

# Customer Segmentation Based on Fuzzy C-Means and Weighted Interval-Valued Dual Hesitant Fuzzy Sets

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## Abstract

Marketing strategies have been one of the top considerations in commercial companies due to the effect on business sales lead to the revenue. In order to build an effective marketing strategy, companies need understand of customer's preferences and needs. With the digital marketing, many companies are able to have customers around the world. Managing and analyzing the customer data in large scale and transform it to useful information is necessary for supporting to business under uncertain environment. This study aims to provide a new two-phase framework for segmenting customers based on the RFM model. Phase 1 implements fuzzy C mean clustering algorithms to segment the customers. Phase 2 focuses on ranking customer segment using weighted interval-valued dual hesitant fuzzy sets which allow to integrated opinions of groups decision makers even different knowledge and points of view when giving judgments. A numerical example in e-commerce is provided to illustrate the proposed method for solving practical problems. The results obtained from the model can be used to develop object-oriented marketing strategies or to develop customer relationship management campaigns.

## Keywords

Customer segmentation, RFM model, fuzzy c-means clustering, weighted interval-valued dual hesitant fuzzy sets, multi-criteria group decision making.

## 1. Introduction

Utilizing the Internet, more individuals can profit from online administrations, including e Commerce. With the support of technology applications, E-commerce has gradually become a daily shopping habit of consumers around the world. According to stastista.com, they predict about 22% of the world's retailers are online and online sales will reach \$5 trillion by the end of 2021 ("Global retail e-commerce market size 2014-2023 | Statista", 2021). The advantage of this form of business makes transactions much easier because of increasing accessibility of the Internet. In addition, online business allows collecting information of huge number of customers that useful for developing and executing marketing strategies.

Customer segmentation is a preferable tool that help business orient strategies for different groups of customers. Based on the customer data, companies can extract the customer information depend on their marketing strategies. Customer data, identifying with distinct needs, characteristics or behaviors, are categorized into homogenous groups. The main purpose of segment is to understand the current customers and to reach potential customers. Companies can bring right products, right time, right price to right customers.

With the huge number of customers, E-commerce, banking, hospitality have been applied to segment customer for different marketing strategies. Moreover, ranking the segments can be used for suggesting group target customers. In ranking process, opinions of group of decision makers directly influence to final ranking results. In recent years, although many the frameworks have been proposed to segment and rank segment customers, but most

of the ranking methods focus on degree of certain in evaluation of decision makers. In reality, group decision makers, various background, knowledge, manpower, work positions, may be hesitant when giving their judgements or giving no idea. How to combine all opinions of group decision makers for these cases.

### 1.1 Objectives

To overcome the above mentioned problem, this paper aims to suggest the two-phase framework for classifying and ranking group of customers based on Recency, Frequency, and Monetary (RFM) model, by using fuzzy C mean (FCM) and weighted interval-valued dual hesitant fuzzy set (WIVDHFS) that can be used to represent degree of certain, uncertain and importance of decision makers. Additionally, a numerical example has been provided to illustrate merits of the proposed framework.

## 2. Literature Review

Customer segmentation is the first step in analyzing customer behavior, which is the information platform to develop marketing strategies to attract potential customers. Depending on the purpose of segmentation, development models use different types of variables to evaluate customers, including demographic, geographic, psychographics and behavioral considerations (Wu and Chou 2011). Later, for the purpose of making objective assessments, researchers focused on analyzing historical data to make predictions about the future demand. Recency, Frequency, and Monetary variables from the purchased product are preferred (Fader et al. 2016). RFM model is widely applied to calculate customer lifelong value, which is an indicator to assess the loyalty of customers (Khajvand et al. 2011).

Clustering algorithms are preferred for dividing customers into subgroups. Hsu et al. (2012) proposed a method of applying hierarchical clustering and demonstrated that the results outperform compared to the traditional methods. However, using hierarchical concept can be cumbersome when it is applied to large data sets. K-means clustering is one of the most popular methods used, because of its simplicity, ease of understanding, and efficiency in giving clustering results (Cheng and Chen 2009).

