Using AHP to deal with Sectorization Problems

Elif Göksu Öztürk
INESC TEC - Technology and Science, Porto, Portugal
Faculty of Economics, University of Porto (FEP.UP), Porto, Portugal
elif.ozturk@inesctec.pt

Ana Maria Rodrigues
INESC TEC - Technology and Science, Porto, Portugal
CEOS.PP/ISCAP/P.PORTO, Porto, Portugal
ana.m.rodrigues@inesctec.pt

José Soeiro Ferreira
INESC TEC - Technology and Science, Porto, Portugal
Faculty of Engineering, University of Porto (FEUP), Porto, Portugal
jose.soeiro.ferreira@inesctec.pt

Abstract

Sectorization refers to partitioning a large territory, network, or area into smaller parts or sectors considering one or more objectives. Sectorization problems appear in diverse realities and applications. For instance, political districting, waste collection, maintenance operations, forest planning, health or school districting are only some of the application fields. Commonly, sectorization problems respect a set of features necessary to be preserved to evaluate the solutions. These features change for different sectorization applications. Thus, it is important to conceive the needs and the preferences of the decision-makers about the solutions. In the current paper, we solve sectorization problems using the Genetic Algorithm by considering three objectives: equilibrium, compactness, and contiguity. These objectives are collected within a single composite objective function to evaluate the solutions over generations. Moreover, the Analytical Hierarchy Process, a powerful method to perceive the relative importance of several objectives regarding decision makers' preferences, is used to construct the weights. We observe the changes in the solutions by considering different sectorization problems that prioritize various objectives. The results show that the solutions' progress changed accurately to the given importance of each objective over generations.

Keywords
Sectorization, Genetic Algorithm, Analytical Hierarch Process

1. Introduction

Sectorization means dividing into sectors or parts, which arises in many contexts and applications, usually to achieve some goal or to facilitate an activity. Most of the time, this division or partition aims to better organize or simplify a massive problem into smaller sub-problems or promote groups with similar characteristics.

Sectorization is a very important matter due to the wide range of application areas. The application areas are diverse and often arise related to geographical issues. Examples include designing political districts, sales territories, school, health and policing zones, forest planning, municipal waste collection or street cleaning zones and maintaining operations (see. Bozkaya et al. 2003, Augstin-Blas et al. 2009, Rios-Mercado and Bard 2019).

The study of sectorization problems is still quite relevant to society and science since they do not have an easy or universal solution. The solution methods used are highly dependent on specific cases. Commonly, they include a set of features that should be preserved in the construction or evaluation of sectors. The examples are equilibrium (identical proportion of the parts in relation to the whole), contiguity (each sector should represent one part instead of a spread of small parts) and, compactness (sectors showing regular shapes, circles, or squares, avoiding tentacle) (Kalcsics et al. 2005). Preserving these features requires involving them in the solution procedure as objective functions. Depending on the selected solution method, these objectives can be evaluated separately or together in a weighted composite single objective function. When the latter is in question, the challenge is to weigh each objective according to the decision-maker's preferences. Moreover, the multiple-criteria nature of these problems make the solution procedures more complex, so the research contributions focus mainly on the advanced heuristics and metaheuristic methods.

In the current paper, we solve sectorization problems, considering the three most common objectives: equilibrium, compactness, and contiguity using a Genetic Algorithm (GA), proposed by Holland (1992). We use a composite weighted single objective function to evaluate each solution. Moreover, we follow the Analytical Hierarchy Process (AHP), presented by Saaty (1980), in the weighting procedure. AHP helps to weigh the different criteria or objectives, for the specific problem, regarding their relative importance according to the decision-maker. The changes in the preferences of the decision-maker on the objectives are expected to affect the solutions. Following this argument, we aim to observe the decision-maker's influence on the solutions. To do this, we create three different scenarios. In each scenario, the relative importance of the objectives is changed, and new weights are assigned following the AHP. As AHP is not a common approach in sectorization, that is also a relevant contribution to the literature.

Moreover, we create and propose new data given the data scarcity. In total, 50 instances are generated, including coordinates, quantities, and contiguity maps. We make these instances available.

The paper is structured as follows. Section 2 describes and revises the sectorization problems. A clear distinction in the application areas is presented in this section. Section 3 offers the method and the objectives that we use. Section 4 comprises detailed information regarding the data and indicates a link to access the data. Moreover, the results and discussions are also presented in Section 4. Finally, Section 5 concludes.

2. Sectorization Problems

Sectorization is much more than merely splitting a map into small pieces. Some characteristics of the final sectors are usually expected, which may be represented by several objectives.

Although the idea of sectorization has always accompanied human activity, the first publication about the subject only appeared in the '60s (Hess et al. 1965), related to the division of territory in political districting. However, in the last decades, many publications appeared, illustrating various applications, solution approaches and different criteria in each case (see. Kalcsics et al. 2005).

In the following, we propose an organization of applications presented in the literature, according to different areas such as Administration, Commerce, and Services, see Figure 1.

