Economic Statistical Design of np Control Chart for Monitoring Attribute

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Abstract

This research proposes an optimization algorithm for the economic statistical design of np chart for shift detection in fraction nonconforming. In the literature, less attention has been paid to the economic statistical design of attribute control charts compared with variable control charts although it's usually easier and faster to handle an attribute quality characteristic rather than a variable quality characteristic. In this model, the charting parameters of the np chart are optimized to satisfy the objective function which aims to reduce the Expected Total Cost (ETC) and in the meantime, ensure that the false alarm rate and the inspection rate do not exceed the allowable levels. The performance of the proposed optimal np chart will be compared with that of the traditional np chart under different circumstances. A sensitivity analysis will be conducted to study the effectiveness of the optimal np chart under different cost parameters and distributions of the p shift. It is found that the proposed optimal np chart outperforms the traditional np chart.

Keywords
Quality Control, Control Chart, np Chart, Expected Total Cost (ETC) and Economic Statistical Design.

1. Introduction
Control charts are extensively used to control a process statistically and maintain it. Control charts can be used to estimate process parameters. Many design approaches have been proposed and discussed in the literature, but generally, they can be categorized into groups: heuristic; statistical; economic; and economic statistical designs of control charts Zhang & Beradi, 1997). As a result of the economic statistical design, the disadvantages of both economic and statistical designs were controlled, the detection effectiveness of control charts was improved, and the cost of being in out-of-control status was reduced. The economic statistical design aims to minimize ETC, subject to statistical constraints, which will in turn reduce the variability in the process and increase the quality of the product. Economic statistical designs are generally more costly than economic designs due to the additional added statistical constraints, but the tight limits applied on the control chart from the statistical properties will lead to a reduction in the process variability and an enhancement in the product quality. Economic statistical designs require that cost parameters that are related to the economic design and the system parameters that are related to statistical design to be estimated. It yields to designs that are at least as good as statistical design in terms of statistical properties and can give protection over a wider range of shifts.

2. Literature Review
The initiation of a designing criterion the combines both economic and statistical models was needed to overcome the disadvantages when using either of them individually. Saniga (1989) developed a design by placing statistical constraints on economic models in order to provide designs that meet industry's demand for low process variability and long-term product quality using $\overline{X}$ and R chart. The resultant design was statistically as good as statistical designs and more costly than economic design, but it gave more protection over wider range of shifts.

Montgomery et al. (1995) proposed economic statistical design approach of EWMA control chart to monitor process mean. Optimum design parameters include n, h, control limit width coefficient and EWMA weight. The optimum design parameters were obtained by minimizing the total cost functions, subject to additional statistical constraints on ATS. Sensitivity analysis was conducted to study how the statistical measure of performance (ATS) affects the

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minimum cost function. It was found that the cost is more sensitive to smaller bound on ATS, but relativity insensitive to large bounds. They also investigated the advantages of using economic statistical design of control chart over economic design. They found that adding the additional statistical constraints on the economic model does not increase the cot significantly and does provide better protection against shift range. Molnau, Montgomery & Runger (2001) designed an economic design constrained by a statistical constraint on multivariate exponentially weighted moving average (MEWMA) control chart. Cost comparison was conducted between optimal economic statistical design and optimal economic design. The results indicated that there is no significant increase in cost. The ARL added constraint improved the design performance significantly. Moreover, the false alarm rate is limited while keeping good shift detection characteristics. Al-Oraini & Rahim (2002) investigated economic statistical design of $\overline{\mathrm{X}}$ control charts by considering Gamma distribution as failure model instead of the widely used exponential model. The study shows that the statistical performance is improved significantly with a slight increase in the cost compared with economic model by adding statistical constraints on the optimization model. The optimized economic statistical model will lower the false alarm rate and in the meantime, give a high probability of detecting process shift. Throughout the literature review, it was observed that the optimization of the charting parameters of any control chart will lead to the best performance for detecting out-of-control scenarios and quality problems. It was also clear that there is the little attention was paid to the economic statistical design of the attribute control charts compared with variable control charts.

