Coordination and Non Coordination Lease Contract Model with Availability Target

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Abstract

A lease contract (LC) model for a fleet of dump truck with availability target is developed in this paper. Under this LC, the lessor provides a full maintenance service during the period LC. To sustain high performance of the fleet, we proposed an availability target as performance of the LC. In addition, the lessor gives some incentive in term of a price discount if the lessee leases more dump trucks. The optimal decisions from the lessor's viewed point (maintenance policy i.e number of periodic maintenance, maintenance level, pricing policy) are to minimize the total cost. The decision problem for the lessee is to select the best option suitable to its requirement. We provide numerical examples for illustrating the optimal decisions for the lessee, and the lessor, which maximizes the expected profit for each party.

Keywords
Lease contract, production price, discount, imperfect repair and game theory.

1. Introduction

Maintenance in lease contract is among important topics in Industrial Engineering and it has increasingly received attention from researchers. Initial investment concern, flexibility on the upgrading of the equipment, and the reduction of maintenance cost, as well as inventory are the main motivations of acquires equipment through leasing rather than purchase it (Jaturenonet al. 2006). Some examples include (Jaturenonet al. 2006; Murthy et al. 2014, Pongpeh et al. 2006) which provides a comprehensive review of LC for new item. The authors in (Pongpech et al. 2006; Yeh et al., 2011) investigated the lease contract associated with used equipment whilst (Aras et al. 2011) considered lease options for equipment resulting from remanufactured items. These authors mentioned that the study can be done from both sides: the lessor's perspective or lessee's perspective. There are two levels of decision problems, that can be viewed from the lessor's perspective as a main concern, i.e. the strategic one and the operational one. In the strategic level issues relevant such as type and number of equipment lease, upgrade options to compensate with technological obsolescence, etc. And the issues of operational level are dealing with consisting of maintenance servicing, spare part stock, crew size, etc.). For the case where the study is done from the lessor and lessee point of views then a game theory formulation is needed to modelling the decision problems (Bhaskaran 2000, Iskandar 2015)).

In many cases, many mining companies used their equipment intensively. It is indicated with high usage per unit of time. The excessive usage affects significantly the deterioration of the equipment. This indicates the need to consider age and usage in the lease contract which involves two parameters –i.e. age and usage limits (called a two dimensional lease contract). We are aware only the works by Husniah et al, 2018 belong to this group. In the period of the contract is always the same with maximum usage rate whilst (Husniah et al. 2018, Hamidi et al. 2016) consider a two dimensional lease contract for maximum (age) or (usage).

As the equipment wear out with age and use, a powerful support of maintenance is expected to keep them in a working condition which thusly gives a high accessibility to the use of them. Many mining organizations think about the accessibility of dump trucks as a basic execution measure in supporting their business. In this regards, the owner would interest on an LC which gives a high availability for the equipment while preserving a reasonable cost. Looking at the current underlying problem of the LC we pose for the dump truck, this approach fit neatly to the current problem.

In this paper, we consider a circumstance where a mining organization works various dump trucks (*N*) to satisfy a daily production target. As a result, the company would like to lease a fleet of dump trucks instead of purchase them. We consider that each truck is leased for a certain period of time or the lease contract is bounded by time but has no usage bound. However, if the usage exceeds a predefined maximum level allowed in the contract, then the lessee has to pay some extra charge. In other words, the lessee incurs some additional costs if the usage exceeds the allowable level of usage. In addition, we consider the price scheme of LC which gives some benefit for the lessee when he leases more than one equipment.

The paper begins with an introductory section followed by model formulation for the two-dimensional lease contract in Section 2. Model analysis is carried out in Section 3 and the optimal decisions for the lessor and the lessee are discussed in Section 4. In section 5, we provide a numerical example and finally followed by brief conclusion together with topics for further research in Section 6.

2. Review of Literature

2.1 Model Formulation

A. Nomenclature

To formulate the model we use symbols and notations which their definitions are given in the following list.

