

# Efficient Cumulative Sum Chart for Monitoring Fraction Nonconforming

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## Abstract

Cumulative Sum (CUSUM) quality control chart is commonly used in many industrial applications. In this research, an improved scheme of the CUSUM chart is introduced to monitor attribute data. In this scheme, the difference between actual and in-control numerals of nonconforming items is raised to an exponent  $w$  to enhance the detection effectiveness. The exponent  $w$  is optimized, along with other charting parameters to minimize the Average Number of Defectives (AND) which is used as an objective function. The proposed scheme outperforms the conventional CUSUM chart for detecting a wide range of shifts in fraction non-conforming.

## Keywords

Control chart, Average Number of Defectives (AND), Quality control, CUSUM chart.

## 1. Introduction

The use of Cumulative Sum (CUSUM) control chart has been gradually increasing across multiple industries for detecting process shifts (Wu et al. 2008, Shu et al. 2008, Zhao et al. 2005). This is because on-line monitoring techniques have become a necessity in modern day Statistical Process Control (SPC) applications (Wu et al. 2008, Woodall and Montgomery 1999). CUSUM charts can be used to monitor attribute and variable quality characteristics. Nowadays, attribute quality control charts are widely utilized in manufacturing and service sectors (Ou et al. 2009). It is accredited to the fact that attribute quality characteristics are relatively easier and simpler to handle in comparison to variable characteristics (Wu et al. 2008). A popular chart to detect a shift in attribute quality characteristics is the binomial CUSUM chart. This chart is designed to detect the number of nonconforming items  $d$  in a sample of size  $n$ . The chart detects the shift by monitoring and plotting the statistic  $C_t$ . The statistic  $C_t$  depends on the difference between the actual number of nonconforming items and the in-control number of nonconforming items ( $d_t - d_0$ ).

$$\begin{aligned} C_0 &= 0 \\ C_t &= \max(0, C_{t-1} + (d_t - d_0) - k) \end{aligned} \quad (1)$$

The best use of this chart is when the actual shift size of the process can be predicted. However, in industry, it is difficult to predict the actual shift size for a lot of processes (Wu et al. 2008). Multiple researchers (Sparks 2000, Zhao et al. 2005) suggested combining two or three control charts to widen the range of mean shifts that can be detected. In this research, an improved scheme of CUSUM chart titled  $w$ CUSUM chart is introduced to monitor attribute data. In this scheme, the difference between actual and in-control numerals of nonconforming items is raised to an exponent  $w$ . The exponential  $w$  will be optimized along with charting parameters  $k$ , and  $H$  to increase the sensitivity of the

control chart to the possible range of shifts. The  $w$ CUSUM chart is expected to have a better overall performance in comparison with the conventional CUSUM chart over the whole shifts range. The process is thought to be in-control when monitoring starts. The number of non-conforming items  $d$  is assumed to follow binomial distribution with a specified in-control fraction non-conforming  $p_0$ . When a shift occurs, the fraction non-conforming will be:

$$p = \delta \times p_0 \quad (2)$$

where  $\delta$  ranges from 1 to  $\delta_{max}$ .  $\delta = 1$  represents an in-control status and ( $1 < \delta \leq \delta_{max}$ ) refers to an out-of-control status.

The Average Time to Signal ( $ATS$ ) is a common performance measure for quality control charts. Commonly, the in-control  $ATS_0$  is used for managing the false alarm rate, while the out-of-control  $ATS$  is a regular indicator for chart's effectiveness (Wu et al. 2008, Haridy et al. 2017). A large value of  $ATS_0$  is always required. On the other hand, a small value of the out-of-control  $ATS$  is necessary. Thus, there is always a trade-off between both of them. The Average Number of Defectives ( $AND$ ) is a popular measure of performance (Haridy et al. 2014). In this research, the  $AND$  is used to evaluate the detection speed of the proposed control chart. The  $AND$  can be determined as follows:

$$AND = \frac{1}{\delta_{max}} \sum_{i=2}^{\delta_{max}} \delta_i \times p_0 \times ATS(\delta_i) \quad (3)$$

where  $ATS(\delta)$  is the value of the Average Time to Signal at a particular shift  $\delta$ . The index  $i$  varies within ( $2 < i \leq \delta_{max}$ ) and it represents the shift size in terms of  $\delta$ . The case of  $i = 1$  (i.e.,  $\delta = \delta_0$ ) indicates an in-control process. The maximum possible shift is indicated by  $i = \delta_{max}$ .

