# Monitoring Multi-Attribute Characteristics using a Combined Scheme of np and EWMA Charts

**Farah Alyassi, Salah Haridy, Mohammad Shamsuzzaman, Imad Alsyouf**
Department of Industrial Engineering and Engineering Management
University of Sharjah
Sharjah 27272, United Arab Emirates
U16101478@sharjah.ac.ae ; sharidy@sharjah.ac.ae; mshamsuzzaman@sharjah.ac.ae; ialsyouf@sharjah.ac.ae

## Abstract

This paper proposes a new combined scheme (termed as Mnp-EWMA scheme) for monitoring multi-attribute characteristics. The proposed Mnp-EWMA scheme combines both the multi-attribute np (Mnp) and multi-attribute Exponentially Weighted Moving Average (MEWMA) charts for effective monitoring fraction nonconforming. In this study, the Average Number of Defectives (*AND*) is used as an objective function in an optimization model to identify the best design variables of the proposed Mnp-EWMA scheme. The performance of the proposed Mnp-EWMA scheme is compared with that of the Mnp and MEWMA charts under five scenarios with the different inputs of design specifications. The results show that the Mnp-EWMA chart outperforms the Mnp and MEWMA charts under different settings.

## Keywords
Control chart, Quality control, Fraction nonconforming, Average time to signal.

## 1. Introduction
When there are multiple processes, which have many associated quality characteristics, there is a need to control & inspect all characteristics of such quality simultaneously because all these characteristics have impact on product. As there are numerous quality characteristics are involved in monitoring the quality of process, only multi-attribute control charts can help the industries nowadays (Jolayemi 2000) . Limited work founded on monitoring multiple quality characteristic, especially on multi-attribute control charts compared to multi-variate control charts (Chong et al. 2019). Haq and Khoo (2019) developed an adaptive multi-variate Exponentially Weighted Moving Average chart (AMEWMA). Similarly, Yue and Liu (2017) proposed an adaptive multi-variate nonparametric EWMA control chart. Furthermore, Park and Jun (2015) examine the process performance using multiple testing approach & multi-variate EWMA control chart. Other multi-variate control charts implemented Hotelling's $T^2$ that caters to the Multi-variate of the business processes (Ahsan et al. 2020, Hadian and Rahimifard 2019). Zhang et al. (2010) designed a new multi-variate control chart called ELR chart; to monitor the mean and variability of the process simultaneously. Whereas Aparisi and Luna (2009) suggested a control chart that has the ability to detect very small shifts showing in the multi-variate process, named Multi-variate Synthetic-$T^2$ control chart. Khoo et al. (2009) developed a multi-variate Synthetic WSD–$T^2$. Adegoke et al. (2018) highlighted the effect of using multi-variate cumulative sum (MCUSUM) control charts on the manufactured product. On the other hand, Niaki and Abbasi (2006) proposed a new technique that assists in monitoring the multi-attribute processes. Haridy et al. (2013) proposed an optimization model for CUSUM chart and np scheme (np-CUSUM). Haridy et al. (2014) illustrated two different charts for monitoring multiple attributes and evaluated their performance. Chong et al. (2019) suggested a binomial multi-attribute cumulative sum (MCUSUM) chart for detecting the upward shifts in the nonconforming units. Shamsuzzaman et al. (2018) designed a multi-attribute Exponentially Weighted Moving Average (MEWMA) chart for processes of products that has multiple defects. Jolayemi (1999) evaluated the performance of the multi-attribute control chart (MACC) and compared it with np control chart.

In this research, a new combined pattern (termed as Mnp-EWMA) of MEWMA and multi-attribute np (Mnp) charts is proposed for monitoring multiple attribute characteristics. The goal of this study to integrate the MEWMA & Mnp features to establish a new combined scheme called Mnp-EWMA with good effectiveness for monitoring increasing shift in fraction nonconforming *p*.