$\Omega_T = [0, \Gamma_0) \times [0, U_0)$	: Lease contract coverage
Δ_y	: Preventive maintenance level
X_i	: Downtime caused by the *i*-th failure and waiting time
$Y(t_{j-1}, t_j)$	:Total downtime in $(0,t]$
F(t)	: Distribution function of downtime
$F^{[k]}(x)$	: The k-fold Stieltjes convolution of $F(x)$.
$\tilde{A}$	: Availability target
Y	: Usage rate
C_r	: Repair cost
C_0	: Preventive maintenance cost
C_v	: Degree of preventive maintenance cost
C_p	: Penalty cost per unit of time
J	: Expected lease contract cost
$r_y(t)$, $R_y(t)$	:Hazard, and Cumulative hazard functions associated with $F(t, \alpha_y)$
$\mathscr{N}$	: Number of fleet
δ	: Preventive maintenance level
$\wp(m), P(m)$	: Profit and Revenue

B. The Lease-contract Model

A mining company that operates a number of lease dump trucks is considered here. In this setting, the lessee uses the equipment with a constant usage rate over the lease contract periods. Furthermore, it is assumed that a different lessee has a different usage rate. In what follows, an LC is studied under the assumption that the equipment is leased for period of $m\Gamma$ ($\Gamma > 0$ and $m = 1, 2, ...$) with a maximum usage (U_{max}) (e.g. km traveled/ time period). It is also assumed that for a given lessee usage rate y, if the total usage at the end of a lease period, U_y exceeds U_{max} (See Fig.1), then the lessee will pay an extra cost assumed to be proportional to $\varphi = Max\{0, U_y - U_{max}\} = Max\{0, y\Gamma - U_{max}\}$ for one period of a lease contract.

Two kind of maintenance actions, CM and PM, are assumed to be done by the lessor without any payment to the lessee. We have made an assumption throughout the paper that if the availability of the equipment for period j, A_j is below the the target $\tilde{A}_j$ then the lessor pay a penalty cost. Moreover, the amount of the penalty cost is assumed to be proportional to $\delta_j = \tilde{A}_j - A_j$. In this case, the penalty cost (C_p) is seen as a penalty given by the lessor. As a logical consequence, the decision issue for the lessor is to decide the ideal number of PM and the level of maintenance such that minimizing the expected cost.

Since under the lease contract all failures are minimally repaired and they also are assumed to follow a non-homogeneous Poisson process (NHPP) with intensity function $r_y(t)$ then the cumulative intensity functions is given by $\int_0^t r_y(x)dx$. Conditional on $Y = y$, PM is done periodically at $k\tau_y$, $k = 1, 2, ...$ where k is an integer value. If Γ is an LC period, then we have k disjoint intervals - $[0, \tau_y)$, …, $[k\tau_y, (k+1)\tau_y = \Gamma)$. After a PM is carried out at $t_j, j \geq 1$ the reduction of the intensity function will be δ_{yj}, $\delta_{yj} = \delta_y$. Again, any failure occurring between PM is assumed minimally repaired, then the expected total number of minimal repairs over the period of $[0, \Gamma_0)$ or $([t_{j-1}, t_j), 1 \leq j \leq k)$ is given by $N = \sum_{j=1}^{k} \int_{t_{j-1}}^{t_j} r_{j-1}(t')dt'$.

2.2 Model Analysis

Let us assume that k units of failed dump-truck exist. They will be served with single service channel by the lessor. We also assume that the service follows "the first come, first served" basis. Furthermore, we consider that the queue of failed trucks follows a Markovian properties with the population ($\mathscr{N}$) is finite.

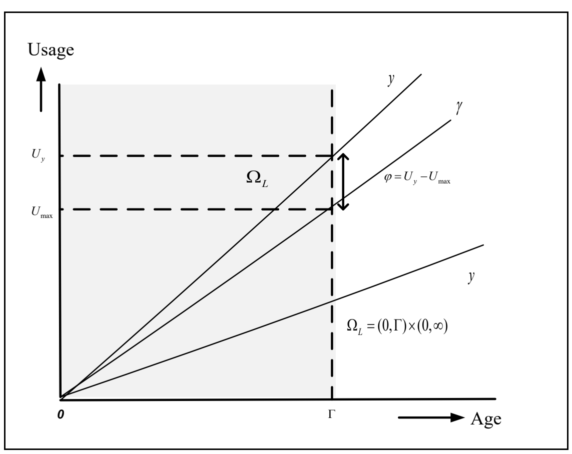

Figure 1. A Usage Lease Contract Region

The sojourn time of a failed truck within the un-operational state can be descibed by the following explanation. For truck $j(1 \leq j \leq N)$, if Z_j is the number of failures in $[0, \tau)$, then T_{ji} is the time to failure after $(i-1)th$ repair