## 2. Literature Review

Lucas (1989) developed two CUSUM schemes are used to spot fraction non-conforming  $p$  when the number of defects is relatively low. The first scheme suggests taking an action at every spotted defect, while the second scheme generates an out-of-control signal when two or more defects are found between a certain number of samples. The same matter has been further investigated by Bourke (1992) where a new Run-Length CUSUM scheme was introduced. The new scheme relies on the number of conforming samples between two back-to-back non-conforming samples. Gan (1993) proposed an optimal CUSUM chart design. The design can be implemented by a 4-step procedure. Gan (1993) relied on optimizing the allowable slack  $k$  under a known shift size. The Poisson approximation for negative binomial counts in the CUSUM charts was studied and investigated by Radaelli (1994). He found that Poisson approximation is not an efficient way to design CUSUM control charts. 100% inspection was conducted using attribute CUSUM chart by Bourke (2001) in which the significance of varying the sample size  $n$  was studied. He used the Average Run Length ( $ARL$ ) as an objective function and the reality that the shift may occur at any point in the sample. Two CUSUM charts were developed by Reynolds and Stoumbos (2000), the first chart depends on finding the number of non-conforming items in a sample of size  $n$ , while the second one inspects each individual item in a given sample of size  $n$ . Reynolds and Stoumbos (2000) found that when the shift in fraction non-conforming is anticipated to be small or moderate, it is advised to have a small sample size  $n$  in order to improve the detection effectiveness of their charts. Sparks (2000) developed an adaptive CUSUM chart that is able to generate an out-of-control signal over a wide range of process shifts. It utilizes multiple CUSUM charts at once or predicts the location of the shift, and then adjust the CUSUM statistic according to the predicted results. Jiao and Helo (2008) raised the difference between the actual process mean and process target mean to an exponent  $w$  to enhance the detection speed of variables CUSUM chart. Wu et al. (2008) investigated the possibility of raising the difference between actual and in-control numbers of non-conforming items for attribute CUSUM chart to an exponent  $w$  while using the  $ARL$  as an objective function. However, the exponent was constrained to three levels only (1, 1.5, 2). The higher the value of  $w$ , the less effective the chart becomes for detecting small shifts and the more effective it becomes for detecting large shifts. Wu et al. (2008) used the relative Average Run Length ( $rARL$ ) to compare between the CUSUM charts with different values of  $w$ . Haridy et al. (2020) also proposed an Exponentially Weighted Moving Average ( $EWMA$ ) chart in which the difference between actual and in-control value of nonconforming units is raised to an exponent  $w$ . The results showed that the developed  $EWMA$  performs better than its traditional counterpart. A complete guide on how to select the charting parameters ( $k$ , and  $H$ ) depending on the allowable false alarm rate, the in-control fraction nonconforming, and the size of downward shift was developed by Bourke (2020).

### 3. Design and Operation of $w$ CUSUM Chart

The  $w$ CUSUM chart has three main charting parameters, the exponent  $w$ , reference parameter  $k$ , and upper control limit  $H$ . The chart's test statistic is  $C_t$ , for a sample, it is updated as follows:

$$C_0 = 0$$

$$C_t = \max(0, C_{t-1} + v_t - k) \quad (4)$$

$$v_t = \begin{cases} (d_t - d_0)^w & \text{if } d_t \geq d_0 \\ -(d_t - d_0)^w & \text{if } d_t < d_0. \end{cases} \quad (5)$$

The  $w$ CUSUM chart can be implemented as follows:

- (1) Take  $C_0 = 0$  as an initial value for the statistic.
- (2) Take a sample of  $n$  items and calculate the corresponding value of  $v_t$  using equation (4).
- (3) Update  $C_t$  according to equation (3).
- (4) If  $C_t \leq H$ , the process is considered to be in control. Go back to step 2 for the next sample.
- (5) If  $C_t > H$ , the process is out of control, and it should be terminated immediately to identify the assignable causes.

