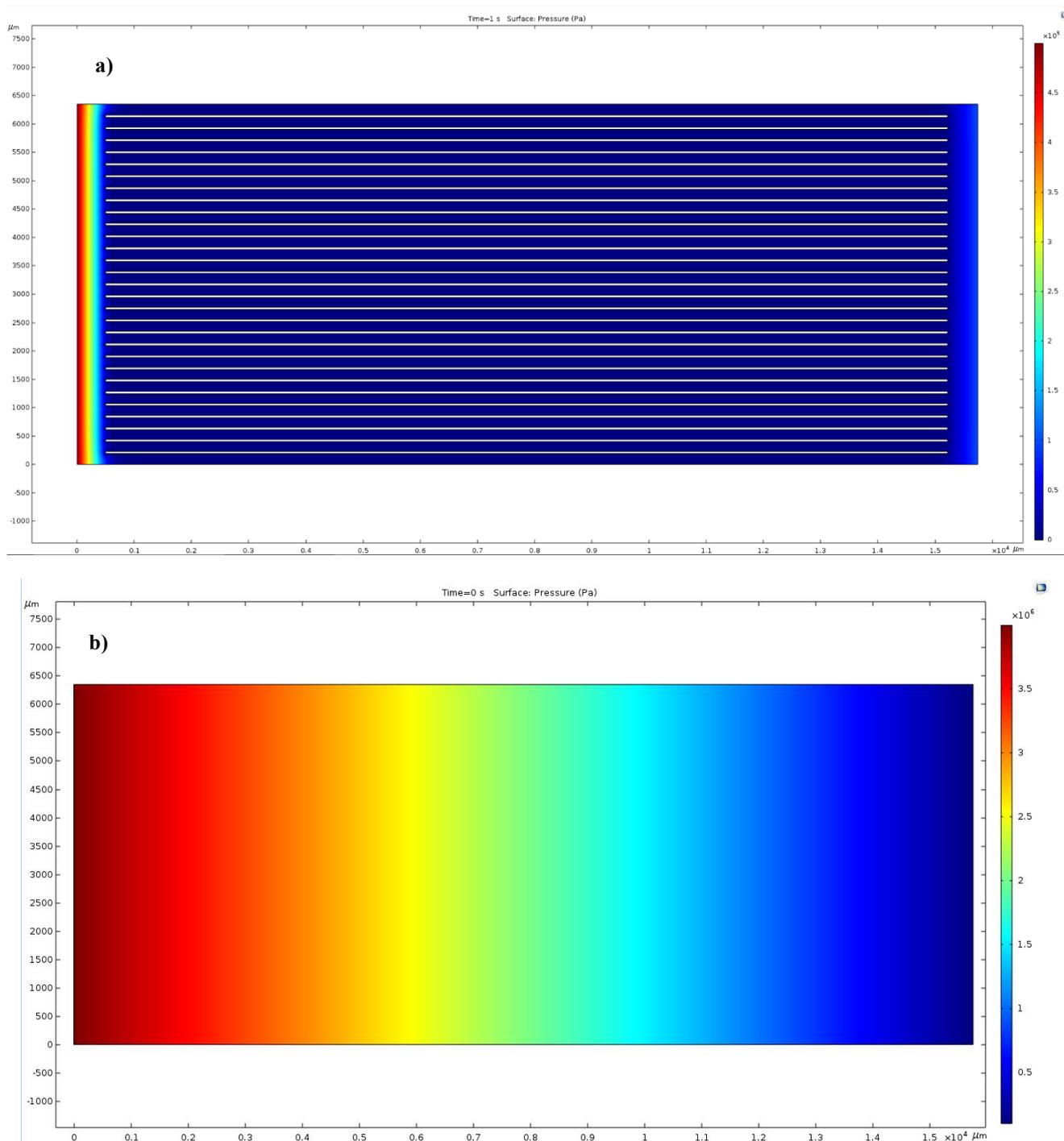


Figure 4. a) Non texturized and b) texturized 2D plot Pressure.