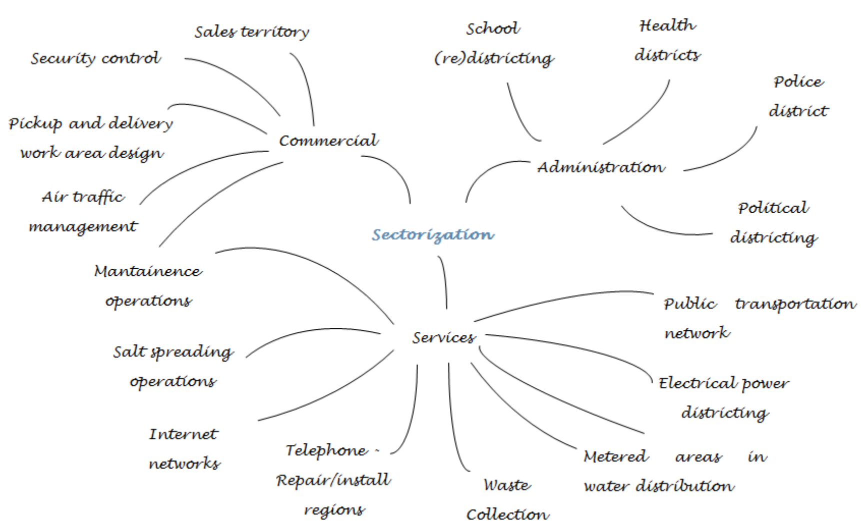


Figure 1. Applications of Sectorization

2.1. Administrative Sectorization

This kind of sectorization includes political districting where the main idea is dividing a certain territory into a given number of sectors (districts), based on the principle of "one man - one vote". Votes are transformed into seats, and a correct partition of the territory cannot admit advantages and/or disadvantages for some sectors. A "bad manipulation" of political districts is known as *gerrymandering*. Objectives to evaluate the sectors are, for instance, the integrity of territories, the respect for the administrative subdivision of the territory, the existence of small communities and the preservation of the minorities' strength (Ricca et al. 2013; Bozkaya et al. 2003; Garfinkel and Nemhause 1970; Hess et al. 1965).

School districting is another administrative sectorization where a region is divided into smaller areas (Caro et al. 2004). Each small part is assigned to a school. Objectives proposed may involve the total travel distance between student residences and schools, racial balance, crossing arterial roads on home-school, and robustness to future developments.

Moreover, the optimal allocation of health services or social services and sectorization related to emergency medical systems in highways (see. Pezzella et al. 1981, Bezarti et al. 2013, Minciardi et al. 1981, Iannoni et al. 2009) are also beneath the administrative field.

Finally, the definition of police command boundaries (see. D'Amico et al. 2002) and the design of sectors related to patrolling operations by vessels of a marine protection agency (see. Lunday et al. 2012) are other examples of this field.

2.2. Commercial Sectorization

Commercial sectorization, which usually refers to the design of sales territories, is one of the most known application fields. The common objective is to maximize the profit while dividing a certain "salesforce" into a given number of smaller areas (sectors). Some examples in the literature are Skiera and Albers (1998), Hess and Samuels (1971), Kalcsics et al. (2005), and Zoltners and Sinha (1983).

In addition, sectorization faced by a pickup and delivery parcel company or the design of work areas for drivers who pick up and deliver hundreds of packages a day are addressed by Gonzalez-Ramirez et al. (2011) and Jarrah and Bard (2012), respectively, that can be grouped within commercial sectorization.

Dynamic situations may also arise in this type of sectorization. For example, Lei et al. (2015) consider dynamic customers that vary over the planning horizon. A certain proportion of clients leaves the territory in each period, and another proportion of new clients arrives.

Finally, airspace (re)sectorization, a concept in air traffic management, can be clustered within the commercial sectorization field (see. Tang et al., 2012).

2.3. Service Sectorization

Service sectorization has a diverse amount of application fields. For instance, the electrical power districting problems (see. Bergey et al. 2003) or security control problems, which aims to design the scheduled tours for the security guards by dividing a vast area into smaller districts for the visits of the guards (see. Prischink et al. 2016) are some applications related to this subject.

Furthermore, solid waste collection problems that aim to facilitate waste collection (see. Rodrigues and Ferreira 2015, Hanafi et al. 2004, Mourão et al. 2009, Male and Liebman 1978, Lamata et al. 1999), and water distribution problems are other fields acknowledged in the literature. For example, Alvisi and Franchini (2014) sectorized the water distribution system by creating the district metered areas.

Finally, defining the metropolitan Internet network areas for installing hubs (Park et al. 2000), districting for salt spreading operations, and partitioning the road network into subnetworks to facilitate the organization of the services (see. Muyldermans et al. 2002, 2003), or splitting the territory to remain the same price in the public transportation (see. Tavares-Pereira et al. 2007) can be exemplified as other fields of application beneath service sectorization.