$(2 \le i \le Z_j)$, $\tilde{T}_j$ is the time from the last repair to the end of the contract period, and X_{ji} $(1 \le i \le Z_j)$ is time needed to make the truck back to the operational state after the i-th failure (including waiting time and repair time). Moreover, if $\mathscr{N}$ is number of truck population and λ is the failure rate, then the arrival rate of failed truck is given by $\lambda_k = (\mathscr{N} - k)\lambda$ for $0 \le k \le \mathscr{N}$, and $\lambda_k = 0$ for $k > \mathscr{N}$. Meanwhile, the service rate is $\mu_k = k\mu$ for $0 \le k \le S$, and $\mu_k = S\mu$ for $k > S$. According to [6] and [7], the expected value of X_{ji} for a given y is

$$E\left[X_{ji}\right] = 1/\mu + \sum_{k=0}^{\mathscr{N}-1} \frac{(k-S+1)}{S\mu} \frac{(\mathscr{N}-k)(\lambda/\mu)^k \left(\mathscr{N}!/(\mathscr{N}-k)!\right)}{\sum_{k=0}^{\mathscr{N}-1}(\mathscr{N}-k)(\lambda/\mu)^k \left(\mathscr{N}!/(\mathscr{N}-k)!\right)} \tag{1}$$

where λ is estimated by the mean value of failure intensity, $\overline{\lambda}$.

Further calculation can be carried out to find the the expected total cost of the lessor during LC period as the following. If $J^1(k_y, \delta_y)$, $J^2(k_y, \delta_y)$ and $J^3(k_y, \delta_y)$ are the expected total PM cost, the expected total repair costs and the expected penalty cost over the LC period $(0, \Gamma]$, respectively, then the expected total cost of the lessor during LC period is the sum of these costs times the numbers of the truck population, given by

$$\Pi\left[k, \delta_y\right] = \mathscr{N}\left[J^1(k, \delta_y) + J^2(k, \delta_y) + J^3(\tau)\right]. \tag{2}$$

Other associated costs are the expected cost of minimal repair and PM and the expected cost of penalty within the period (0, Γ_0] which are derived as follows.

Expected cost with PM and minimal repair:

It is obvious and easy to understand that the expected cost of PM and minimal repair, conditional on Y=y, is given by

$$J\left(k_y, \tau_y\right) = J^1(k, \delta_y) + J^2(k, \delta_y) = \mathscr{N}\begin{bmatrix} C_r R\left(\Gamma_0\right) + k_y C_0 \\ -\sum_{j=1}^{k_y+1}\left[C_r\left(L - j\tau_y\right) - C_v\right]\left[r\left(j\tau_y\right) - r\left((j-1)\tau_y\right)\right] \end{bmatrix} \tag{3}$$

Expected penalty cost:

Recall that $\tilde{A}_j, j = 1, ..., \Gamma_0$ denote the availability target at t_j. Let $Y(t_{j-1}, t_j)$ and $A(t_{j-1}, t_j)$ denote the total of down time and availability of the equipment in (t_{j-1}, t_j), then $A\left(t_{j-1}, t_j\right) = 1 - Y\left(t_{j-1}, t_j\right) / \left(t_j - t_{j-1}\right)$. Again, that as assumed earlier, the penalty incurs the OEM at time t_j if $A\left(t_{j-1}, t_j\right) < \tilde{A}_j$ or $Y\left(t_{j-1}, t_j\right) > \zeta_j$ (the total down time in $\left(t_{j-1}, t_j\right)$ is greater than ζ_j), where $\zeta_j = \left(1 - \tilde{A}_j\right)\left(t_j - t_{j-1}\right)$. Hence, the probability that the penalty incurs at t_j is given by

$$\mathrm{P}\left\{Y(t_{j-1},t_j) > \zeta_j\right\} = \sum_{k=1}^{\infty}\left\{1 - F^{[k]}\left(\zeta_j\right)\right\}\frac{R(t_{j-1},t_j)^k\, e^{-R(t_{j-1},t_j)}}{k!}. \qquad (4)$$