### 3.2. Statistical Analysis

As seen in Table 5 the design of experiments reduced from 81 runs to 27 runs, in Table 5 we can see the results of the COF depending on the factors and the level used. These results are placed in Minitab to analyze statistically the results and obtain which factors are the most significant in the second order model as shown in Equation 1.

Table 5. 27 Runs with the factors and the resultant COF simulation in COMSOL.

Depth $\mu\text{m}$	Width $\mu\text{m}$	Texture Density	Lubricant Film Thickness $\mu\text{m}$	COF
20	45	30	3	0.00374
20	45	5	3	0.00403
45	20	16	6	0.00211
45	70	16	6	0.00259
45	20	16	1	0.00284
45	45	16	3	0.00347
45	70	5	3	0.00403
45	70	16	1	0.00581
70	45	30	3	0.00050
20	45	16	6	0.00243
20	45	16	1	0.00977
45	20	30	3	0.00019
45	70	30	3	0.00284
45	45	16	3	0.00347
45	45	5	1	0.01016
70	45	16	6	0.00235
70	45	5	3	0.00385
70	45	16	1	0.00668
20	20	16	3	0.00375
20	70	16	3	0.00410
45	45	30	6	0.00230
45	45	5	6	0.00237
45	45	16	3	0.00347
45	20	5	3	0.00385
45	45	30	1	0.01164
70	20	16	3	0.00100
70	70	16	3	0.00317

We use 4 parameters, A is depth ( $\mu\text{m}$ ), B is texture density (%), C is width ( $\mu\text{m}$ ) and D is the lubricant film thickness, as shown in Equation 1. The R square confirms that our model has a reliability of 80.41%, which means that the study has good reliability. In Fig. 6 the Pareto chart shows the absolute values of the standardized effects from the largest effect to the smallest effect. The standardized effects are t-statistics that test the null hypothesis that the effect is 0. The chart also plots a reference line to indicate which effects are statistically significant., implying that the factors that have a T value greater than the Critical Value T are the most significant in the model, in this case all the factors statistically are significant in the model but the one that has the most significance is the thickness of the lubrication film, and the interactions are not significant statistically.