For each type of business there are different preferences about variables, and clustering is ignored the level of importance of segments, some studies combined multi-criteria decision making (MCDM) methods on the purpose of conforming different situations. Safari and Aggarwal (2020) used Fuzzy AHP to aggregate the opinions of decision makers to determine the weightage value of the RFM variables proved the combination of genetic algorithms and FCM gives a lower means square error compared to the methods that use only FCM.

Many papers proposed different methods to execute customer segmentation, notably clustering, classifying, artificial neural network (ANN), and self-organizing maps (SOM) techniques (Ozan & Ithme, 2019), (Kameoka et al., 2015). Besides, the RFM model is widely applied by the scholars to accomplish the segmentation.

### 2.1 RFM Model

RFM model, an analysis based on three parameters Recency (R), Frequency (F), and Monetary (M), has been widely used in many practical areas for its high applicability. RFM model can be used to calculate the customer lifetime value (CLV), segment customers, observe customer behavior, evaluate responsiveness, and evaluate customer online reviews. Liu and Shil (2005) integrated AHP in the model to determine the relative weightage value of RFM parameters. This model is referred as Weighted RFM or WRFM as weight is an influential variable.

The RFM model can be extended by combining with other variables or other models. On the attempt to improve the RFM model, some papers have added more variables to test whether the new method performs better than the original. Buckinx and Poel (2005) proposed a model to predict the partial disappearance of behavioral loyalty customers of FMCG industry using three classification techniques: logistic regression, automatic relevance determination (ARD) Neural Network and Random Forest. Yeh et al. (2009) introduced the RFMTC model, considering time since first purchase and churn probability. The study concluded that the extended method provides more accurate results. Peker et al. (2017) applied LRFMP in grocery retail industry which the two addition variables are length and periodicity.

### 2.2 Clustering algorithm

Clustering is a method of dividing a dataset of  $n$  objects into  $c$  groups. Condition for grouping is similar properties. Clustering can be achieved by various algorithms, depends on the efficiency of the result. FCM algorithm is an extension of K-means algorithm using fuzzy numbers instead of real numbers. The fuzzy logic allows an object belong to more than one clusters, but assign membership degree. Bezdek (1973) the generalized concept of fuzzification value  $m$  is greater than 1. Later, Dunn (1974) considered a special case that  $m$  is equal to 2.

## 2.3 Fuzzy Sets and its developed methods

Since Zadeh (1965) established the term of fuzzy sets, it has been widely applied in many fields of study, including multi-criteria decision making. At the same time, he developed the concept of interval-valued fuzzy sets (IVFS), which determines the membership value by an interval. Atanassov (1986) suggested the definition of intuitionistic fuzzy set (IFS), and Atanassov and Gargov (1989) extended it extended by combining IVFS and IFS and denoted it as interval-valued intuitionistic fuzzy sets (IVIFS). In reality, hesitancy as evaluating an issue is frequently happen because of limitation knowledge or different points of view.

In 2010, Torra proposed the concept of hesitant fuzzy set (HFS) which then opened to many studies about the development and applications of HFS. The advantage of HFS is to allow decision makers to give a set of membership degrees. A year later, Xia and Xu (2011) investigated the aggregation operators of HFS and proposed its application in decision-making. The introduction of concept of dual hesitant fuzzy sets (DHFS) was done by Zhu, Xu and Xia (2012), which was the combination of HFS and intuitionistic fuzzy sets (IFS).

Similar to IFS, DHFS contains membership and non-membership function, where each function is based on the concept of HFS. Zhu & Xu (2014) investigated some operations and properties of DHFS. Yu, Li, & Merigó (2015) applied DHFS in resolving a group decision making problem. Chen et al. (2013) developed the term of interval-valued hesitant fuzzy set (IVHFS), which allows the membership function consisting of interval elements. The weightage values are involved when determining the preferences among assessments are put in consideration. Zeng et al. (2019) introduced the term of weighted dual hesitant fuzzy sets and provided an application in group decision making.