3. Methodology and Criteria

We used the Genetic Algorithm (GA) in the solution process of the sectorization problems. GA is an approximate and flexible method, sufficiently good to deal with practical problems in which data and objectives are not completely rigorous. Due to this advantage of GA, several authors implemented it in the literature to solve sectorization and related problems (Konak 2012; Agustín-Blas 2009; Baçao et al. 2005).

GA is a population-based metaheuristic. The solution set is called population, and each solution represents an individual in this population. Solutions are encrypted into chromosomes (Ruiz 2005). The evaluation of these solutions is performed according to their fitness values. These values show the convenience of the solution to the problem.

In an analogy with Darwin's theory of evolution, which inspired GA, the solutions are expected to improve/evolve over generations (i.e., iterations). A complete iteration consists of several steps: selection, crossover, mutation, and elitism. Selection picks two existing individuals according to some procedure. Although this procedure may be random, it usually depends on the fitness value of each solution. This way, it increases the possibility of mating the two relatively better solutions. The process of mating is called crossover. Depending on the selected crossover procedure, one or two new solutions are generated. These new solutions are called offspring. Mutation follows the crossover to give some randomness to the population. Usually, some offspring are mutated before being merged with the population. Finally, elitism is used to eliminate inadequate solutions from the population. This procedure keeps the population size fixed by removing as many solutions as the offspring generated. Whether a solution is good is decided according to its fitness value.

This work utilizes a suitable “matrix form genetic encoding system”. In this encoding system, rows and columns represent the nodes and sectors, respectively. Each row is a binary set taking the value one when a node is assigned to a sector and zero otherwise. There are two restrictions in this encoding system: (i) each node is assigned to only one sector, and (ii) each sector has to have at least one node. Moreover, we use tournament selection for the crossover. In tournament selection, randomly selected two solutions (individual) are evaluated according to their fitness values, and the one with better fitness value is chosen for mating. We use random mutation in the algorithm that we developed and mutate only half of the offspring. Python 3.7 is adopted to implement the algorithm and visualize the solutions. It is possible to see the pseudocode below.

Generate N feasible solutions and inset to the population ($Pop_{size} = N$)
Evaluation of the solutions according to the fitness function.
while $Generation \; != \; T$ **do** (T = *number of generations*):
    while $Pop_{size} \; != 2 \times N$:
        Select parents to mate.
        Crossover to create two offspring in each time
        $Pop_{size} := N + 2$
    end while
    Mutate half of the offspring
    Join the offspring into all population
    Evaluation of the solutions according to the fitness function.
    while $Pop_{size} \times 2 \; != N$:
        Delete the worst solution from the population
        $Pop_{size} := N - 1$
    end while
    $Generation := Generation + 1$
end while

The fitness value exhibits the information regarding the objective or objectives of the problem. Albeit a single objective function with several restrictions can be used, a single composite objective function which contains several objectives or criteria can also be used. The composite single objective function is illustrated as follows.

$$F(x) = \omega_1 f_1(x) + \omega_2 f_2(x) + \cdots + \omega_k f_k(x), \qquad \sum_{k=1}^{K} \omega_k = 1$$

In this equation, $F(x)$ represents the fitness value for solution x. The weights for k objectives are ω_k, and the sum of the weights has to be equal to one. Finally, $f_k(x)$ indicates the normalized values of solution x for k objectives.

In the composite objective functions, normalization is necessary if the measurement units of separate objectives are different. Moreover, weighting requires careful consideration given the effect of the weights in the final solutions. In the current paper, we used min-max normalization method to set the values between 0 and 1. Furthermore, Analytical Hierarch Process (AHP) is selected to construct the weights for each objective.

AHP is presented by Thomas L. Saaty in 1970s (Saaty 1977; Saaty 1980; Saaty and Vargas 2012). AHP sets weights considering the preference factor of the decision-makers. In this method, the decision-maker is expected to make selections considering a comparison scale from 1 to 9, where each value represents a factor from equal importance to extreme importance. S/he needs to make this comparison for every two objectives unilaterally. Thus, when there are k objectives, there will be $k(k-1)\ /\ 2$ comparisons to be made.

In this work, we consider three objectives or criteria: (i) equilibrium, (ii) compactness, and (iii) contiguity. It is possible to see the selected measures for the three objectives during the rest of this section.

3.1. Equilibrium

Equilibrium refers to creating a balance in each sector regarding the specific problem to be optimized (e.g., demand in customers, the similarity in working hours). To measure equilibrium, we adopt the method proposed by Rodrigues and Ferreira (2015). The formula can be seen in Equation 1.

$$\bar{q} = \frac{\sum_{j=1}^{K} q_j}{K}, \text{ where } q_j = \sum_i x_{ij},\ \forall j \text{ and } s'_q = \sqrt{\frac{1}{K-1}\sum_{i=1}^{k}(q_i - \bar{q})^2} \tag{1}$$

Here $\bar{q}$ shows the mean value in each sector under the assumption of K is the total number of sectors. In other words, dividing the sum of basic units by the total number of sectors gives each sector's mean value. Then, this information is used to construct the standard deviation from the mean value in each sector. Ultimately, we used the standard deviation to detect the equilibrium in each sector. The smaller this value is, the more balance the sectors are.