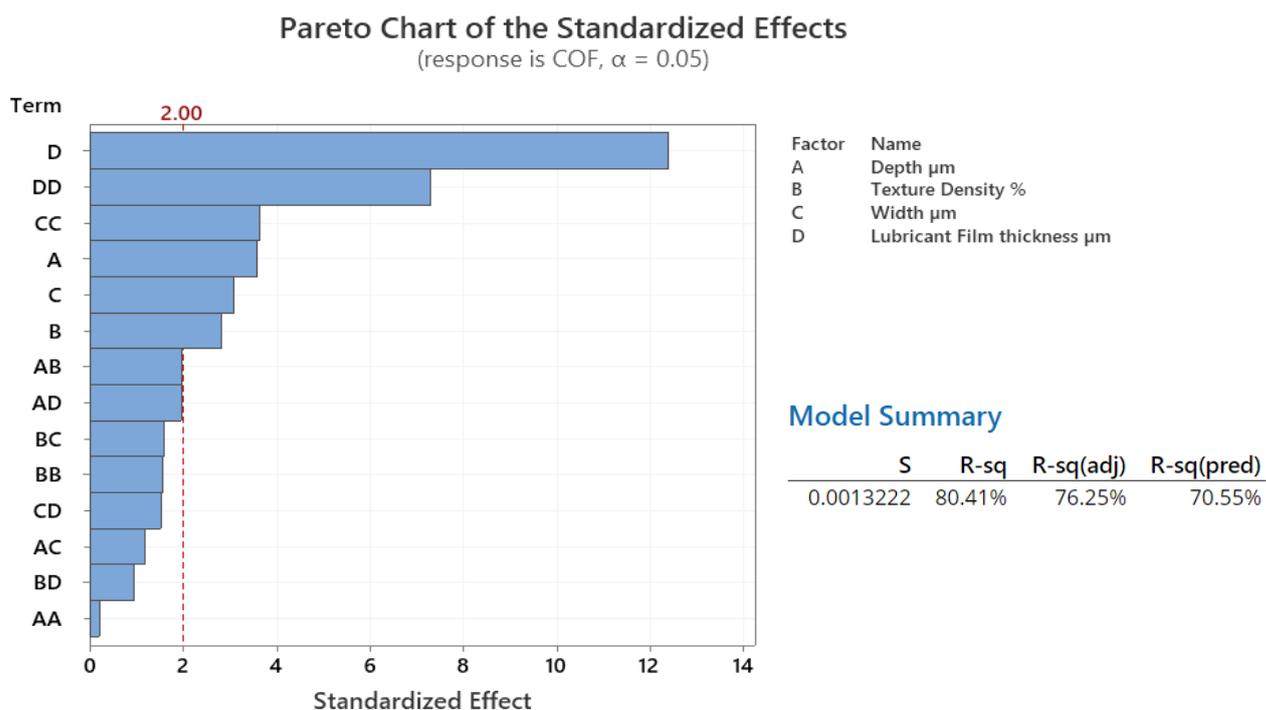


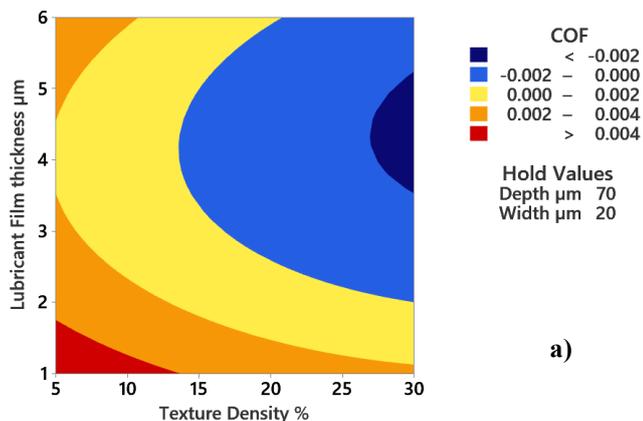
Figure 6. Model Summary and Pareto Chart of Standardized Effects.

$$COF = 0.001038 - 0.0000555 * A - 0.000108 * B + 0.000168 * C - 0.003862 * D - 0.00000001 * A^2 + 0.00000003 * B^2 - 0.000002 * C^2 + 0.000406 * D^2 - 0.000002 * AB + 0.000001 * AC + 0.000012 * AD + 0.000002 * BC - 0.000011 * BD - 0.000009 * CD$$

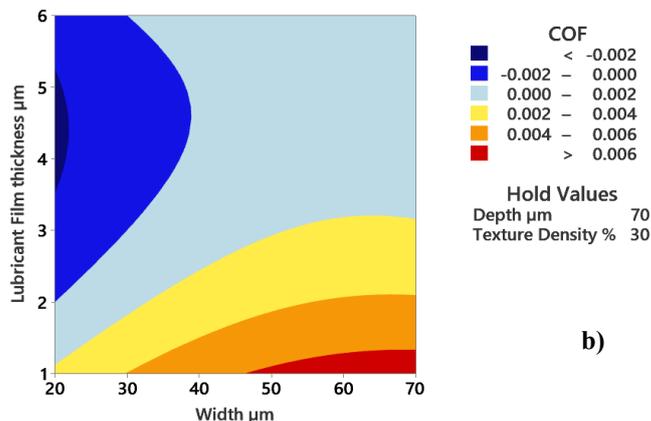
Equation 1. Equation of COF obtained by Minitab.

As shown in Equation 1, to obtain the expected value of the COF a formula was made with Minitab in which, we enter these 4 parameters, the A is depth ( $\mu\text{m}$ ), B is texture density (%), C is width ( $\mu\text{m}$ ), and D is the lubricant film thickness. Using these parameters on the formula you can obtain an expected value of COF.