Four design specifications need to be determined before implementing the  $w$ CUSUM chart and are required for the design of the chart. (1) The minimum allowable average time to signal  $\tau$ , (2) The in-control fraction non-conforming  $p_0$ , (3) The maximum shift  $\delta_{max}$ , and (4) the sample size  $n$ .

The value of  $\tau$  is determined based on acceptable false alarm rate. The in-control fraction non-conforming  $p_0$  is identified based on in-control historical data. The maximum shift  $\delta_{max}$  is decided based on the maximum tolerable shift. The sample size  $n$  depends on the available resources.

The  $w$ CUSUM chart will be designed using the following optimization model:

$$\text{Objective function:} \quad \text{Minimize } AND \quad (6)$$

$$\text{Constraint:} \quad ATS_0 > \tau \quad (7)$$

$$\text{Design variables:} \quad w, k, \text{ and } H$$

The  $ATS_0$  is calculated using Markov-chain method and it is the average time taken till the chart generates a false alarm. It should be as close as possible to  $\tau$ . Identifying the optimal design variables  $w$ ,  $k$  and  $H$  is implemented using two-level optimization algorithm. Initially, the four design specifications are identified. After that, a very large number is used as a preliminary value for the term  $AND_{min}$ . This term is used to store the minimum value of  $AND$ . At the top level, the optimal value of  $w$  is explored. The search at this level is terminated when  $AND$  can no longer be reduced. At the lower level, the optimal  $k$  and  $H$  are determined so that the constraint  $ATS_0 > \tau$  is satisfied. After finding all charting parameter,  $AND$  is calculated and compared to  $AND_{min}$ . If the calculated  $AND$  is smaller than  $AND_{min}$ , the corresponding charting parameters are considered temporarily as optimal solutions.

## 4. Comparative Study

### 4.1 Comparison Under a General Case

In this section, the performance of the conventional CUSUM and the  $w$ CUSUM charts is analyzed and compared.

The study is conducted only for an increase in fraction nonconforming.

The conventional CUSUM chart is designed to modify  $k$  and  $H$  such that  $ATS_0$  is always larger than and as close as possible to  $\tau$ . On the other hand, the  $w$ CUSUM chart is designed such that the optimal  $w$ ,  $k$  and  $H$  are found using two-level optimization algorithm while maintaining the constraint  $ATS_0 > \tau$  satisfied.

The two charts are tested under one case with the following design specifications:

$$\tau = 500, p_0 = 0.005, \delta_{max} = 8, n = 20 \quad (8)$$

The values of in-control  $ATS_0$  (when  $p = p_0$ ) and out-of-control  $ATS$  (when  $p_0 < p \leq 8p_0$ ) of the conventional CUSUM and the  $w$ CUSUM charts are presented in Table 1. The normalized  $ATS$  is calculated by dividing the chart's  $ATS$  value of the CUSUM chart at any shift by that of the  $w$ CUSUM chart at the same specified shift. The normalized  $ATS$  curves are shown in Figure 1.

The following observations can be noticed from Table 1 and Figure 1:

- (1) Firstly, each of the two charts generates an  $ATS_0$  value larger than  $\tau$  when the process is in control. This ensures that both charts satisfy the requirement on the false alarm rate.
- (2) The  $w$ CUSUM chart outperforms its conventional counterpart over the whole specified shift range ( $p_0 \leq p \leq 8p_0$ ). This means that the proposed  $w$ CUSUM scheme is more sensitive than the conventional CUSUM to small and large shifts.
- (3) The superiority of the  $w$ CUSUM chart decreases when the shift size increases. When  $p = 1.5p_0$ , the conventional CUSUM chart produces an out-of-control  $ATS$  larger than that of the  $w$ CUSUM chart by 61%. Conversely, when  $p = 8p_0$  the conventional CUSUM chart generates an out-of-control  $ATS$  that which is only 18% larger than that of the  $w$ CUSUM chart.
- (4) The  $w$ CUSUM chart offers more flexibility in design as it has three optimizable charting parameters, in comparison to two in its conventional equal. This allows the  $w$ CUSUM chart to be more adaptable under different design specifications.