3.2. Compactness

Compactness shows the density in each sector. We use a measure that observes the distance between the centroid of the sector and the furthest point to the centroid to detect the compactness level. It is possible to see the formula in Equation 2.

$$d = \sum_{j=1}^{K} dist(o_j, p_j) \tag{2}$$

Here o_j is the centroid point and p_j represents the furthest point from this centroid in sector j. Compactness is gauged by the sum of the furthest distances to the centroids in each sector. The smaller this value is, the more compact the sectors are.

3.3. Contiguity

Contiguity implies to the linked sectors. This criterion evaluates the possibility of movement within each sector. We adopted the measurement presented by Rodrigues and Ferreira (2015) to calculate the contiguity. The authors considered a matrix that stores the information regarding the links between the basic units in the territory. This matrix is designed as a symmetric square matrix in the size of the basic units (BUs), and each cell takes the value 'one' if there is a link between two BUs and 'zero' otherwise. It is essential to mention that BUs in the same sector are assumed connected if they both link to another (i.e., a common) BU, although they are not directly connected.

It is possible to see this matrix below.

$$M^j = \begin{bmatrix} 0 & m_{12}^j & \cdots & m_{1n_i}^j \\ m_{21}^j & 0 & & m_{2n_i}^j \\ \vdots & & \ddots & \vdots \\ m_{n_i1}^j & m_{n_i2}^j & \cdots & 0 \end{bmatrix}$$

where $m_{wi}^j = \begin{cases} 1 & if\ there\ is\ a\ direct\ or\ indirect\ path\ between\ point\ w\ and\ point\ i\ in\ sector\ j \\ 0 & otherwise \end{cases}$

By considering this matrix, contiguity in each sector can be calculated using Equation 3. In Equation 3, the numerator shows the total number of paths among the BUs in sector j. Moreover, the denominator shows the maximum number of possible links within the sector j. Thus, c_j represents the contiguity in sector j and is equal to one when all the points located in the same sector are linked.

$$c_j = \frac{\sum_{i=1}^{n_j}(\sum_{w}^{n_j} m_{\mathrm{wi}}^j)}{n_j \times (n_j - 1)} \tag{3}$$

Contiguity is evaluated using Equation 4, which represents the weighted average of isolated contiguity. N represents the total number of BUs and n_j is the number of BUs in sector j.

$$\bar{c} = \frac{\sum_{j=1}^{k} c_j n_j}{N} \tag{4}$$

Equation 4 occurs always between 0 and 1. The higher the value is, the better the contiguity. We use $(1 - \bar{c})$ to evaluate all the objectives as minimization.

4. Data and Results

The computational results are based on instances created. We provide 50 instances with a minimum size of 25 and a maximum of 1000 nodes, including coordinates, quantities and contiguity maps for each instance.

All instances are created using a gamma distribution. We use different shape and scale parameters for each node created in a single instance. This approach helped to develop an adequate allocation of the nodes in a large area. The quantities are generated considering uniform random distribution. Finally, connectivity maps are made under the assumption of connected graph theory. It is possible to find these instances through the following link: https://drive.inesctec.pt/s/oLdcR2CJzNcff4c

The remainder of this section includes the results obtained for two instances (i.e. Gamma 4 with the size of 882 nodes, and Gamma 33 with the size of 450 nodes) among 50, presented in above mentioned link.

We assume three different sectorization problems that all consider three selected objectives: equilibrium, compactness, and contiguity. However, the relative importance of the objectives is designed differently for each case or scenario.

The first scenario is a health districting problem. One of the essential objectives for health districting is the equilibrium or balance in caregivers' work hours. Contiguity is also an important objective, given that home care services or emergency services are in question. Compactness may be relatively less important for the health districting problem. It is possible to observe the pairwise comparisons among the three objectives for the first scenario (S_1) in Table 1.

S_1	Equilibrium	Compactness	Contiguity
Equilibrium	1	4	2
Compactness	1/4	1	1/3
Contiguity	1/2	3	1

Table 1. Pairwise Comparisons under Scenario 1

The second scenario is on maintenance operations. Among the three objectives, compactness is relatively more important for this type of sectorization problem since it would be difficult for the maintenance personnel if the customers are located diversely. Of course, equilibrium and contiguity are also necessary features to be kept. It is possible to observe the pairwise comparisons among the three objectives for the second scenario ($\boldsymbol{S_2}$) in Table 2.

S_2	Equilibrium	Compactness	Contiguity
Equilibrium	1	1/3	1/2
Compactness	3	1	2
Contiguity	2	1/2	1

Table 2. Pairwise Comparisons under Scenario 2

Finally, the third scenario is created for a waste collection problem. Contiguity appears as the most critical objective among the three for this type of problems. Given the cost of using a vehicle, equilibrium is considered relatively more important than compactness. Given these reasons, the pairwise comparisons can be observed in Table 3 for the third scenario (S_3).