The Mnp-EWMA scheme charting parameters are optimized using the Average Number of Defectives (*AND*) as an objective function to be reduced. The charting parameters of the proposed Mnp-EWMA scheme are:

1. The number of attributes $k$ .
2. The acceptable minimum value $\tau$ of the in-control average time to signal.
3. The in-control fraction nonconforming $p_0$ .
4. The sample size $n$.
5. The maximum allowable shift $\delta_{max}$ .

The *AND* can be computed as follows:

$$AND = \int_1^{\delta_{max}} \delta \times p_0 \times ATS(\delta) \times f_\delta(\delta) \;\; d\delta \qquad (1)$$

Where $p_0$, $\delta$ and $ATS(\delta)$ are the in-control fraction nonconforming, shift in fraction nonconforming and out-of-control Average Time to Signal, respectively, while $f_\delta$ is the density function of the distribution of $\delta$ which is assumed to be uniform. ($ATS_0$), in-control fraction nonconforming $p_0$, sample size n, and maximum shift $\delta_{max}$ in the overall fraction nonconforming (Haridy et al. 2013).

## 2. Assumptions

The proposed Mnp-EWMA scheme is operated under following assumptions:

1. Binominal distribution of the in-control fraction non-conforming.
2. Uniform distribution of the process shifts $\delta$.
3. known overall in-control fraction non-conforming $p_0$.
4. Increasing process shift is merely considered in this study.

## 3. Implementation of the Mnp-EWMA Scheme

The Mnp-EWMA scheme is implemented as shown in Figure 1.

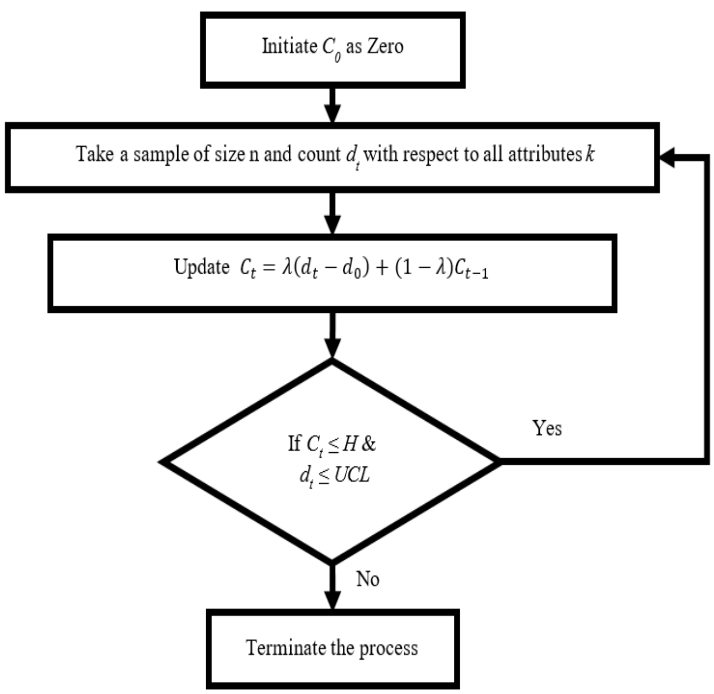


Figure 1. Implementation of an Mnp-EWMA scheme

where $C_t$ is a monitoring statistic, $\lambda$ ($0 < \lambda \leq 1$) is a constant parameter, $d_t$ is the total of non-conforming units detected in the $t$-th sample, $H$ is the control limit of the MEWMA element, $UCL$ is the control limit of the Mnp element, and $d_0$ is the in-control value of $d_t$.

## 4. Optimization Model of the Mnp-EWMA Scheme

The following optimization model is expressed to identify the optimal design of the Mnp-EWMA scheme:

Objective: Minimize *AND*
Constraint: $ATS_0 \approx \tau$
Design Variables: UCL, λ, H

## 5. Optimization Design of Mnp-EWMA Scheme

The optimization design of the Mnp-EWMA scheme can explained particularly as shown in Figure 2.