## 3. Methodology

In this section, the three-phase framework is proposed for segmenting and ranking fuzzy numbers (see Figure 1). In Phase 1, the raw customer data is transformed under R, F, M variables using RFM model. In Phase 2, Fuzzy C-means is applied to divide customers into clusters, where customers have similar characteristics within one cluster and the PBMF indicates the optimal number of clusters. Finally, WIVDHFS is applied to rank the customer clusters and choose the most potential cluster in Phase 3.

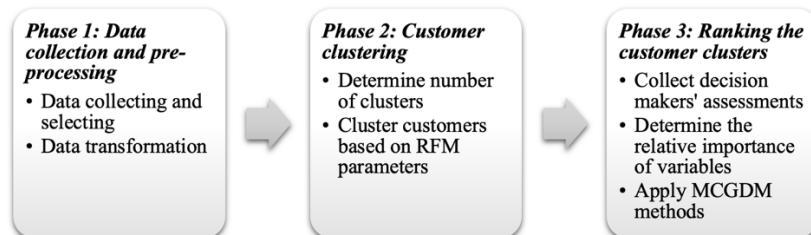


Figure 1: Proposed methodology flowchart

### 3.1 RFM model

In the first phase, the information is used for the transformation including customer ID, date / time, and amount of transaction. The value of Recency is measured by the time difference from the last purchase to the present. Frequency is the number of transactions a customer made in the examined duration. Monetary is the average amount that a customer made on each transaction. We then aggregate the records based on customer ID. Hence, each customer is considered as one data point and the value of each data point is the 3 variables R, F, M. The data after being transformed become inputs of the second phase.

### 3.2 Fuzzy C-means clustering

In the second phase, the RFM data set is clustered by FCM algorithm. The purpose of this stage is to classify customers into groups, whereby customers in each group have similar R, F and M values.

Let  $X = \{x | x \in X\}$  be the set of records that are going to be segmented.  $N$  is the number of objects in  $X$ ,  $c$  is the number of clusters,  $m$  is the fuzzy partition matrix exponent, with  $m > 1$  ( $m = 1$  is for crisp clustering). In this calculation, we set  $m = 2$ .

The goal of FCM method is to minimize the objective function:

$$J_m(U, v) = \sum_{k=1}^N \sum_{i=1}^c u_{ik}^m d_{ik}^2 \quad (1)$$

The value of  $J_m$  indicates the sum of intra-cluster distances, where the intra-cluster distance is the variance within a cluster, therefore, minimizing the  $J_m$  is equivalent to increasing the similarity among elements in one cluster. In the opposite direction, the clustering result can be obtained by maximizing the inter-cluster difference, using another objective function.

Euclidean distance is calculated by:

$$d_{ik}^2 = \|x_k - v_i\|^2 \quad (2)$$

The membership value of data point  $k^{\text{th}}$  to the  $i^{\text{th}}$  cluster is identified as:

$$u_{ik} = \frac{1}{\sum_{j=1}^c \left(\frac{d_{ik}}{d_{jk}}\right)^{\frac{2}{m-1}}} \quad (3)$$

The cluster centroids are updated according to:

$$v_i = \frac{\sum_{k=1}^n u_{ik}^m \times x_k}{\sum_{k=1}^n u_{ik}^m} \quad (4)$$

### 3.3 PBMF index

The objective of PBMF index is to determine an appropriate number of clusters, as if there are too few clusters, the differences of members in a cluster are too big, but if there are too many clusters, it would be difficult to distinguish one cluster to the others. Maximization of the PBMF index ensures that the partition has a small number of clusters with large separation between at least 2 of them. PBMF index is calculated by:

$$PBMF = \left(\frac{1}{c} \times \frac{E_1}{J_m} \times D_c\right)^2 \quad (5)$$