Contour Plot of COF vs Lubricant Film thickness  $\mu\text{m}$ , Texture Density %



Contour Plot of COF vs Lubricant Film thickness  $\mu\text{m}$ , Width  $\mu\text{m}$



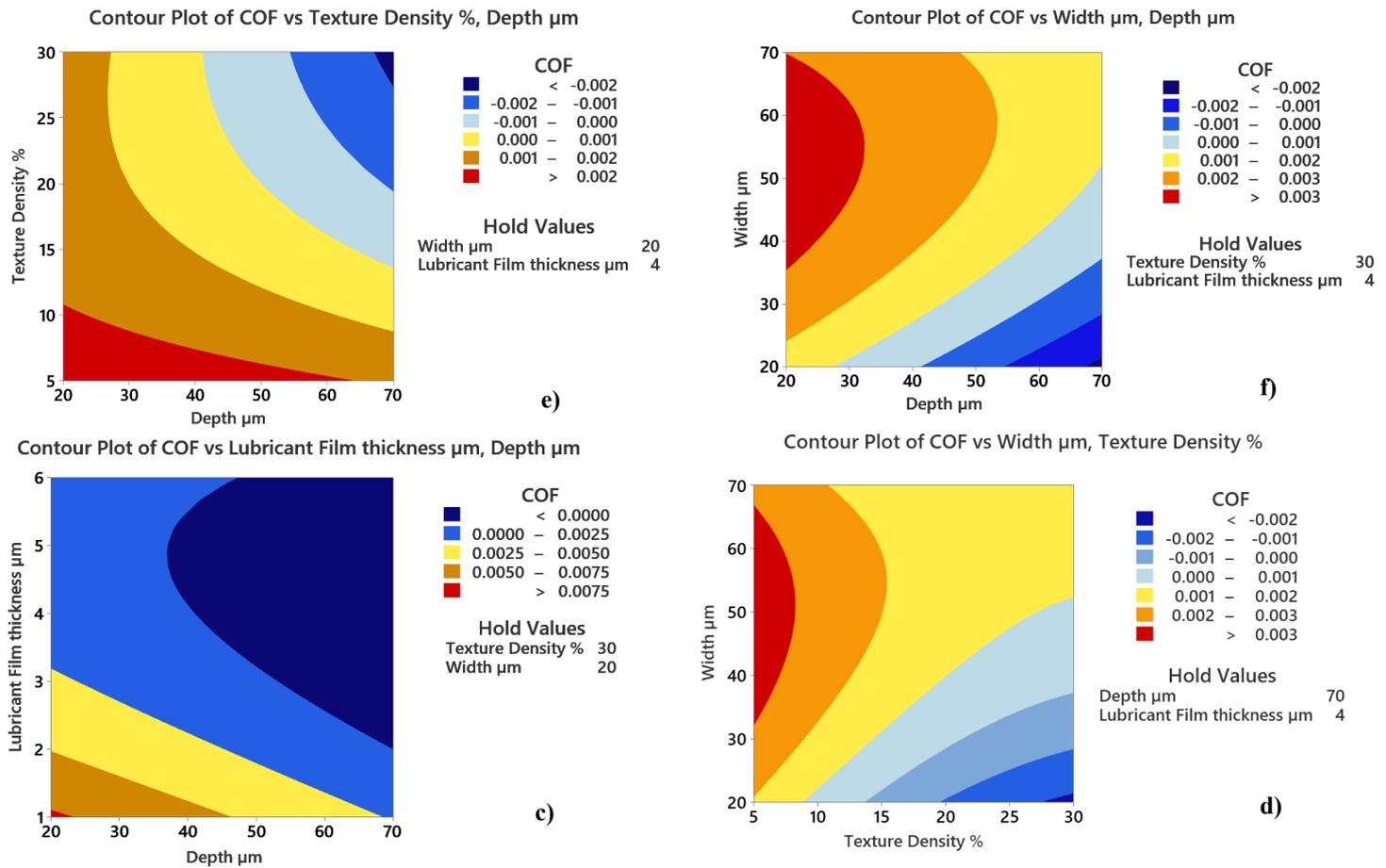


Figure 7. Contour Plots COF vs. Input Parameters.