S_3	Equilibrium	Compactness	Contiguity
Equilibrium	1	3	1/3
Compactness	1/3	1	1/5
Contiguity	3	5	1

Table 3. Pairwise Comparisons under Scenario 3

Regarding the pairwise comparison matrices, the weights for three objectives can be computed using AHP. It is possible to see those weights in Table 4. Moreover, Table 4 also includes the *Consistency Ratio* (CR) for pairwise comparisons conducted in each scenario. CR should be less than 0.10 for approved comparisons.

$\boldsymbol{Scenario}/\boldsymbol{Weigths}$	*Equilibrium*	*Compactness*	*Contiguity*	CR
$\boldsymbol{S_1}$	0.55714286	0.12261905	0.3202381	0.015797
$\boldsymbol{S_2}$	0.16378066	0.53896104	0.2972583	0.007938
$\boldsymbol{S_3}$	0.26049796	0.10615632	0.63334572	0.033374

Table 4. The weighting scenarios

The rest of the section includes the results conducted using the weights presented in Table 4.

GA requires the selection of some parameters such as the size of the population, number of generations or mutation rate. It is possible to see the parameters that we used in Table 5.

Instance Tested	Number of Nodes	Number of sectors	Mutation Rate	Size of the Population	Number of Generation
Gamma 33	450	20	0.05	50	1000
Gamma 4	882	30	0.05	50	1000

Table 5. The parameters necessary for GA

It is possible to observe the adjustments in the results when the weighting scheme is changed in Table 6 and Table 7 for both test instances, respectively. As is seen, when the decision-maker's judgements change on the objectives, the results approach differently over generations.

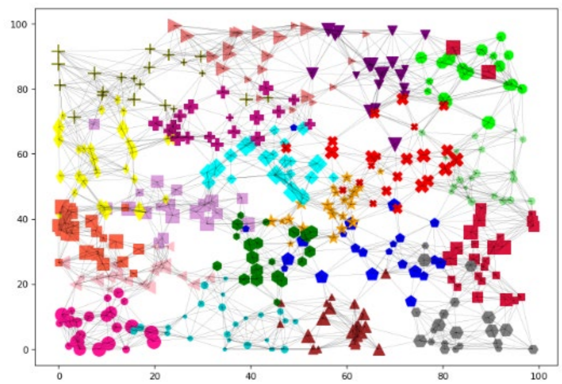
Gamma 33	Equilibrium	Compactness	Contiguity	Computation Time (min)
S_1	1.7917	446.0356	0.0342	~ 21 min
S_2	3.0521	322.0305	0.0	
S_3	2.2594	369.8635	0.0044	

Table 6. The numerical results of instance Gamma 33 under three scenarios

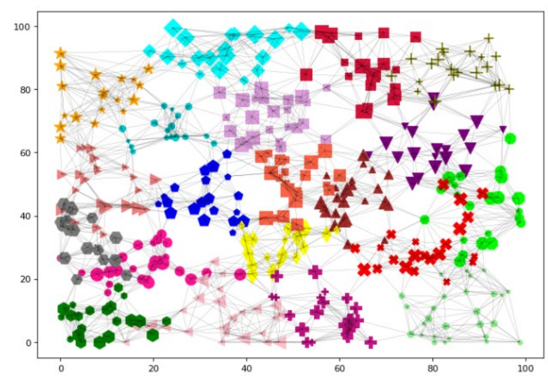
Gamma 4	Equilibrium	Compactness	Contiguity	Computation Time (min)
S_1	2.1172	563.0587	0.0111	~ 49 min
S_2	2.9594	453.6929	0.0068	
S_3	2.5997	454.6984	0.0	

Table 7. The numerical results of instance Gamma 4 under three scenarios

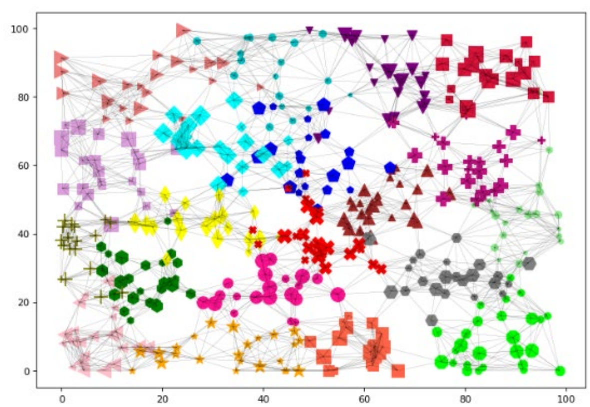
It is also possible to observe the illustrations of the results for the three scenarios in Figure 2 for Gamma 33 with 450 nodes.



