Supply chain inventory optimization: An alternative approach

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Abstract

In this paper, an alternative approach is proposed to deal with the inventory positioning problem in a supply chain network. In particular, the problem is modelled as a mixed integer programming model. In this model, the locations and amount of stock at each location are decision variables, while the total holding inventory cost is the objective function. The optimal solution for such a model can be obtained by using a commercial. The applicability of the proposed method, especially the mathematical model, is validated by a few numerical experiments. The results indicates that our approach is suitable for various types of supply chain networks and the optimal solutions for these networks are easily obtained via a commercial solver.

Keywords
Inventory placement; base stock policy; stochastic demand; mixed-integer programming.

1. Introduction
In a highly competitive market of today, matching supply with demand is of great concern for supply chain managers/practioners. Since uncertainty always exists in both demand and supply, mismatches between them are unavoidable. Therefore, the management often relies on inventory, especially safety stock, as a buffer for these mismatches. With the supply chain network growing more complex, optimizing safety inventory at a location in the network become an ineffective practice. As a result, modern supply chain management is interested in effectively managing the safety inventory at not one location but across multiple locations in a supply chain network. This management issue is known as the inventory positioning (IP) problem and is a main research topic for many researchers.

According to Simchi-Levi et al. (2008), a typical IP problem is concerned with the stocking locations and the amount of safety inventory at each location within a supply chain network. These concerns are initially addressed in the study of Simpson (1958). The author develops a model, from which the level of safety stocks in a serial supply chain are analytically determined. As a result, Simpson's (1958) model becomes the foundation, based on which several models are developed to deal with IP problem under various supply chain settings. For instance, Graves (1988) incorporates multiple products and production flexibility in an analytical model for a single-stage supply chain. Following Simpson (1958) and Graves (1988), Graves and Willems (2000) construct a safety stock placement model for a spanning tree supply chain network. Since the model is a non-linear optimization, the authors provide a solution algorithm based on dynamic programing. The model of Graves and Willems (2000) is extended to accommodate different aspect in supply chain network, e.g., new products (Graves & Willems, 2005), clusters of commonalities (Humair & Willems, 2006), adaptive inventory policy (Bossert & Willems, 2007), nonstationary demand (Graves & Willems, 2008; Neale, & Willems, 2009), dual sourcing (Klosterhalfen et al., 2014), and capacity constraints (Graves & Schoenmeyr, 2016).

From the literature, it is observed that IP problems are extensively explored in many research articles. However, most of these studies often employs non-linear modelling approach. This results in a non-linear optimization model. To solve such a model, a dynamic programming algorithm, developed by Graves and Willems (2000), are adopted. However, this algorithm is quite computational extensive. In addition, the solution obtained from the algorithm is not guaranteed to be optimal as elaborated in the study of Magnanti et al. (2006). In fact, the authors demonstrates that by including a set of redundant constraints and repetitively refining the approximation of the total cost function, a medium-size IP problems can be solved to optimality. Inspired by the study of Magnanti et al. (2006), this research proposes an alternative approach to model an IP problem as a mixed-integer linear programming (MIP) model. The solution for this MIP model can be easily obtained by using a commercial solver. The applicability of our approach, particularly the MIP model, is demonstrated via a numerical experiment.

The remainder of this paper is organized as follows. The problem statement is provided in Section 2. The problem modeling is presented in Section 3. Numerical experiments for validation are illustrated and discussed in Section 4. Conclusions are made in Section 5.

2. Problem Statement

In this reaserch, a supply chain network is modeled as a graph, including vertices and arcs. Each vertex (or node) represents a stage (or facility) while each arc represents the flow of materials (raw material, component, work-in-process or subassembly, and finished goods) from one stage to another. An example graph for a supply chain network is illustrated in the following figure.