3. Optimization Design of np Control Chart

In this research, an optimization model for the economic statistical design of the np chart is proposed for monitoring a wide range of shifts in fraction nonconforming p. The model is mainly derived from Duncan's model and has the strength of the economic design to consider the cost of the assignable causes and the power of the statistical model to improve the speed of detecting an out-of-control signal of the np control chart. The np control chart is mostly used to investigate the number of nonconforming units (d) in a sample. While monitoring a process using an np chart, the process is in control if d satisfies LCL ≤ d ≤ UCL, where LCL and UCL are the lower control limit and upper control limit of the np chart, respectively. However, the goal of this study is creating a one-sided upper np chart that only detect the increasing shift in the number of nonconforming units since decreasing number of nonconforming units where d $<LCL$ is a desirable target. The upper control limit of the 3σ np chart is calculated using the following formula:

$$UCL = np_0 + 3\sqrt{np_0(1-p_0)} \qquad (1)$$

where p_0 is the in-control fraction nonconforming. When a shift occurs in the fraction nonconforming, the fraction nonconforming p will change to:

$$p = \delta \times p_0 \qquad (2)$$

where (δ > 1) means that a shift occurred, and the process is out of control.

Assumptions

The following principles were taken into consideration in developing the optimal np chart:

1- The process starts from an in-control state. The number of nonconforming units is assumed to follow a binomial distribution. If an assignable cause occurs, the in-control mean p_0 will change to out-of-control mean p where $p = \delta \times p_0$ and δ is the p shift resulted by an assignable cause.
2- The random shift δ follows a uniform distribution.
3- In-control fraction nonconforming p_0 is assumed to be known.
4- Increasing shifts in p will only be considered, as decreasing shifts in p means improvement.
5- Whenever an out-of-control status occurs, only a single assignable cause is considered. An assignable cause occurs follows a homogenous Poisson distribution with mean λ.
6- During the search for the assignable cause, the process continues to run.

The design algorithm optimizes the decision variables of the np chart to minimize ETC which is used as an objective function subject to constraints. The proposed economic statistical model of np control chart can be described as follows:

Objective Function: Minimize ETC (3)

Constraints: $ATS_0 \geq \tau$ (4)

$r = \frac{n}{h}$ (5)

Design variables: n, h and UCL

The objective function is to minimize ETC with two constraints on the in-control average time to signal (ATS_0) and inspection rate (r). The sample size (n) is an independent variable, while the sampling interval (h) and control limit (UCL) are the dependent variables on n and the specified value of r and τ respectively. The above-mentioned model can find the optimal values of n, h and UCL that minimize ETC over a shift range of $(1 \leq \delta \leq \delta_{max})$ while ensuring that the in-control Average Time to Signal (ATS_0) is greater than or equal to a predefined value of τ. The optimization algorithm for np chart is explained as shown in Figure 1.

Abbreviations

p_0 In-control fraction nonconforming
r Inspection rate
τ Minimum allowable in-control ATS_0.
δ_{max} Maximum shift in fraction nonconforming
λ Rate of occurrence of the assignable cause
e Time to estimate an observed data of a sample
D Time period from the detection of the out-of-control state and removal of the assignable cause
B Fixed component of sampling cost
c Variable component of sampling cost
W Cost of finding and fixing an assignable cause
T Cost of examining a false alarm
M Cost per hour incurred due to the increasing p shift

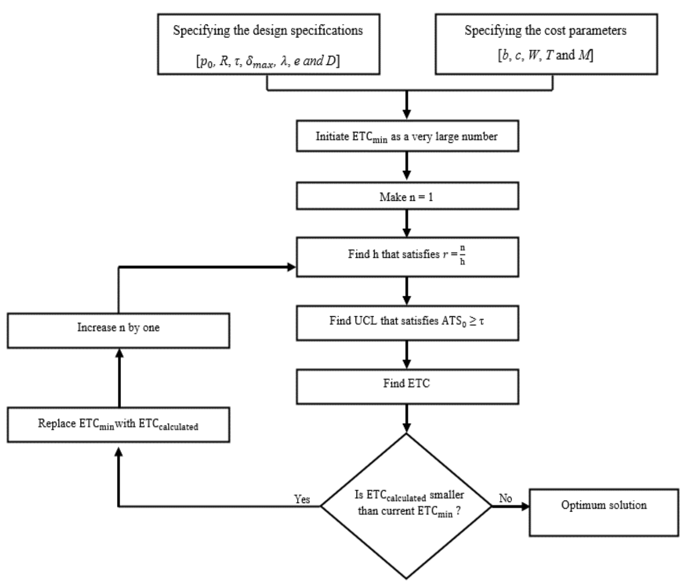

Figure 1- Optimization algorithm flowchart

The ETC can be calculated by:

$$ETC=\int_{1}^{\delta max} L(\delta).f_{\delta}(\delta)d\delta \quad (6)$$

where, $L(\delta)$ is the loss cost per unit time for a given p shift and $f_{\delta}(\delta)$ is the probability density function of the shift. A computer program coded in C language is used to implement the above-mentioned algorithm.

4. Numerical Studies

The performance of the traditional np chart and optimal np chart is compared using one general case with the following cost parameters and design specifications:

$$p_0=0.01,\ \delta_{max}=10,\ \tau=1000,\ R=16,\ B=1,\ c=0.01$$

$$M=10000,\ W=300,\ T=600, \lambda=0.05,\ e=0.125,\ D=3$$

The values of *ATS* for the traditional np and optimal np chart are presented in Table 1.

Table 1. Comparison of both charts under a general case

	ATS	
δ	Traditional np chart	Optimal np chart
1	1968.727	1029.477
2	270.8444	147.6441
3	88.15674	49.95349
4	40.74644	23.91757
5	22.78947	13.81319
6	14.36335	8.96528
7	9.8219	6.29947
8	7.12488	4.68793
9	5.40437	3.64394
10	4.24494	2.93131

To compare between the *ATS* of both charts, a normalized *ATS* is calculated for each shift by dividing the *ATS* of the traditional np chart by the optimal np chart. The normalized *ATS* curves are presented in Figure 2.

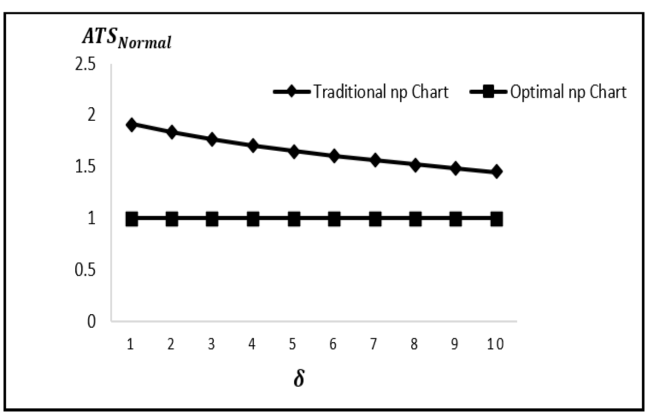

Figure 2. Normalized *ATS* for both charts

From Figure 2 and Table 1 the following results were obtained:

1- Both charts result in an ATS_0 magnitude that is larger than τ when the process is in control, which satisfies constraint (4) and indicates meeting the false alarm requirement.
2- The values of ATS for both charts indicates that optimal np chart excels the traditional np chart over the range shift (1 ≤ δ ≤10). Through the 10 observations, the ATS of the optimal np chart is always smaller than ATS of the traditional np chart. This confirms that the optimal np chart is more sensitive to small and large shifts than traditional np chart.
3- The optimal np chart effectiveness decreases when the shift increases. For example, when the *p*=2 the optimal np chart detects the shift faster than the traditional by 83%. But when *p*=10, the optimal np chart is only faster by 45%.

The results of implementing the optimization algorithm for optimal np and traditional np charts for the studied case are shown below:

Traditional np chart:
UCL=2, n=16, h=1 and $ETC_{Traditional}$=276.3111
Optimal np chart:
UCL=2, n=22, h=1.375 and $ETC_{Traditional}$=204.91307

The ratio $ETC_{Traditional}/ETC_{Optimal}$= 276.3111/204.91307= 1.35 indicates that the performance of the optimal np chart is better than that of the traditional np chart by 35%.

5. Conclusion

Through the investigation of the economic statistical design of attribute control charts and how it can exceed the performance of other control chart designs, it was found that the economic statistical design of optimal np chart outperforms the traditional design of np chart significantly. The same model might be used in future to propose economic statistical design of other control charts such as CUSUM and EWMA charts.

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Biography

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