(a) The solution obtained by Scenario 1



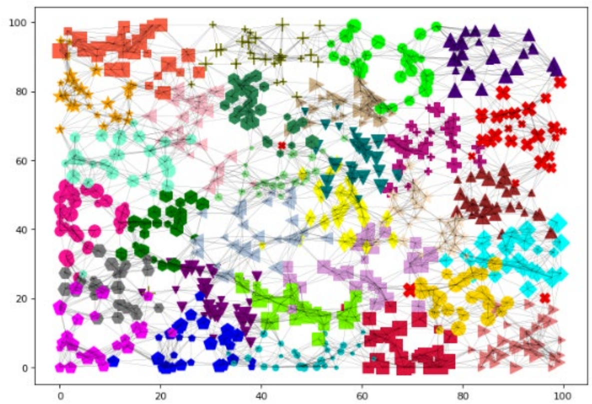
(b) The solution obtained by Scenario 2



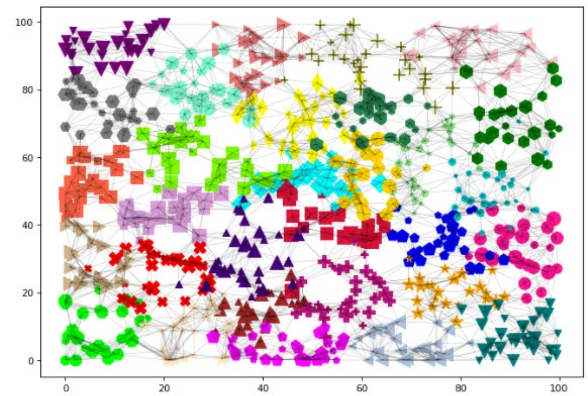
(c) The solution obtained by Scenario 3

Figure 2. Solutions obtained under three scenarios, for instance-Gamma 33

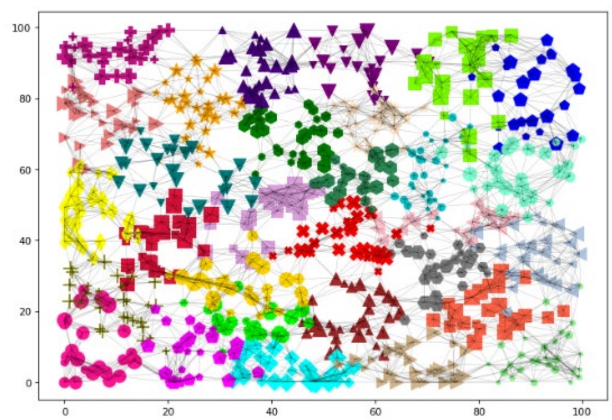
Likewise, Figure 3 illustrated the solutions for Gamma 4 with 882 nodes. In both Figure 2 and Figure 3, every colored group represents a sector.



(a) The solution obtained by Scenario 1



(b) The solution obtained by Scenario 2



(c) The solution obtained by Scenario 3

Figure 3. Solutions obtained under three scenarios, for instance-Gamma 4

Different sectorization problems require a separate evaluation of the objectives. The results that we represent show the influence of the decision-makers on the final solutions if their preferences are reflected in the problematic correctly. AHP is an easy and fast method to adjust the weights according to the decision makers' preferences. It helps the decision-makers re-evaluate each objective for the specific sectorization problem and assign new weights.

5. Conclusion

Sectorization refers to dividing a large area, territory or network into smaller parts considering one or more objectives. If there is more than one objective, the solution procedure requires simultaneous evaluation of each of them. To do that, either a multi-objective optimization method can be used, or all the objectives can be collected within a composite weighted single objective function. The latter requires normalization when the data has different measurement units and weighting to determine each objective's importance to the problematic situation.

We solved different sectorization problems in the current paper considering three objectives: equilibrium, compactness, and contiguity by evaluating them within a composite single objective function. We assumed three different sectorization scenarios, and each prioritizes a different objective. We assigned the weights for each scenario following AHP.

AHP is a solid method to impose the decision makers' preference on a problem's weighting scheme with several dimensions. The obligation in this method is the pairwise comparisons of each objective with each other. These comparisons depend on the relative importance of each objective according to the decision-maker.

We used GA to solve sectorization problems under different weighting schemes. The results show that the solutions advance in a different direction, more proper to each objective's given importance, over generations. These results show the relevance of AHP to demonstrate the decision-makers' influence on the solutions when the weighing system is altered regarding their preferences.

Furthermore, given the data scarcity, we generate our instances and make them available. In total, 50 instances, including coordinates, quantities, and contiguity maps, are produced following gamma distribution.

Acknowledgements

This work is financed by the ERDF - European Regional Development Fund through the Operational Programme for Competitiveness and Internationalization - COMPETE 2020 Programme and by National Funds through the Portuguese funding agency, FCT - Fundação para a Ciência e a Tecnologia within the project "POCI-01-0145-FEDER-031671".