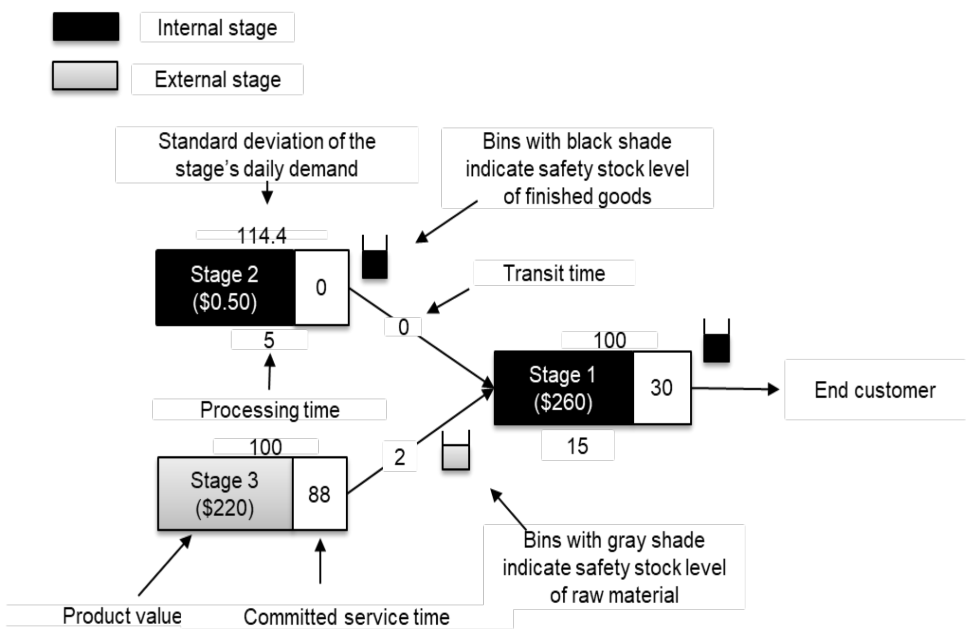

Figure 1. Illustration of a supply chain

In Figure 1, the supply chain contains three stages, in which there are two internal stages (dark nodes) and one external stage (grey node). Internal stages are eligible for holding inventory, while that is not allowed for the external stage. There are two types of inventories, i.e., finished goods and raw materials. They are illustrated by dark and gray buckets, which are placed before and after a stage, respectively. In addition, it is assumed that each internal manages its inventory by using a base stock policy (Simpson, 1958; Grave & Williems, 2000).

In such a supply chain, customer orders are received at Stage 1 and immediately sent to other stages, i.e., 2 and 3, respectively. Therefore, each stage can determine the demand of its downstream customers. The customer demand (or order quantity) experienced by Stage 1 is assumed to be bounded and normally distributed with a mean μ and standard deviation σ. The bounded part of the demand is determined by $\mu + k\sigma$, where k is the safety factor for a given cycle service level (Simpson, 1958). For the unbounded part of the customer demand, it is presumed to be satisfied by extraordinary measures, such expediting, subcontracting, or over-time. It is worth to note that $k\sigma$ is the primary concern of an inventory positioning problem because the average demand μ is covered by cycle inventory. Therefore, the "bounded demand" is simply referred to $k\sigma$ in the rest of this paper.

For every order received, each stage quotes a service time to its immediate downstream stages/customers. The service time from an internal stage can be adjusted while that from an external stage are fixed and given. In addition to service time, each stage has a constant processing time, during which an order is prepared. This processing time is independent of the order quantity. Moreover, there occurs a transportation time between two stages if they are in different geographical locations. In case that a stage is supplied by multiple upstream stages, it must wait for all items arrive before it can start preparing its orders.

3. Mathematical Model

With a network structure and operational characteristics as described in previous section, the inventory positioning for such a supply chain is modelled as a mixed-integer programming model. The following notations are used in the development of the model.