In Figure 7 the results made in COMSOL are shown in Contour Plots showing the graphic representation of the COF with the different combinations of the parameters. The blue zone is the interesting zone in which the minor COF is shown.

Table 6. Conclusions of Contour Plots Results.

Graphic	Parameters	Conclusion
A	Lubricant Film thickness vs. Texture Density	For a lubricant film thickness between 2 and 6 Micrometers, the texture density recommended is 15 to 30%.
B	Lubricant Film thickness vs. Width	For a lubricant film thickness between 2 and 6 Micrometers, the width recommended is from 20 to 40 Micrometers
C	Lubricant Film thickness vs. Depth	For a lubricant film thickness between 2 and 6 Micrometers, the depth recommended is 40 to 70 Micrometers
D	Texture Density vs. Width	For a Texture Density between 20 and 30%, the width recommended is 20 to 30 Micrometers.
E	Texture Density vs. Depth	For a Texture Density between 20 and 30%, the depth recommended is 55 to 70 Micrometers.
F	Width vs. Depth	For a Width between 20 and 30 Micrometers, the recommended depth is from 55 to 70 micrometers.

To obtain the lowest COF, the Minitab Response Optimizer tool is used, where the objective is selected to minimize the COF with its own factors, in Table 7 the solutions that Minitab yields are shown, in this case having a low COF with a small film thickness requires large dimensions of the texturing so that more lubricant can be contained. With a higher film thickness smaller dimensions are needed since the film thickness contains enough lubricant to reduce the COF.

Table 7. Response Optimizer minimize COF.

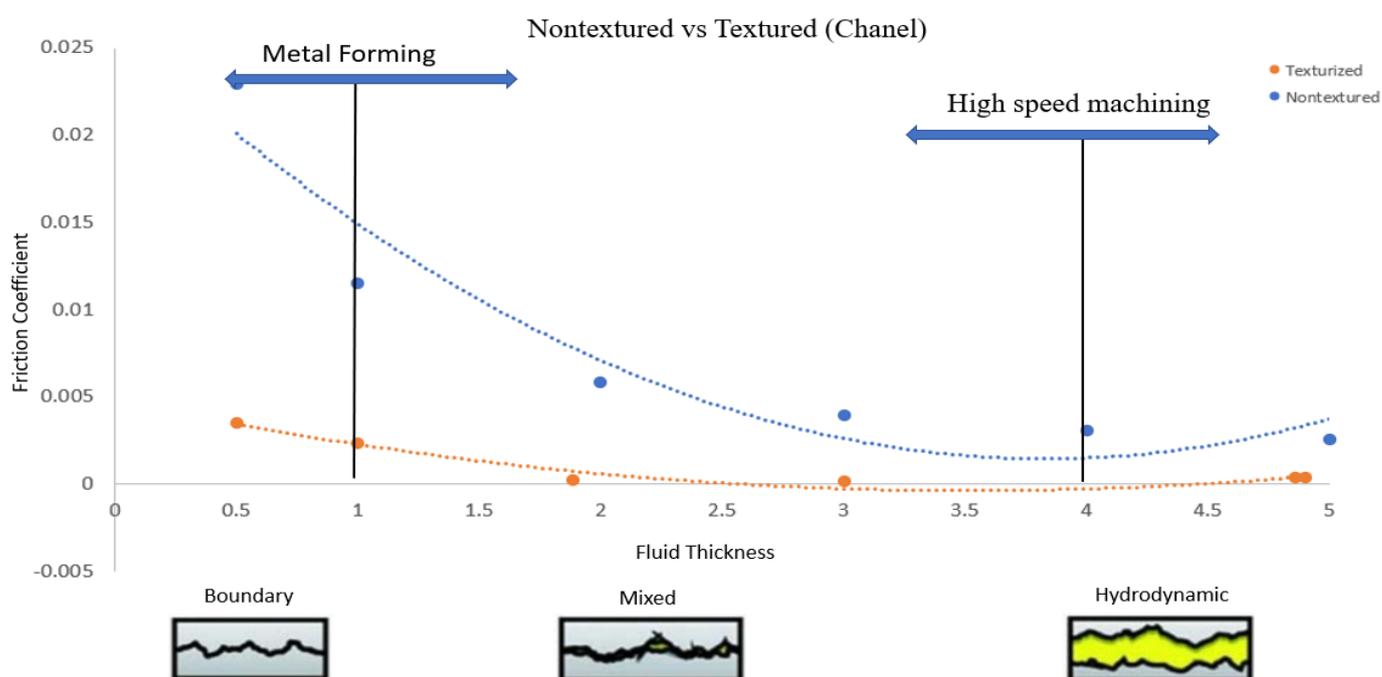
Depth	Texture Density %	Width $\mu\text{m}$	Lubricant Film Thickness $\mu\text{m}$	COF Fit	Composite Desirability
70.0000	30.0000	20.0000	1.88332	0.0002431	0.99501