References

Agustín-Blas, L.E., Salcedo-Sanz, S., Ortiz-García, E.G., Portilla-Figueras, A., and Pérez-Bellido, A.M., A Hybrid Grouping Genetic Algorithm for Assigning Students to Preferred Laboratory Groups, *Expert Systems wit Application,* vol. 36, pp. 7234-7241, 2009.

Alvisi S. and Franchini M., A heuristic procedure for the automatic creation of district metered areas in water distribution systems, *Urban Water Journal*, vol. 11, no.2, pp. 137–159, 2014.

Bação F., Lobo, V. And Painho, M., Applying genetic algorithms to zone design, *Sof Comput,* vol. 9, pp.341-348, 2005.

Benzarti E., Sahin E., and Y. Dallery, Operations management applied to home care services: Analysis of the districting problem, *Decision Support Systems*, vol. 55, pp. 587–598, 2013.

Bergey, P.K., Ragsdale, C.T. and Hoskote, M., A decision support system for the electrical power districting problem, *Decision Support Systems*, vol. 36, no. 1, pp.1-17, 2003.

Bozkaya, B., Erkut, E. and Laporte, G., A Tabu Search Heuristic and Adaptive Memory Procedure for Political Districting, *European Journal of Operational Research*, vol. 144, no.1, pp. 12-26, 2003.

Caro, F., Shirabe, T., Guignard, M. and Weintraub, A., School redistricting: Embedding GIS tools with integer programming, *Journal of the Operational Research Society*, vol. 55, no. 8, pp. 836-849, 2004.

D'Amico, S.J., Wang, S., Batta, R. and Rump, C.M., A simulated annealing approach to police district design, *Computer & Operations Research*, vol. 29, no. 6, pp.667-684, 2002.

Garfinkel R. S. and Nemhauser G. L., Optimal Political Districting by Implicit Enumeration Techniques, *Management Science*, vol. 16, no.8, pp. 495–508, 1970.

Gonzalez-Ramirez R. G., Smith N. R., Askin R. G., Miranda P. A., and J. M. Sanchez, A Hybrid Metaheuristic Approach to Optimize the Districting Design of a Parcel Company, *Journal of Applied Research and Technology*, vol. 9, pp. 19–35, 2011.

Hanafi S., Freville A., and Vaca P., Municipal Solid Waste Collection: An effective data structure for solving the sectorization problem with local search methods, *INFOR*, vol. 37, no. 3, pp. 236–254, 1999.

Hess S. W. and Samuels S. A., Experiences with A Sales Districting Model: Criteria and Implementation, *Management Science*, vol. 18, no. 4, pp. 41–54, 1971.

Hess S. W., Weaver J. B., Siegfeldt H. J., Whelan J. N., and Zitlau P. A., Nonpartisan Political Redistricting By Computer, *Operations Research*, vol: 13, no.6, pp. 998–1006, 1965.

Holland, J. H. Genetic Algorithms, *Scientific American,* vol. 267, no. 1, pp 66-73, 1992.

Iannoni A. P., Morabito R., and Saydam C., An optimization approach for ambulance location and the districting of the response segments on highways, *European Journal of Operational Research*, vol. 195, pp. 528–542, 2009.

Jarrah A. I. and Bard J. F., Large-scale pickup and delivery work area design, *Computers and Operations Research*, vol. 39, no. 12, pp. 3102–3118, 2012.

Kalcsics, J., Nickel, S., and Schröder, M., Towards a unified territorial design approach—Applications, algorithms, and GIS integration, *Top*, vol.13, no. 1, pp. 1-56, 2005.

Konak, A., Network Design Problem with Relays: A Genetic Algorithm with a Path-Based Crossover and a Set Covering Formulation, *European Journal of Operational Research,* vol. 218, pp. 829-837, 2012.

Lamata M., Pcláez J., Sierra J., and Bravo J., A Sectoring Genetic Algorithm for the Urban Waste Collection Problem, in: Reusch B. (ed.), *Computational Intelligence: Lecture Notes in Computer Scienc*e, vol. 1625, pp. 642–646, Springer, Berlin/Heidelberg, 1999.

Lei H., Laporte G., Liu Y., and Zhang T., Dynamic design of sales territories, *Computers and Operations Research*, vol. 56, pp. 84–92, 2015.

Lunday B. J., Sherali H. D., and Lunday K. E., The coastal sea space patrol sector design and allocation problem, *Computational Management Science*, vol. 9, pp. 483–514, 2012.

Male J. W. and Liebman J. C., Districting and Routing for Solid Waste Collection, *Journal of the Environmental Engineering Division*, vol. 104, no. 1, pp. 1–14, 1978.

Minciardi R., Puliafito P. P., and Zoppoli R., A districting procedure for social organizations, *European Journal of Operational Research*, vol. 8, no.1, pp. 47–57, 1981.

Mourão M. C., Nunes A. C. and Prins, C., Heuristic methods for the sectoring arc routing problem, *European Journal of Operational Research*, vol. 196, no. 3, pp. 856–868, 2009.

Muyldermans L., Cattrysse D., and Van Oudheusden D., District design for arc-routing applications, *Journal of the Operational Research Society*, vol. 54, pp. 1209–1221, 2003.