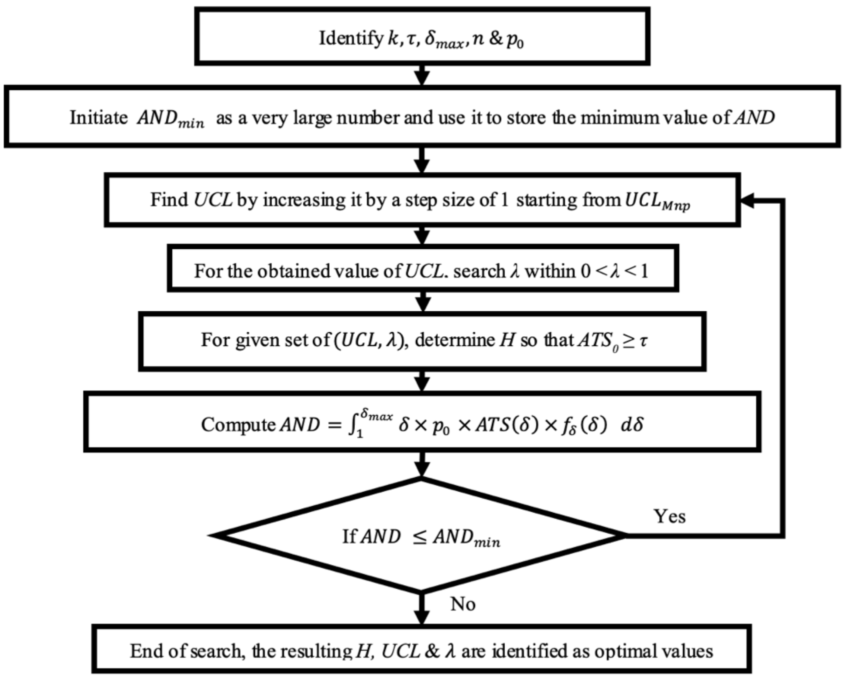


Figure 2. Optimization model of Mnp-EWMA Scheme

## 6. Comparative Analysis

The detection effectiveness of the three control charts (Mnp, MEWMA & Mnp-EWMA scheme) are compared first under the following design specifications:

$p_0 = 0.03$
$\delta_{max} = 6$
$n = 20$
$\tau = 1000$

The design algorithm of the three control charts is established and coded using C program software. The charts are investigated and compared within process shift range of $1 < \delta \leq 10$.
The output of the three charts are shown in Table 1 & Table 2. Figure 3 shows the normalized $ATS_0$ curves of the Mnp, MEWMA & Mnp-EWMA control charts.

Table 1. $ATS_0$ of the three control charts

| $\delta$ | $ATS_0$ | | |
|---|---|---|---|
| | Mnp chart | MEWMA chart | Mnp-EWMA chart |

| | | | |
|---|---|---|---|
| 1 | 2396.64647 | 994.34074 | 984.53090 |
| 2 | 37.33514 | 8.22177 | 7.40551 |
| 3 | 5.41570 | 2.44034 | 2.48063 |
| 4 | 1.81041 | 1.35243 | 1.32358 |
| 5 | 0.93822 | 0.91581 | 0.84454 |
| 6 | 0.65106 | 0.69000 | 0.63710 |
| 7 | 0.54862 | 0.58240 | 0.54430 |
| 8 | 0.51388 | 0.52900 | 0.50000 |
| 9 | 0.50343 | 0.50000 | 0.50000 |
| 10 | 0.50073 | 0.50000 | 0.50000 |

Table 2. Overall performance of the three charts in terms of *AND*

| *AND* | | |
|---|---|---|
| Mnp chart | MEWMA chart | Mnp-EWMA chart |
| 0.64054 | 0.22677 | 0.21300 |