Where  $E_1$  is the value of the objective function  $J_m$  when the number of clusters is set as 1. Hence, it is a constant for a given data set and is set for the purpose of preventing the index too small in value.

$$E_1 = \sum_{k=1}^n u_k \|x_k - v\| \quad (6)$$

$D_c$  denotes the largest value of the distance between 2 clusters in the set of  $c$  clusters.

$$D_c = \max_{i,j=1}^c \|d(v_i - v_j)\| \quad (7)$$

As can be seen from the equation, the PBMF validity index uses 3 factors: (1) divisibility of clusters ( $1/c$ ), (2) intra-cluster distances ( $E_1/J_m$ ), (3) maximum separation ( $D_c$ ).

The first factor decreases as  $c$  increases, hence, it reduces the value of the index. The second factor has the  $E_1$  value is constant for a fixed data set, and  $J_m$  decreases as  $c$  increases, and then the ratio  $E_1/J_m$  increases as  $c$  increases. The third factor consists only  $D_c$  increasing as  $c$  increases. However,  $D_c$  may be stable if the increase of  $c$  does not influence the two most separated clusters. The case happens when the division of clusters occurs in the middle of them. Overall, we want to increase the similarity within one cluster and the separation among clusters, on the other hand, the number of clusters should be as small as possible to keep the division simple.

### 3.4 Weighted interval-valued dual hesitant fuzzy sets

In the last phase, the clusters are ranked based on experts' assessments using WIVDHFS. This method helps to aggregate expert judgments for each cluster and gives a real number for each cluster to compare with each other. For a given fixed set  $X$ , the WIVDHFS on  $X$  can be described as

$$D = \{(x, h_D(x), g_D(x)) \mid x \in X\} \quad (8)$$

Where  $h_D(x) = \cup_{(\gamma, w_\gamma) \in h_D(x)} \{(\gamma, w_\gamma)\}$  and  $g_D(x) = \cup_{(\eta, w_\eta) \in g_D(x)} \{(\eta, w_\eta)\}$  denote the satisfaction and dissatisfaction degrees, respectively, of the element  $x \in X$  to the set  $D$ , in which  $\gamma = [\gamma^l, \gamma^u]$  and  $\eta = [\eta^l, \eta^u]$  are two sets of possible interval values in  $[0, 1]$ .  $w_\gamma$  and  $w_\eta$  are the corresponding weights for the two types of degrees.

An element of a WIVDHFS is called weighted interval-valued dual hesitant fuzzy element, denoted as  $d(x) = \langle h_d(x), g_d(x) \rangle$  or marked by  $d = \langle h_d, g_d \rangle$  for short.

### The comparison of WIVDHFES

Let  $d = \langle h_d, g_d \rangle$  be a WIVDHFES, then the score function is given as follows:

$$S(d) = \frac{1}{2} \left( \frac{1}{\#h_d} \sum_{\gamma \in h_d} (\gamma^l + \gamma^u) w_\gamma - \frac{1}{\#g_d} \sum_{\eta \in g_d} (\eta^l + \eta^u) w_\eta \right) \quad (9)$$

The accuracy function is given as:

$$A(d) = \frac{1}{2} \left( \frac{1}{\#h_d} \sum_{\gamma \in h_d} (\gamma^l + \gamma^u) w_\gamma + \frac{1}{\#g_d} \sum_{\eta \in g_d} (\eta^l + \eta^u) w_\eta \right) \quad (10)$$

Let  $d_1 = \langle h_{d_1}, g_{d_1} \rangle$  and  $d_2 = \langle h_{d_2}, g_{d_2} \rangle$  be the two WIVDHFES of set  $D$ , the comparison between them using the score function and accuracy function is explained as follows:

- (i) If  $S(d_1) > S(d_2)$ , then  $d_1 > d_2$
- (ii) If  $S(d_1) < S(d_2)$ , then  $d_1 < d_2$
- (iii) If  $S(d_1) = S(d_2)$ , then:
  - (a) If  $A(d_1) > A(d_2)$ , then  $d_1 > d_2$
  - (b) If  $A(d_1) < A(d_2)$ , then  $d_1 < d_2$
  - (c) If  $A(d_1) = A(d_2)$ , then  $d_1 = d_2$

The weighted interval-valued dual hesitant fuzzy weighted averaging (WIVDHFWA) operator is defined as follows:

$$WIVDHFWA(d_1, d_2, \dots, d_n) = \bigoplus_{j=1}^n (\omega_j d_j) = \left\langle \bigcup_{(\gamma_j, \omega_j) \in h_{d_j}} \left\{ \left( [1 - \prod_{j=1}^n (1 - \gamma_j^l)^{\omega_j}, 1 - \prod_{j=1}^n (1 - \gamma_j^u)^{\omega_j}] \right) \right\}, \bigcup_{(\eta_j, \omega_j) \in g_{d_j}} \left\{ \left( [\prod_{j=1}^n (\eta_j^l)^{\omega_j}, \prod_{j=1}^n (\eta_j^u)^{\omega_j}], \prod_{j=1}^n w_{\eta_{d_j}} \right) \right\} \right\rangle \quad (11)$$

The weighted interval-valued dual hesitant fuzzy weighted geometric (WIVDHFVG) operator is defined as follows:

$$WIVDHFVG(d_1, d_2, \dots, d_n) = \bigotimes_{j=1}^n (d_j)^{\omega_j} = \left\langle \bigcup_{(\gamma_j, \omega_j) \in h_{d_j}} \left\{ \left( [\prod_{j=1}^n (\gamma_j^l)^{\omega_j}, \prod_{j=1}^n (\gamma_j^u)^{\omega_j}], \prod_{j=1}^n w_{\gamma_{d_j}} \right) \right\}, \bigcup_{(\eta_j, \omega_j) \in g_{d_j}} \left\{ \left( [1 - \prod_{j=1}^n (1 - \eta_j^l)^{\omega_j}, 1 - \prod_{j=1}^n (1 - \eta_j^u)^{\omega_j}], \prod_{j=1}^n w_{\eta_{d_j}} \right) \right\} \right\rangle \quad (12)$$

## 4. Numerical example

To illustrate the segment and ranking in the proposed method, this study collects 4317 customer transaction records from UK-based online retail in the duration of 21 months (from January 12, 2010 to September 12, 2011), ("E-Commerce Data", 2021). Table 1 shows 8 types of factors considered for segmenting customers. These factors are integrated into R, F, M variables in RFM model, as can be in Table 2. Assume that 4 experts  $E_k$  ( $k = 1, 2, 3, 4$ ) are invited to judge the 4 segments of customers denoted as  $A_i$  ( $i = 1, 2, 3, 4$ ) based on 3 attributes:  $C_1$ - customers purchase recently;  $C_2$ - customers purchase frequently; and  $C_3$ - the value of customer purchase. The attribute weight vector of  $C_j$  ( $j = 1, 2, 3$ ) is  $\omega = (0.4, 0.3, 0.3)^T$  and the weight vector of 4 experts is  $\lambda = (0.3, 0.2, 0.3, 0.2)^T$  based on the importance of experts  $E_k$  assess of alternative  $A_i$  under attribute  $C_j$  to be constructed as an interval-valued intuitionistic fuzzy number  $r_{ij}^k = \left( [(\gamma_{ij}^k)^l, (\gamma_{ij}^k)^u], [(\eta_{ij}^k)^l, (\eta_{ij}^k)^u] \right)$ . The assessment information given by the experts for the four customer segments are shown in Table 3, 4, 5 and 6.