Table 1. Comparison between CUSUM and  $w$ CUSUM schemes in terms of  $ATS$

$\delta$	$ATS$	
	CUSUM scheme ( $w = 1$ )	$w$ CUSUM scheme ( $w = 1.04$ )
1	910.291	539.248
1.5	287.286	177.946
2	129.727	82.514
2.5	70.705	47.397
3	44.087	30.855
3.5	29.761	21.924
4	22.460	16.819
4.5	17.403	13.021
5	13.626	10.551
5.5	11.365	8.813
6	9.271	7.579
6.5	8.022	6.654
7	7.104	5.810
7.5	6.144	5.624
8	5.592	4.748

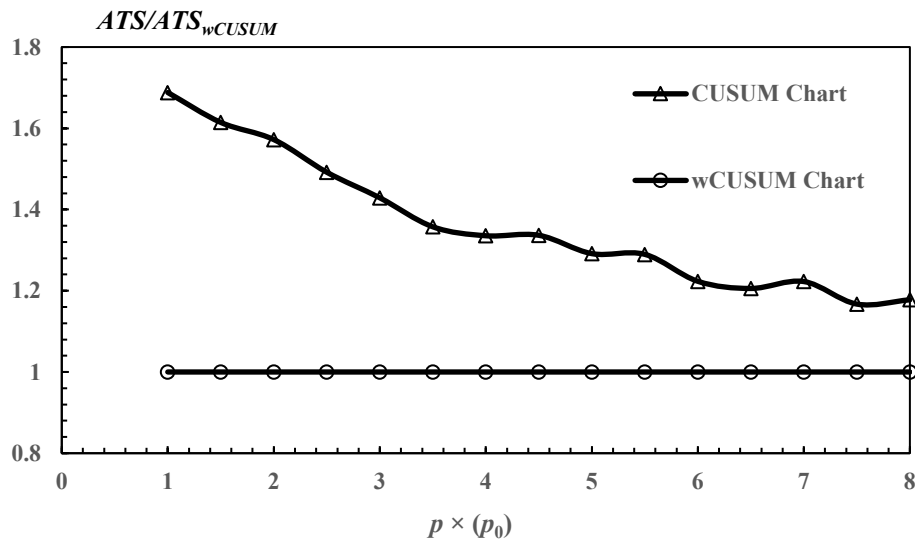


Figure 1. Normalized  $ATS$  of the two quality control charts

The results of implementing the optimization algorithm for the conventional CUSUM and wCUSUM charts are as follows:

CUSUM chart:  $w=1, k=0.485, H=1.426, AND = 0.49654$

wCUSUM chart:  $w=1.04, k=0.38, H=1.572, AND = 0.35882$

The ratio of  $AND_{CUSUM}/AND_{wCUSUM}$  for the case  $\tau = 500, p_0 = 0.005, \delta_{max} = 8$ , and  $n = 20$  is 1.38. This emphasizes a 38% improvement in the detection effectiveness of the wCUSUM chart when compared with the conventional CUSUM chart.

#### 4.2 Comparison Under Multiple Cases

After that, both charts are studied under different design conditions in which the design parameters ( $\tau, p_0, \delta_{max}$  and  $n$ ) are considered as input factors. Each parameter has two levels as follows:

$\tau$ : 200, 1000.  
 $p_0$ : 0.001, 0.01.  
 $\delta_{max}$ : 3, 10.  
 $n$ : 30, 100.

The levels are based on a combination of what frequently has been used by multiple authors (Haridy et al. 2020, Haridy et al. 2014, Haridy et al. 2017, Haridy et al. 2014).