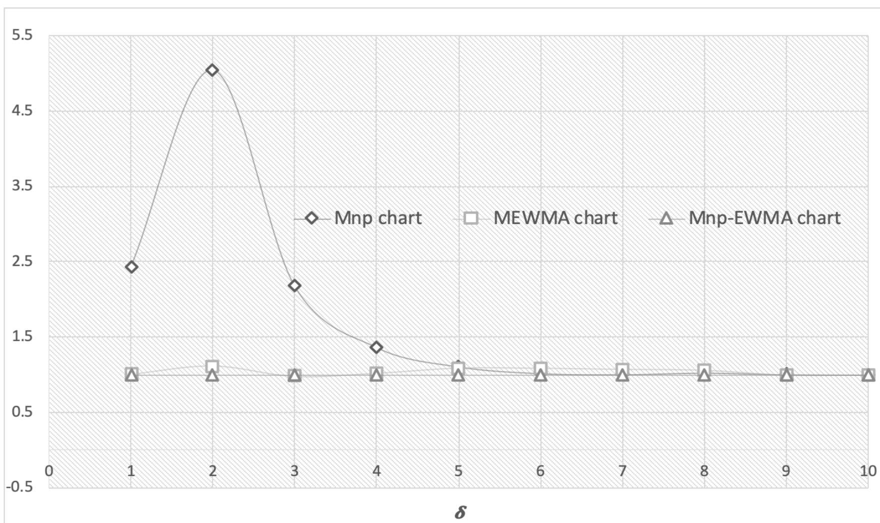


Figure 3. Normalized $ATS_0$ for the three control charts

Based on the results from Table 1, Table 2 & Figure 3, the followings can be noticed:

1. It is clear that Mnp chart generates $ATS_0$ (= 2396.6) which is greater than the specified $\tau = 1000$, this reveal the low effectiveness of Mnp chart.
2. The average time to signal values of both MEWMA & Mnp-EWMA charts are quite close to $\tau = 1000$.This indicates that these charts have more flexibility in design parameter to meet the Constraints, $ATS_0 \geq \tau$.
3. Both, MEWMA & Mnp-EWMA charts are successful in meeting the requirement of the false alarm rate than the Mnp chart.
4. From overall point of view, the MEWMA & Mnp-EWMA charts perform better than the Mnp chart over the entire range of shift $1 < \delta \leq 10$. At $\delta < 5$, the Mnp control chart is more sensitive to detect the large shift.
5. The *AND* values of the three charts are obtained and indicate that Mnp-EWMA scheme has a superiority over Mnp & MEWMA charts about 200 % & 7 %, respectively.
6. Mnp-EWMA scheme superiority over the Mnp and Mnp-EWMA charts decreases with increasing $\delta$ in $p$.

The overall performance of the proposed Mnp-EWMA scheme, the Mnp and MEWMA charts is examined and compared under different circumstances. The *AND* is used as main function to measure the performance of the control charts.

The three charts are examined under five case scenarios with the following different inputs of design specifications:

| | | |
|---|---|---|
| $\tau$ : | 200 | 600 |
| $p_0$ : | 0.0050 | 0.030 |
| $\delta_{max}$ : | 3 | 10 |

where $k$ and $n$ are set at 4 and 100, respectively.
The design variables as well as the overall performance of the three charts are summarized in Table 3.