Table 1: A part of the transaction data

Invoice No	Stock Code	Description	Quantity	Invoice Date	Unit price	Customer ID	Country
536365	85123A	White hanging heart T-light holder	6	12/1/2010	2.55	17850	United Kingdom

536365	71053	White metal lantern	6	12/1/2010	3.39	17850	United Kingdom
536365	84406B	Cream cupid hearts coat hanger	8	12/1/2010	2.75	17850	United Kingdom
536365	84429G	Knitted union flag hot water	6	12/1/2010	3.39	17850	United Kingdom
536365	84029E	Red woolly hottie white heart	6	12/1/2010	3.39	17850	United Kingdom

Table 2: Data transacted into RFM model

Customer ID	Day since (R)	Count of Invoice No (F)	Sum of Amount (M)
12347	24	182	4310
12348	97	31	1797.24
12349	41	73	1757.55
12350	332	17	334.4
12352	58	95	1545.41

Table 3: The assessments of expert  $E_1$  for every cluster under each criterion

	$C_1$	$C_2$	$C_3$
$A_1$	$\{([0.5, 0.6], [0.2, 0.3]), 0.3\}$	$\{([0.4, 0.6], [0.3, 0.4]), 0.3\}$	$\{([0.4, 0.5], [0.4, 0.5]), 0.3\}$
$A_2$	$\{([0.3, 0.4], [0.5, 0.6]), 0.3\}$	$\{([0.3, 0.5], [0.3, 0.5]), 0.3\}$	$\{([0.3, 0.5], [0.3, 0.4]), 0.3\}$
$A_3$	$\{([0.6, 0.7], [0.2, 0.3]), 0.3\}$	$\{([0.5, 0.6], [0.1, 0.3]), 0.3\}$	$\{([0.7, 0.8], [0.1, 0.2]), 0.3\}$
$A_4$	$\{([0.1, 0.3], [0.3, 0.5]), 0.3\}$	$\{([0.3, 0.4], [0.4, 0.5]), 0.3\}$	$\{([0.3, 0.5], [0.4, 0.5]), 0.3\}$

Table 4: The assessments of expert  $E_1$  for every cluster under each criterion

	$C_1$	$C_2$	$C_3$
$A_1$	$\{([0.6, 0.7], [0.2, 0.3]), 0.2\}$	$\{([0.5, 0.6], [0.2, 0.3]), 0.2\}$	$\{([0.5, 0.6], [0.2, 0.3]), 0.2\}$
$A_2$	$\{([0.3, 0.4], [0.5, 0.6]), 0.2\}$	$\{([0.3, 0.5], [0.3, 0.4]), 0.2\}$	$\{([0.4, 0.6], [0.3, 0.4]), 0.2\}$
$A_3$	$\{([0.6, 0.8], [0.1, 0.2]), 0.2\}$	$\{([0.6, 0.7], [0.2, 0.3]), 0.2\}$	$\{([0.7, 0.8], [0.1, 0.2]), 0.2\}$
$A_4$	$\{([0.2, 0.4], [0.4, 0.5]), 0.2\}$	$\{([0.4, 0.6], [0.3, 0.4]), 0.2\}$	$\{([0.4, 0.5], [0.4, 0.5]), 0.2\}$

Table 5: The assessments of expert  $E_1$  for every cluster under each criterion

	$C_1$	$C_2$	$C_3$
$A_1$	$\{([0.4, 0.6], [0.1, 0.3]), 0.3\}$	$\{([0.4, 0.5], [0.3, 0.4]), 0.3\}$	$\{([0.3, 0.5], [0.2, 0.3]), 0.3\}$
$A_2$	$\{([0.3, 0.5], [0.4, 0.5]), 0.3\}$	$\{([0.5, 0.6], [0.3, 0.4]), 0.3\}$	$\{([0.3, 0.4], [0.4, 0.5]), 0.3\}$
$A_3$	$\{([0.5, 0.7], [0.2, 0.3]), 0.3\}$	$\{([0.6, 0.8], [0.1, 0.2]), 0.3\}$	$\{([0.6, 0.7], [0.1, 0.3]), 0.3\}$
$A_4$	$\{([0.2, 0.4], [0.3, 0.5]), 0.3\}$	$\{([0.3, 0.5], [0.4, 0.5]), 0.3\}$	$\{([0.3, 0.4], [0.4, 0.6]), 0.3\}$