Next, we define $G_j(\zeta_j) = \mathrm{P}\left\{Y\left(t_{j-1},t_j\right) \le \zeta_j\right\}$ which from (4) is equivalent to

$$G_j(\zeta_j) = \sum_{k=1}^{\infty} F^{[k]}\left(\zeta_j\right)\frac{R(t_{j-1},t_j)^k\, e^{-R(t_{j-1},t_j)}}{k!}. \qquad (5)$$

If further we assume that $F\left(x\right)$ has an exponential distribution form with parameter λ, then after some algebraic calculation, we end up to

$$G_j(\zeta_j) = \mathrm{P}\left\{Y(t_{j-1},t_j) \le \zeta_j\right\} = \sum_{k=1}^{\infty}\frac{R(t_{j-1},t_j)^k\, e^{-R(t_{j-1},t_j)}}{k!}\frac{(\lambda\zeta_j)^k}{k!}e^{-\lambda\zeta_j}.$$

It is obvious that the density function of $g_j\left(\zeta_j\right)$ is given by $g_j(\zeta_j) = \dfrac{dG_j(\zeta_j)}{d\zeta_j}$.

Consequently, the expected penalty cost in $\left(t_{j-1},t_j\right)$ is given by

$$EP_j\left(t_{j-1},t_j\right) = \frac{c_p E\left[Max\left\{0, Y\left(t_{j-1},t_j\right) - \zeta_j\right)\right\}\right]}{\left(t_j - t_{j-1}\right)},$$

where C_P is the penalty cost per unit time. The last expression is equivalent to

$$EP_j\left(t_{j-1},t_j\right) = \frac{c_p \int_{\zeta_j}^{\infty}\left[1 - G_j\left(y\right)\right]dy}{\left(t_j - t_{j-1}\right)}. \qquad (6)$$

This yield in the expected penalty cost within $(0,\Gamma_0)$ which is given by

$$J^3(0,\Gamma_o) = \begin{cases} \dfrac{c_p \int_{\zeta_j}^{\infty}\left[1 - G_j\left(y\right)\right]dy}{\Gamma_o}. & \text{for } \quad A_j < \tilde{A}_{\mathrm{j}} \\ 0 & \text{otherwise} \end{cases}$$

To complete the economic aspect of the LC, we should calculate the price of the LC given in the subsequent derivation.

<u>The price of lease contract:</u>
It is conceivable and logical that the unit price of the lease contract $P(m)$ is a function of the number of truck (m). We may define a unit price that decreasing with the respect to the number of the truck leased. An example of the function may look like

$$P(m) = P\left\{e^{-\varphi m}/e^{-\varphi}\right\} = P\left\{e^{-\varphi(m-1)}\right\} \qquad (7)$$

where $0 \le \varphi < 1$ denotes a discount parameter. This scheme of pricing facilitates some incentive for the lessee when more than one truck is leased (or $m \ge 2$). In other words the lessee get a discounted price whenever $m \ge 2$. Other form of decreasing function of m can also be used. Note: We use a different form of $P(m)$ compared to others, e.g. those in [8].

2.3 Optimization

Understanding a system, such as the description on how it works, the relation between technical and economical aspects of the system etc, is usually done as the first attempt in studying the system. Moreover, the result of this kind of study is usually fruitful in understanding the effect of certain intervention to the system, but often does not give explicit recommendation that yields in the best performance of the system. Accordingly, in the case of LC system, further attempt that relevant to do is to choose the best decision that give a maximum profit for both the lessor and the lesee. Many researchers have devoted their works to attack this problem form various perspective using different tools and different models. For the case of our LC model in the paper, we will look for the best values of the number of preventive maintenance, the lenght of maintennace period, and the level of the preventive maintenance that would give minimum total cost of the maintenance. Mathematically, we seek the optimal values of $k_y^*, \tau_y^*,$ and Δ_y^* minimizing the total cost function $\Pi\left[k_y, \delta_y\right]$. This problem can be written as,

$$\begin{array}{ll} \min\limits_{\delta_y, \tau_y, k_y} & \Pi\left[k_y, \delta_y\right] \\ s.t. & \delta_y, \tau_y, and\ k_y \end{array} \qquad (8)$$

subject to the constraint $0 \leq \Delta_j \leq r(t_{j-1}) - \sum_{i=0}^{j} \Delta_i$.