Sets:

I: set of internal stages in the supply chain network;

E: set of external stages in the supply chain network;

N: set of nodes that represent supply chain stages, and node $i, j \in N$;

M: set of options for length of net replenishment time, $M = \{1, 2, 3, \dots\}$;

Parameters:

φ_i: Product value at stage i;

A_{ij}: Binary parameter, which takes a value of 1 if there is an arc that goes from stage i to stage j, 0 otherwise;

T_{ij}: Transit time from stage i to stage j, which can take any values if there is an arc that goes from stage i to stage j;

B_i: Binary parameter, which takes a value of 1 if stage i is an internal stage, 0 if stage j is an external stage;

V_i: Quoted service time of external stage i, which takes a value of 0 if stage i is an internal stage;

P_i: Processing time of stage i, which can take any values if stage i is an external stage;

$H_i = h \times \varphi_i$: Inventory holding cost per unit per year of the finished part coming out of stage i;

R: Service time committed to the end customers; and

Q_m: Bounded demand during the replenishment time of m periods;

Decision variables:

S_i^{in}: Incoming service time of stage i ;

S_i^{out}: Outgoing service time of stage i;

x_i^+: Net replenishment time of finished goods at stage i;

x_i^-: Time allowance for processing finished goods at stage i;

x_{im}: Binary variable, which takes a value of 1 if net replenishment time of finished part at stage i is m periods, or 0 otherwise. When $x_{im} = 1$, it means that the finished goods inventory is kept at stage i.

y_{ij}^{+}: Net replenishment time of finished goods coming out of stage i. The finished goods is held as raw material inventory for stage j;

y_{ij}^{-}: Time allowance for replenishing finished goods coming from stage i. The finished goods is held as raw material inventory for stage j;

y_{ijm}: Binary variable, which takes a value of 1 if net replenishment time of finished goods, coming from stage i and held as raw material inventory at stage j, is m periods, or 0 otherwise.

Objective Function of the Model:
In this model, the total inventory cost, including holding cost of raw material and finished goods, are considered as the objective function. It is formulated as follows.

$$Minimize\ Z = \sum_{i \in N: B_i = 1} H_i \sum_{m \in M} x_{im} Q_m + \sum_{i,j \in N: A_{ij} = 1} H_i \sum_{m \in M} y_{ijm} Q_m$$

Constraints:
Sets of constraints are presented as follows:

- Incoming service time: All internal stages without predecessor must have a zero incoming service time.
$$S_i^{in} = 0 \quad \text{for } i \in N,\ B_i = 1, \text{and } \sum_{j \in N} A_{ij} = 0$$
- Outgoing service time: Each stage must have an outgoing service time that is above its corresponding quoted service time.
$$S_i^{out} \geq V_i \quad for\ i \in N$$
- Net replenishment time and time allowance for having raw material kept as inventory.
$$S_i^{out} + T_{ij} - S_i^{in} = y_{ij}^{+} - y_{ij}^{-} \quad \text{for } i, j \in N, \text{and } A_{ij} = 1$$
- Net replenishment time and time allowance for having finished goods kept as inventory.
$$S_i^{in} + P_i - S_i^{out} = x_i^{+} - x_i^{-} \quad \text{for } i \in N, \text{and } B_i = 1$$
- Outgoing service time for customer: service time chosen by the stage receiving customer orders must not exceed the service time committed to them.
$$S_i^{out} \leq R \quad \text{for } i \in N, \text{and } \sum_{j \in N} A_{ij} = 0$$
- Correspondence between net replenishment time and the option of net replenishment time that is selected.
$$x_{ij}^{+} = \sum_{m \in M} m x_{im} \quad \text{for } i \in N, \text{and } B_i = 1$$
$$y_{ij}^{+} = \sum_{m \in M} m y_{ijm} \quad \text{for } i, j \in N, \text{and } A_{ij} = 1$$
- One option of net replenishment time is selected for each inventory type at a stage.
$$\sum_{m \in M} x_{im} \leq 1 \quad \text{for } i \in N, \text{and } B_i = 1$$
$$\sum_{m \in M} y_{ijm} \leq 1 \quad \text{for } i, j \in N, \text{and } A_{ij} = 1$$
- Non-negativity
$$S_i^{in}, S_i^{out}, x_i^{+}, x_i^{-} \geq 0 \quad \text{for } i \in N$$
$$y_{ij}^{+}, y_{ij}^{-} \geq 0 \quad \text{for } i, j \in N, \text{and } A_{ij} = 1$$