Table 6: The assessments of expert  $E_1$  for every cluster under each criterion

	$C_1$	$C_2$	$C_3$
$A_1$	$\{([0.5, 0.7], [0.1, 0.3]), 0.2\}$	$\{([0.4, 0.6], [0.3, 0.4]), 0.2\}$	$\{([0.4, 0.6], [0.2, 0.3]), 0.2\}$
$A_2$	$\{([0.4, 0.6], [0.2, 0.3]), 0.2\}$	$\{([0.4, 0.6], [0.2, 0.3]), 0.2\}$	$\{([0.3, 0.5], [0.2, 0.4]), 0.2\}$
$A_3$	$\{([0.6, 0.8], [0.1, 0.2]), 0.2\}$	$\{([0.5, 0.6], [0.2, 0.3]), 0.2\}$	$\{([0.6, 0.7], [0.1, 0.2]), 0.2\}$
$A_4$	$\{([0.3, 0.5], [0.3, 0.4]), 0.2\}$	$\{([0.3, 0.5], [0.3, 0.5]), 0.2\}$	$\{([0.3, 0.4], [0.5, 0.6]), 0.2\}$

## 5. Results

From Phase 2, the PBMF index values and cluster centroids are obtained by using fuzzy C-mean clustering. As can be seen from Figure 2, the PBMF value increases from the beginning and reaches the peak at 4 clusters. Since  $k = 5$  onwards, the index value tends to decrease gradually and is close to 0 at  $k = 10$ . Obviously, the optimal number of clusters is 4. The centroids of 4 clusters are shown in Figure 3 with 4 coordinates:  $\{(40,420, 15120), (60,190, 3380), (30,1910, 123420), (130,50, 680)\}$

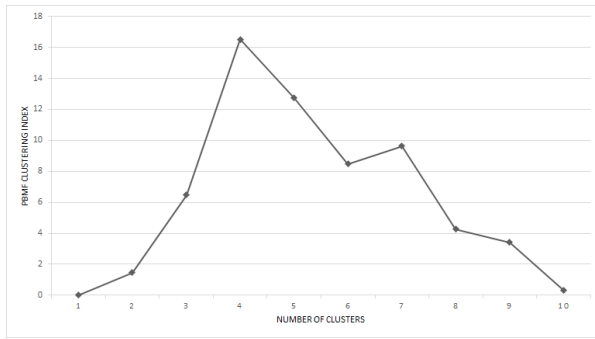


Figure 2: PMBF index values for different numbers of cluster

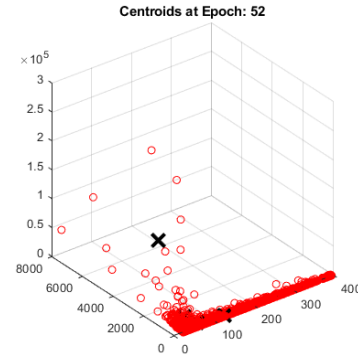


Figure 3: Position of cluster centroids in the dataset

In Phase 3, WIVDHFWA and WIVDHFWD operators are applied to aggregate the assessments on 3 criteria and define a WIVDHFV to each cluster value. The fused values of each cluster follow the format  $A_i = \{\{\text{satisfaction interval values, weights}\}, \{\text{dissatisfaction interval values, weights}\}\}$ .