Various combinations of the design parameters ( $\tau, p_0, \delta_{max}$  and  $n$ ) are used to generate six different cases in which the performance of CUSUM and wCUSUM charts are compared. The six cases are as follows:

- Case 1:  $\tau = 200, p_0 = 0.001, \delta_{max} = 3$ , and  $n = 30$ .
- Case 2:  $\tau = 200, p_0 = 0.001, \delta_{max} = 10$ , and  $n = 30$ .
- Case 3:  $\tau = 200, p_0 = 0.01, \delta_{max} = 3$ , and  $n = 100$ .
- Case 4:  $\tau = 1000, p_0 = 0.001, \delta_{max} = 3$ , and  $n = 30$ .
- Case 5:  $\tau = 1000, p_0 = 0.001, \delta_{max} = 10$ , and  $n = 30$ .
- Case 6:  $\tau = 1000, p_0 = 0.01, \delta_{max} = 3$ , and  $n = 100$ .

Both charts are designed to generate an in-control  $ATS_0$  close to  $\tau$ . Table 2 illustrates the relative  $AND$  ratios ( $AND_{CUSUM}/AND_{wCUSUM}$ ) for both schemes under the six cases. The relative  $AND$  ratios are used to compare the performance of both schemes.

Table 2: Comparison of the CUSUM and wCUSUM charts under different conditions

Case	$\tau$	$p_0$	$\delta_{max}$	$n$	Scheme	$w$	$k$	$H$	$AND$	$AND_{CUSUM}/AND_{wCUSUM}$
1	200	0.001	3	30	CUSUM	1	0.485	0.499	0.3599	1.554
					wCUSUM	1.19	0.455	0.510	0.2316	1.000
2	200	0.001	10	30	CUSUM	1	0.485	0.485	0.2092	1.455
					wCUSUM	1.13	0.440	0.536	0.1438	1.000
3	200	0.01	3	100	CUSUM	1	0.470	4.588	0.1179	1.003
					wCUSUM	1.01	0.485	4.590	0.1175	1.000
4	1000	0.001	3	30	CUSUM	1	0.455	1.031	0.7536	1.469
					wCUSUM	0.5	0.455	0.953	0.5129	1.000
5	1000	0.001	10	30	CUSUM	1	0.455	1.032	0.3699	1.364
					wCUSUM	0.68	0.335	1.252	0.2712	1.000

6	1000	0.01	3	100	CUSUM	1	0.530	6.204	0.1777	1.007
					wCUSUM	1.04	0.485	6.883	0.1764	1.000

The values of the ratio  $AND_{CUSUM}/AND_{wCUSUM}$  are always greater than one, this indicates that the proposed scheme always outperforms the conventional scheme. It is observed that the superiority of the wCUSUM chart over the conventional CUSUM chart is not constant. In some cases, such as cases (1, 2, 4, and 5) the performance difference between the two charts is quite high and can exceed 50%. On the other hand, in cases 3, and 6, the performance of both charts is almost identical. According to the performance difference between both schemes under the selected levels for the six cases, the maximum shift  $d_{max}$  and the sample size  $n$ , seem to have the most significant effect on the relative  $AND$  ratios. The average of the  $AND_{CUSUM}/AND_{wCUSUM}$  is calculated for the six cases presented in Table 2, and it represented as the grand average  $AND_{CUSUM}/AND_{wCUSUM}$ . The result is  $AND_{CUSUM}/AND_{wCUSUM} = 1.31$ . This indicates that the proposed scheme under multiple different settings is more effective than the conventional CUSUM chart by 31%, in average. A remark that worth mentioning is that the conventional CUSUM chart has not been more effective than the proposed wCUSUM under any circumstances. This is because the conventional chart is just a special case of the newly proposed wCUSUM chart where  $w = 1$ .