Table 3. Charts under performance under different design specifications

| case | $\tau$ | $p_0$ | $\delta_{max}$ | chart | *UCL* | *H* | $\lambda$ | *AND* | $AND/AND_{Mnp\text{-}EWMA}$ |
|---|---|---|---|---|---|---|---|---|---|
| | | | | 0 | 6 | - | - | 0.0282 | 1.206193 |
| 1 | 200 | 0.005 | 10 | 1 | - | 3.916 | 0.50 | 0.0235 | 1.006606 |
| | | | | 2 | 7 | 3.638 | 0.42 | 0.0234 | 1.000000 |
| | | | | 0 | 22 | - | - | 0.0942 | 1.016316 |
| 2 | 200 | 0.03 | 10 | 1 | - | 14.33 | 0.50 | 0.0931 | 1.004700 |
| | | | | 2 | 22 | 14.53 | 0.50 | 0.0927 | 1.000000 |
| | | | | 0 | 23 | - | - | 0.0960 | 1.019886 |
| 3 | 600 | 0.03 | 10 | 1 | - | 15.35 | 0.51 | 0.0948 | 1.007448 |
| | | | | 2 | 23 | 15.65 | 0.50 | 0.0941 | 1.000000 |
| | | | | 0 | 22 | - | - | 0.0566 | 1.133116 |
| 4 | 200 | 0.03 | 3 | 1 | - | 14.43 | 0.51 | 0.0517 | 1.034824 |
| | | | | 2 | 22 | 14.51 | 0.50 | 0.0500 | 1.000000 |
| | | | | 0 | 23 | - | - | 0.0646 | 1.149017 |
| 5 | 600 | 0.03 | 3 | 1 | - | 15.48 | 0.53 | 0.0588 | 1.046558 |
| | | | | 2 | 23 | 15.79 | 0.51 | 0.0562 | 1.000000 |
| 0 = Mnp chart | | | | 1 = MEWMA chart | | | | 2 = Mnp-EWMA chart | |

The overall performance of the three control charts under five different cases are reflected by *AND*. It is clearly observed that the ratio of $AND/AND_{\text{Mnp-EWMA}}$ for the three charts are always more than 1. It is interesting to note that the control chart that produces a smaller number of defectives, has high strength of detection effectiveness. This indicates that the proposed Mnp-EWMA scheme always outperforms the Mnp and MEWMA control charts.

## 7. Conclusion

This paper shows the design and implementation of a combined Mnp-EWMA scheme for monitoring multiple attributes. The proposed scheme is compared with the Mnp and MEWMA charts using the average number of defectives (*AND*) as an objective function to be minimized. The overall performance of the proposed Mnp-EWMA is examined and compared with Mnp & MEWMA control charts. The results of comparative study indicate that the proposed Mnp-EWMA scheme always performs better than the Mnp and MEWMA charts. For further optimization of the proposed Mnp-EWMA control chart, $n$ and $h$ might be involved in the optimization model as design variables in a future work.

## References

Adegoke, N. A., Smith, A. N., Anderson, M. J., Abbasi, S. A., & Pawley, M. D. (2018). Shrinkage estimates of covariance matrices to improve the performance of multivariate cumulative sum control charts. *Computers & Industrial Engineering*, 117, 207-216.

Ahsan, M., Mashuri, M., Lee, M. H., Kuswanto, H., & Prastyo, D. D. (2020). Robust adaptive multivariate Hotelling's T2 control chart based on kernel density estimation for intrusion detection system. *Expert Systems with Applications*, 145, 113105.

Akhavan Niaki, S., & Abbasi, B. (2006). On the monitoring of multi-attributes high-quality production processes. *Metrika*, 66(3), 373-388. doi: 10.1007/s00184-006-0117-0

Aparisi, F., & de Luna, M. (2009). The Design and Performance of the Multivariate Synthetic-T2Control Chart. *Communications In Statistics - Theory And Methods*, 38(2), 173-192. doi: 10.1080/03610920802178413

Chong, N. L., Khoo, M. B., Haridy, S., & Shamsuzzaman, M. (2019). A multiattribute cumulative sum-np chart. *Stat*, 8, no. 1, 2019. Available: 10.1002/sta4.239.

Hadian, H., & Rahimifard, A. (2019). Multivariate statistical control chart and process capability indices for simultaneous monitoring of project duration and cost. *Computers & Industrial Engineering*, 130, 788-797.

Haq, A., & Khoo, M. (2019). An adaptive multivariate EWMA chart. *Computers & Industrial Engineering*, 127, 549-557. doi: 10.1016/j.cie.2018.10.040

Haridy, S., Wu, Z., Abhary, K., Castagliola, P., & Shamsuzzaman, M. (2014). Development of a multiattribute synthetic-np chart. *Journal of Statistical Computation and Simulation*, 84(9), 1884-1903.