### 3.3 Non-Texturized vs. Texturized

The thickness of the lubrication film is the variable with the greatest impact on the COF, this is clearly seen with the Stribeck curve which tells us that according to the process conditions, such as RPM, viscosity and applied load, the thickness of the film can be made thinner. In an Stribeck curve we can see how the thickness of the lubrication film is very small so the COF is very high, the more the film thickness rises, the COF will decrease, when the film reaches being very thick occurs a phenomenon called pitting which creates a localized accelerated dissolution in the metal, and this causes a break in the passive protective film on the metal surface (Frankel, 1998). In a study made by C. Gachot (Gachot, 2017), he mentions in his results charts that when the texturized is used, a material that is normally in a boundary lubrication, it passes to mixed lubrication, in which the same parameters are still taken such as RPM, viscosity and the same pressure.

In Figure 8 we can see the results of our project where the blue graph is the non-textured piece and the orange one is the textured one, depending on the lubrication film thickness, the Stribeck curve has a tendency. Where at the beginning we have a boundary condition with the highest COF which tends to go down and then it goes to mixed since the lubrication film is stored more in the channels, so the COF is reduced.

Figure 8: Graph Non-textured vs. Textured samples



We can see evidence of this in the study by Maldonado, D (Maldonado, 2021) where 1 MPa is hydrodynamic, 2.5 mixed and at very high pressures it becomes boundary, where, as we can see, the lines are the best results, this because they are open geometries, and this makes them contain more lubricant and less burr. It should be noted that our COF is very low since there is no roughness, we tried to add it, but we realized that it was not feasible since it did not have a significant change in the comparison between the plane and the texturing since the whole study was carried out without roughness, this means that there will be no difference in the results compared to other studies (Maldonado, 2021).

### 3.4 Design Guide

A design guide was obtained which relates the COF vs. the film thicknesses and the possible manufacturing processes using Comsol and the optimization obtained in Minitab. Some processes found in the literature such as that of Recklin V. and Aktürk, D. and it was found that for forming processes such as die cutting, punching and bending, the thicknesses tend to be thin. If you have Metal Forming, the recommended thing to have a good COF would be to have a density between 20% to 30% with a width of 20 Microns and a depth between 40 to 70 Microns (Recklin, 2018) (Aktürk, 2015).