Hypothesis testing is conducted to validate that the proposed wCUSUM chart outperforms the CUSUM chart. The hypothesis testing is performed by comparing the average  $AND$  of the wCUSUM chart with that of the conventional CUSUM chart under a confidence interval of 95%. The data available from the six cases presented in Table 2 is insufficient to perform the hypothesis testing, and due to the high computational time, a hundred values of  $AND_{wCUSUM}$  are generated using Minitab statistical software using the mean and the standard deviation of the  $AND_{wCUSUM}$  values. Similarly, a hundred  $AND_{CUSUM}$  values are generated using the mean and standard deviation of the  $AND_{CUSUM}$  values. The mean and standard deviation of  $AND_{wCUSUM}$  and  $AND_{CUSUM}$  can be obtained using their respective  $AND$  values shown Table 2, and they are as follows:

$$\begin{aligned} \mu_{wCUSUM} &= 0.2422 & \sigma_{wCUSUM} &= 0.1315 \\ \mu_{CUSUM} &= 0.3314 & \sigma_{CUSUM} &= 0.2110 \end{aligned}$$

After that, the hypothesis testing is conducted using 2-sample-t test. The null and alternative hypotheses are respectively as follows:

$$\begin{aligned} H_0: \mu_{CUSUM} - \mu_{wCUSUM} &= 0 \\ H_1: \mu_{CUSUM} - \mu_{wCUSUM} &> 0 \end{aligned}$$

If the P-value is less than or equal to the level of significance  $\alpha = 0.05$ , the null hypothesis is rejected and the alternative hypothesis is accepted, otherwise the null hypothesis is accepted. The hypothesis testing results obtained from Minitab statistical software are as follow:

Null hypothesis	$H_0: \mu_1 - \mu_2 = 0$
Alternative hypothesis	$H_1: \mu_1 - \mu_2 > 0$
<b>T-Value</b>	<b>DF</b>
5.40	157
<b>P-Value</b>	0.000

Where  $\mu_1$  and  $\mu_2$  are the mean of the generated  $AND$  values of the CUSUM chart and wCUSUM chart, respectively. The obtained P-value is less than  $\alpha = 0.05$ , as a result, the null hypothesis is rejected, and the alternative hypothesis is accepted. The objective function is to minimize  $AND$ , and accepting the null hypothesis proves that  $AND_{wCUSUM}$  is smaller than the  $AND_{CUSUM}$ .

## 5. Conclusion

This article investigates the possibility of raising the difference between the actual and in-control numbers of non-conforming items to an exponent  $w$ . Also, this research reflects the importance of the optimization design of the control charts in general, and specifically, the wCUSUM chart. Adding  $w$  as an additional charting parameter to be optimized provides little more complications to the design of CUSUM charts, but it allows more design flexibility and enhances the detection effectiveness. The wCUSUM chart can perform reasonably good and outperform the conventional CUSUM chart for detecting a wide range of shifts. The results of the conducted comparative study

demonstrate the superiority of the  $w$ CUSUM chart over its conventional counterpart considering different shift sizes. It is noticeable that at a small shift, the difference in performance between both charts in terms of  $ATS$  is higher than that at a large shift. At small shifts, the conventional CUSUM generates a higher  $ATS$ , in comparison with the  $w$ CUSUM chart by 61%. This percentage drops to 18% at the largest shift point. The ratio  $AND_{CUSUM}/AND_{wCUSUM}$  indicates a significant improvement in the detection effectiveness by 38%.

The performance of both charts has also been studied under different design specifications considering six different cases. The proposed  $w$ CUSUM chart is 31% more effective than the traditional one, on average in terms of  $AND$ , under different circumstances. The advantage of the  $w$ CUSUM chart over the conventional CUSUM chart is further proved using 2-sample-t test. It is perceived that the value of the relative  $AND$  ratio ( $AND_{CUSUM}/AND_{wCUSUM}$ ) is never below 1, this indicates that the conventional CUSUM chart is simply a unique case of the  $w$ CUSUM chart in which the exponent  $w$  equals one. If the values of the optimizable parameters  $k$ , and  $H$  of the  $w$ CUSUM chart are set to those of the conventional CUSUM chart, and  $w$  is set one, then both charts generate identical results.

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