Haridy, S., Wu, Z., Yu, F. J., & Shamsuzzaman, M. (2013). Optimization design of the combined np-CUSUM scheme for attributes. *European Journal of Industrial Engineering*, 7(1), 16-37.

Haridy, Salah, Zhang Wu, Ka Man Lee, and Nadia Bhuiyan. 2013. Optimal Average Sample Number of the SPRT Chart for Monitoring Fraction Nonconforming. *European Journal of Operational Research*, 229: 411–21. doi:10.1016/j.ejor.2013.03.026

Jolayemi, J. (1999). A Statistical Model for the Design of Multiattribute Control Charts. *Sankhyā: The Indian Journal of Statistics*, Series B (1960-2002), 61(2), 351-365. Retrieved April 27, 2020, from www.jstor.org/stable/25053089

Jolayemi, J. K. (2000). An optimal design of multiattribute control charts for processes subject to a multiplicity of assignable causes. *Applied Mathematics and Computation*, 114(2-3), 187-203.

Khoo, M., Atta, A., & Wu, Z. (2009). A Multivariate Synthetic Control Chart for Monitoring the Process Mean Vector of Skewed Populations Using Weighted Standard Deviations. *Communications In Statistics - Simulation And Computation*, 38(7), 1493-1518. doi: 10.1080/03610910903019905

Park, J., & Jun, C. H. (2015). A new multivariate EWMA control chart via multiple testing. *Journal of Process Control*, 26, 51-55.

Shamsuzzaman, M., Haridy, S., & Zhang, L. (2018). Design of multiattribute EWMA chart. *Ieomsociety.org*, 2018. [Online]. Available: http://ieomsociety.org/ieom2018/papers/125.pdf. [Accessed: 10- Apr- 2021].

Yue, J., & Liu, L. (2017). Multivariate nonparametric control chart with variable sampling interval. *Applied Mathematical Modelling*, 52, 603-612. doi: 10.1016/j.apm.2017.08.005

Zhang, J., Li, Z., & Wang, Z. (2010). A multivariate control chart for simultaneously monitoring process mean and variability. *Computational Statistics & Data Analysis*, 54(10), 2244-2252. doi: 10.1016/j.csda.2010.03.027

## Biographies

**Farah Alyassi** is an Engineer in the Sharjah Electricity and water Authority, UAE. She got her Bachelor degree in Sustainable and renewable Energy from the University of Sharjah. Her research concerns on quality engineering & statistical process control.

**Salah Haridy** is an associate professor in the Department of Industrial Engineering and Engineering Management at the University of Sharjah, UAE. He received his M.Sc. and Ph.D. degrees from Benha University, Egypt and Nanyang Technological University, Singapore in 2008 and 2014, respectively. He is the recipient of the 2013 Mary G. and Joseph Natrella Scholarship awarded by the American Statistical Association (ASA) and the 2014 Richard A. Freund International Scholarship awarded by the American Society for Quality (ASQ).

**Mohammad Shamsuzzaman** is currently a professor in the Department of Industrial Engineering and Engineering Management at the University of Sharjah, UAE. He obtained his Ph.D. in Systems and Engineering Management in 2005 from Nanyang Technological University, Singapore. His current research focuses on quality control and improvement, reliability, simulation, and multi-criteria decision-making. He is a member of the American Society for Quality.

**Imad Alsyouf** is a professor of Industrial Engineering, employed by the University of Sharjah, UAE. He is the founder and coordinator of the Sustainable Engineering Asset Management (SEAM) Research Group. He has produced more than 67 conference and journal papers. He has about 30 years of industrial and academic experience in various positions in Jordan, Sweden, and UAE. His research interests include reliability, quality, maintenance, and optimization. He has developed and taught more than 25 post and undergrad courses. He delivered training courses in Kaizen, TQM, and organizational excellence.