The optimal values are obtained involving two stages. In the first stage, for a fixed *k*, minimize $\Pi\left[k_y, \delta_y\right]$ to obtain the optimal values of $\left\{ j\tau_y^*, 1 \leq j \leq k_y + 1 \right\}$. Later on, in the second stage, the optimal *k* is obtained using the results of the first stage. Details steps of the calculation can be refered to [X]. The following section gives a numerical example for this optimization problem.

2.4 Numerical example

For a given usage rate y the failure distribution is given by the Weibull distribution. Let the parameter values be as follows. β=2.25, Γ_0=2 years, U=2(1x10^{4}Km), $C_r = 100$, $C_0 = C_v = 0.5C_r$, $\mathfrak{I}$ = 80 hours or 4 days or $C_P = 3P$ and $P = 5 \cdot 10^2 \cdot 2025$ \$. Further, we choose the down time is given by the exponential distribution with $1/\lambda = 1/300$ (years). The resulting optimal number of PM and improvement level of lease contract with two usage types, namely, medium $(1.0 \leq y < 1.4)$ and heavy $(1.4 \leq y < 1.8)$ are shown in Table 1.

The table shows three different values of ρ (1.2, 2.0, and 2.2) corresponding to light incline, high incline and very hilly land contours, respectively. It is observed that for a given *y* and α_0 (reliability level), the optimal expected lease cost increases as the usage rate *y* increases. It means that under lease contract coverage, larger values of the usage rate result in shorter periods of time between PM actions τ_y^*, which is plausible. It indicates that the reliability of the equipment has been decreased. As a result the average cost of lease contract, $\Pi\left[k_y, \delta_y\right]$, increases with the increasing in *y*, since the penalty cost increases when the number of failures increases. We also observe the same behavior as the operating condition is more severe (ρ is bigger), since the reliability of equipment decreases as the unit deteriorates rapidly with time. And it influenced into the profit as the expected maintenance cost increased then the profit is also decreased.

Table 1. Expected cost and expected profit for $\mathscr{K} = 25$ with $P(25) = 1.647892263 \times 10^8$ and $\alpha_0 = 4$ months

ρ = 1.2					ρ = 2.0				ρ = 2.2			
$\overline{y}$	k_y^*	τ_y^*	Δ_y^*	$\Pi\left[k_y, \delta_y\right]; \wp(m)$	k_y^*	τ_y^*	Δ_y^*	$\Pi\left[k_y, \delta_y\right]; \wp(m)$	k_y^*	τ_y^*	Δ_y^*	$\Pi\left[k_y, \delta_y\right]; \wp(m)$
Medium												
1.00	13	1.70	0.32	26401.71; 1.65x10^8	13	1.70	0.32	26401.71; 1.65x10^8	13	1.70	0.32	26401.71; 1.65x10^8
1.20	18	1.04	0.41	43936.21; 1.65x10^{11}	21	0.89	0.47	57406.09; 1.65x10^{11}	22	0.85	0.49	61716.88; 1.65x10^{11}
Heavy												

1.40	22	0.7 3	0.5 7	67428.65; $1.65x10^{11}$	32	0.5 1	0.6 4	$1.19x10^5$; $1.65x10^{11}$	35	0.4 6	0.6 7	$1.38.10^5$; $1.65x10^{11}$
1.60	28	0.5 0	0.6 8	$1.03x10^5$; $1.65x10^{11}$	45	0.3 2	0.8 6	$2.34x10^5$; $1.65x10^{11}$	51	0.2 8	0.9 1	$2.89.10^5$; $1.65x10^{11}$
1.80	38	0.3 3	0.6 9	$1,58x10^5$; $1.65x10^{11}$	60	0.2 1	1.1 4	$4.36x10^5$; $1.65x10^{11}$	70	0.1 8	1.2 1	$5.74.10^5$; $1.65x10^{11}$

3. Conclusions

In this paper, we have devised a mathematical model for a lease equipment, in which the lease is sold for a fleet of the equipment. The fleet is leased for a period of time and during the lease contract, imperfect PM is performed. We find the optimal value of maintenance parameters such that minimizing expected cost of the lessor. Since nowadays, two-dimensional lease are abound in the market, it is important to consider these two-dimensional approach in dealing with failures. A bivariate distribution of those failures can be estimated using real data from related industries, such as mining industry. A game theory approach can be also considered as one alternative to address optimization in that two-dimensional problem.

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