4. Numerical experiments

In this section, the applicability of the model developed in the previous section is validated by using two supply chain networks. They are illustrated in Figures 2 and 3, respectively. Most of the important information of the supply chain networks, i.e., transportation time T_{ij}(the value on an arc), product value φ_i(the value in parenthesis under a stage's name), processing time P_i(the value under each retangular shape representing a stage), service time quoted by a stage V_i or R (the value in the box), is included in these illustrations.

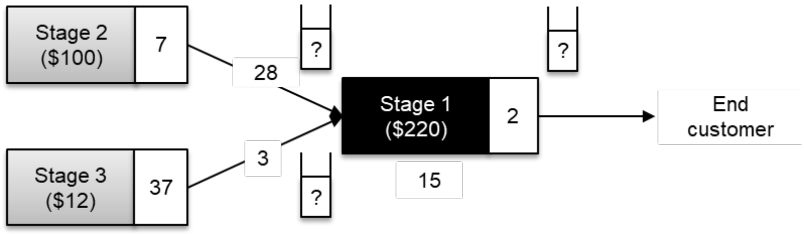

Figure 2. Illustration of an assembly supply chain

The supply chain in Figure 2 is known as an assembly network, where a downstream internal stage, i.e., Stage 1, is supplied by two upstream external ones, i.e., Stages 2 and 3. In this network, there are two possible locations for holding inventory, either before or after Stage 1. In addition, two types of inventories can be kept in storage. They include the finished goods from Stage 1(in the bucket on the arc linking Stage 1 to the End customer), raw material from either Stage 2 (in the bucket on the arc linking Stage 2 to Stage 1) or Stage 3 (in the bucket on the arc linking Stage 3 to Stage 1). The holding costs for these inventories are fractions of their corresponding values. For example, if the raw material from Stage 2 is held as stock before Stage 1, its holding cost is a fraction of $100 because it is considered as the product of Stage 2.

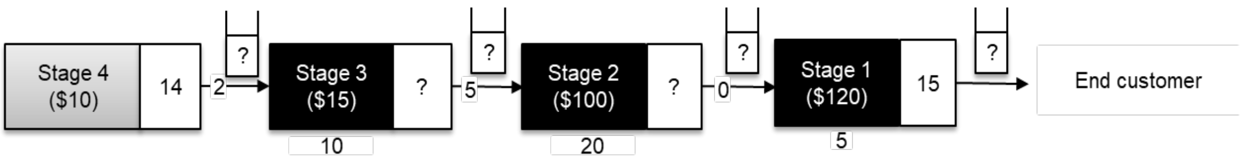

Figure 3. Illustration of a serial supply chain

Similar to the above figure, Figure 3 illustrates a serial supply chain, in which one stage is supplied by another. In such a network, the places for keeping inventory are located before and after each internal stage. In addition, two types of inventories, i.e., finished goods and material, are considered for placement. The holding costs for these inventories are interpreted the same as in the case of assembly network. For parameters other than those included in Figures 2 and 3, they are presented in the following table.

Table 1. Parameters for supply chain networks

Parameter	**Description**
Holding cost at each stage i (H_i)	$h \times \varphi_i$, where $h = 20\%$
Customer demand	Normally distributed with $\mu = 11.76$ and $\sigma = 11.91$
Bounded demand during the m periods (Q_m)	$Q_m = k\sigma\sqrt{m}$, where $k = 1.65$ (safety factor for a service level of 95%), $m \in M$

From Table 1, it should be noted that the upper bound for the set of replenishment options M depends on the structure of the supply chain. Specifically, the upper bound of M is 55 days for the assembly network in Figure 2 and 56 days for the serial network in Figure 3.