Muyldermans L., Cattrysse D., Oudheusden D. V., and Lotan T., Districting for salt spreading operations, *European Journal of Operational Research*, vol. 139, no. 3, pp. 521–532, 2002.

Park K., Lee K., Park S., and Lee H., Telecommunication Node Clustering with Node Compatibility and Network Survivability Requirements, *Management Science*, vol. 46, no. 2, pp. 363–374, 2000.

Pezzella F., Bonanno R., and Nicoletti B., A system approach to the optimal health-care districting, *European Journal of Operational Research*, vol. 8, no. 2, pp. 139–146, 1981.

Prischink M., Kloimullner C., Biesinger B., and Raidl G., Districting and Routing for Security Control, in: Blesa M. J.(ed.), *Hybrid Metaheuristics*, vol. 2, pp. 87–103, Springer International Publishing, Switzerland, 2016.

Ricca F., Scozzari A., and Simeone B., Political Districting: from classical models to recent approaches, *Annals of Operations Research*, vol. 204, pp. 271–299, 2013.

Ríos-Mercado, R. Z. and Bard, J. F., An exact algorithm for designing optimal districts in the collection of waste electric and electronic equipment through an improved reformulation, *European Journal of Operational Research*, vol. 276, no. 1, pp. 259–271, 2019.

Rodrigues, A.M. and Ferreira, J.S., Sectors and Routes in Solid Waste Collection, in: Póvoa, A.P.F.D.B. and Miranda, J.L. (eds.), *Operational Research and Big Data, Springer Studies in Big Data*, vol.15, pp. 353-375, 2015.

Ruiz, R., Maroto, C. and Alcaraz, J., Solving the flowshop scheduling problem with sequence dependent setup times using advanced metaheuristics, *European Journal of Operational Research*, vol. 165, no. 1, pp.34-54, 2005.

Saaty, T.L., A Scaling Method For Priorities In Hierarchical Structures, *Journal of Mathematical Psychology*, vol. 15, no. 3, pp. 234-281, 1977.

Saaty, T.L., *The Analytical Hierarchy Process: Panning, Priority Setting, Resource Allocation*, New York, London: McGraw-Hill International Book Co.,1980.

Saaty, T. L., and Vargas, L. G., *Models, methods, concepts & applications of the analytic hierarchy process*, Springer, Boston, MA, 2012.

Skiera B. and Albers S., COSTA: Contribution Optimizing Sales Territory Alignment, *Marketing Science*, vol. 17, pp. 196–213, 1998.

Tang J., Alam S., Lokan C., and Abbass H. A., A multi-objective evolutionary method for Dynamic Airspace Re-sectorization using sectors clipping and similarities, *IEEE Congress on Evolutionary Computation*, pp. 1–8, 2012.

Tavares-Pereira F., Figueira J. R., Mousseau V., and Roy B., Multiple criteria districting problems, The public transportation network pricing system of the Paris region, *Annals of Operations Research*, vol. 154, pp. 69–92, 2007.

Zoltners A. A. and Sinha P., Sales Territory Alignment: A Review and Model, *Management Science*, vol. 29, no. 11, pp. 1237–1256, 1983.

Biographies

Öztürk E.G. is a PhD candidate on Economics at School of Economics and Management, University of Porto. She holds BSc and MA degrees on Economics from Hacettepe University. Her research interests are Development Economics, Welfare Economics, Environmental Economics, Applied Economics, Innovation, Technological Change, Operations Research, and Sectorization. She held a research grant from CEF.UP (Center for Economics and Finance at UPorto) in 2018. She is a research assistant at INESC-TEC (Institute for Systems and Computer Engineering, Technology and Science) since 2019.

Rodrigues A.M. is a Senior Lecturer at ISCAP-P.Porto (Porto Accounting and Business School-Polytechnic Institute of Porto). She holds a BSc degree in Applied Mathematics and Computation, a Master in Applied Quantitative Methods for Management, and a Ph.D. in Engineering and Industrial Management from the Faculty of Engineering, University of Porto. She is a researcher at INESC TEC – Technology and Science and at CEOS.PP (Centre for Social and Organizational Studies). Her research interests are related to Sectorization Problems, Arc Routing, and Waste Management.

Ferreira J. S. is a Professor at FEUP – Faculty of Engineering, University of Porto (Department of Engineering and Industrial Management) and Researcher Coordinator at INESC TEC – Technology and Science (Centre for Enterprise Systems Engineering). He holds a Chair Aggregation from FEUP, a PhD degree in Operational Research from the Technical University of Denmark and a degree in Electrotechnical Engineering from FEUP. He has been President of APDIO - the Portuguese Operational Research Society. The main field of activity is Operational Research/Management Science. Specific interests cover Problem Structuring Methods and Decision and Optimization Methods. He authored many articles in international journals and international conferences, coordinated various research and applied projects developed in close cooperation with industrial and service companies.