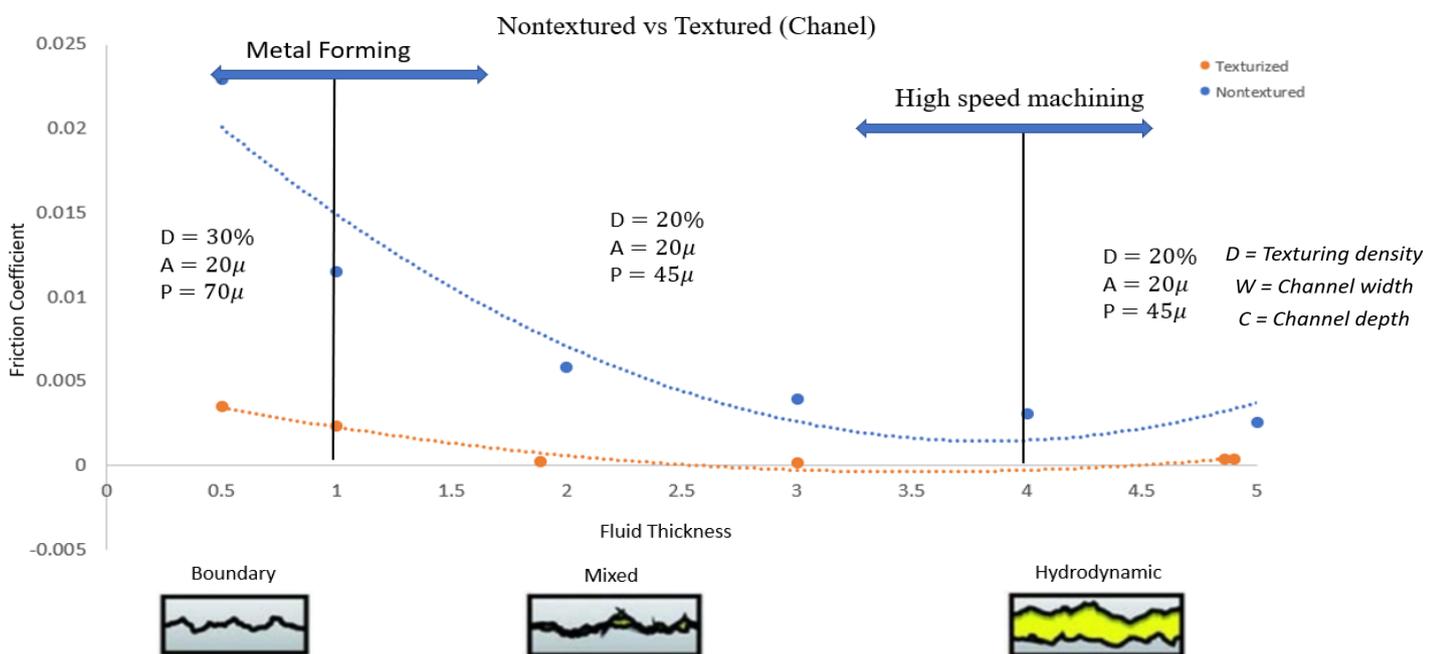


Figure 9. Graph Non-textured vs. Textured, the design guide is obtained based on the Design of Experiments data from MINITAB and COMSOL.

These recommendations come from studies and literature, which tells us that a boundary lubrication regime requires a lot of texturing since the lubrication film thickness is very low and a higher percentage is occupied in the geometries so that more lubricant is retained. For machining with high speeds, thicker thicknesses are needed since there is no need to texturize as much, with 20% texturing and with a width of 20 Microns a lower COF can be obtained, in addition, it is cheaper, and it uses less laser (Aldana, 2014).

### 4. Conclusions

In this project we carried out a simulation of the tribological test T-05 through COMSOL (Thin-Film Flow), then we carried out a Design of Experiments (Box Bhenken) that later we analyzed statistically by Minitab, both were

used to reduce time and laboratory resources. Through these tests it was possible to determine that all the established parameters are significant, however the lubricant film thickness is the most statistically significant parameter, also through Minitab results showed a second order equation, to obtain an expected value of the COF where our  $R^2$  gave us a value of 80.41%, which tells us that it is a reliable model. As shown in Figure 8, the graph of the nontextured versus the textured, shows an improvement in percentage of approximately 60 to 95%, where in the boundary lubrication regime it was approximately 90%, in the mixed lubrication it was between 85 to 95%, and in hydrodynamic it was between 60 to 80%. A design guide shown in Figure 9 was made, in which the optimized parameters were obtained according to the lubrication regime and the manufacturing process. Our recommendation would be to carry out these same tests in the laboratory to validate what was obtained in our simulations and in our Design of Experiments, our study could also be implemented for different geometries searching to improve the COF.

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