By using the above data with the model in Section 3, the inventory positing solution for each supply chain network is obtained through IBM Ilog Cplex optimization suite. The results are presented in Figures 3 and 4.

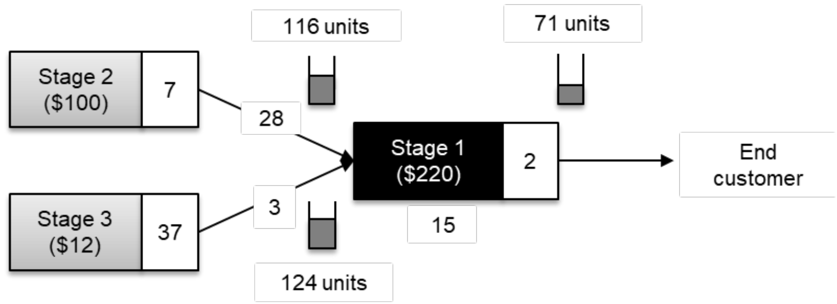

Figure 4. Inventory placement for assembly supply chain

In Figure 3, it is observed that to accommodate a short service time quoted to a customer, i.e., 2 periods, and a relatively long service time quoted by one of the suppliers, i.e., 37 periods, inventory should be held in both locations of Stage 1, i.e., before and after. Particularly, 71 units of finished goods (the bucket after Stage 1), 116 units of raw material from Stage 2 (the bucket on the arc between Stages 1 and 2), and 124 units of raw material from Stage 3 (the bucket on the arc between Stages 1 and 3) are kept as inventory. This placement of inventory results in a total inventory cost of $5721.35, including $3106.88 for finished goods and $2614.47 for raw material.

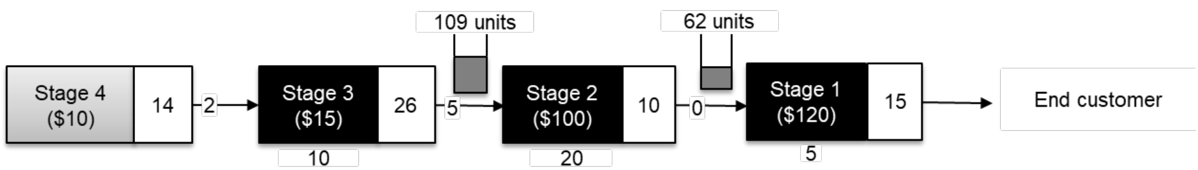

Figure 5. Inventory placement for serial supply chain

A similar interpretation is applied for the results in Figure 5. Among eligible stocking locations in the serial supply chain, two are selected for holding inventory, i.e., before and after Stage 2. Specifically, 62 units of finished goods and 109 units of raw material are held as inventory. This leads to a total inventory cost of $1565.72, of which, $1238.60 is for finished goods and $327.12 for raw material.

5. Conclusion

In this paper, an alternative approach for modelling a supply chain inventory positioning (IP) problem is introduced. Conventionally, the problem is formulated as non-linear mathematical model. Solving this type of model involves the development of complex algorithm. The implementation of such an algorithm is also a challenge. As an attempt to overcome this difficulty, the IP problem is constructed as mixed-integer linear programming model. The solution for this kind model is obtained by using a commercial solver. Indeed, this is demonstrated by a numerical study, in which the inventory position problem is solved for an assembly and a serial supply chain. In addition to illustrating the ease of having an optimal solution, the numerical study highlights the flexibility of our approach to model different structures of a supply chain network. As a result, the outcome of this research is served as the groundwork for future studies, including the consideration of uncertainty in processing and transportation, or an experiment with non-normal distributions for customer demand.

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Biographies

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