Applying WIVDHFWA operator, the fused values of each cluster indicating as below:

$$A_1 = \left\{ \begin{array}{l} ([0.4422, 0.5723], 0.045), ([0.4719, 0.6], 0.03), ([0.4158, 0.5723], 0.045), \\ ([0.4422, 0.6], 0.03), ([0.4719, 0.5723], 0.018), ([0.5, 0.6], 0.012), \\ ([0.4469, 0.5723], 0.018), ([0.4719, 0.6], 0.012), ([0.4422, 0.5427], 0.027), \\ ([0.4719, 0.5723], 0.018), ([0.4158, 0.5427], 0.027), ([0.4422, 0.5723], 0.018), \\ ([0.4898, 0.6188], 0.03), ([0.5170, 0.6435], 0.02), ([0.4657, 0.6188], 0.03), \\ ([0.4898, 0.6435], 0.02), ([0.5170, 0.6188], 0.012), ([0.5427, 0.6435], 0.008), \\ ([0.4941, 0.6188], 0.012), ([0.5170, 0.6435], 0.008), ([0.4898, 0.5924], 0.018), \\ ([0.5170, 0.6188], 0.012), ([0.4657, 0.5924], 0.018), ([0.4898, 0.6188], 0.012), \\ ([0.4, 0.5723], 0.045), ([0.4319, 0.6], 0.03), ([0.3716, 0.5723], 0.045), \\ ([0.4, 0.6], 0.03), ([0.4319, 0.5723], 0.018), ([0.4622, 0.6], 0.012), \\ ([0.4050, 0.5723], 0.018), ([0.4319, 0.6], 0.012), ([0.4, 0.5427], 0.027), \\ ([0.4319, 0.5723], 0.018), ([0.3716, 0.5427], 0.027), ([0.4, 0.5723], 0.018), \\ ([0.4422, 0.6188], 0.03), ([0.4719, 0.6435], 0.02), ([0.4158, 0.6188], 0.03), \\ ([0.4422, 0.6435], 0.02), ([0.4719, 0.6188], 0.012), ([0.5, 0.6435], 0.008), \\ ([0.4469, 0.6188], 0.012), ([0.4719, 0.6435], 0.008), ([0.4422, 0.5924], 0.018), \\ ([0.4719, 0.6188], 0.012), ([0.4158, 0.5924], 0.018), ([0.4422, 0.6188], 0.012) \\ \left. \begin{array}{l} ([0.2781, 0.3812], 0.12), ([0.2259, 0.3270], 0.28), ([0.2462, 0.3497], 0.03), \\ ([0.2, 0.3], 0.07), ([0.2107, 0.3812], 0.12), ([0.1712, 0.3270], 0.28), \\ ([0.1866, 0.3497], 0.03), ([0.1516, 0.3], 0.07) \end{array} \right\}$$

$$\begin{aligned}
 A_2 = & \left\{ \begin{aligned} & ([0.3, 0.4622], 0.125), ([0.3316, 0.4970], 0.05), ([0.3, 0.4319], 0.075), \\ & ([0.3672, 0.4970], 0.075), ([0.3958, 0.5296], 0.03), ([0.3672, 0.4687], 0.045), \\ & ([0.3316, 0.4970], 0.05), ([0.3618, 0.5296], 0.02), ([0.3316, 0.4687], 0.03), \\ & ([0.3, 0.5], 0.075), ([0.3316, 0.5324], 0.03), ([0.3, 0.4719], 0.045), \\ & ([0.3672, 0.5324], 0.045), ([0.3958, 0.5627], 0.018), ([0.3672, 0.5061], 0.027), \\ & ([0.3316, 0.5324], 0.03), ([0.3618, 0.5627], 0.012), ([0.3316, 0.5061], 0.018), \\ & ([0.3419, 0.5427], 0.05), ([0.3716, 0.5723], 0.02), ([0.3419, 0.5170], 0.03), \\ & ([0.4060, 0.5723], 0.03), ([0.4319, 0.6], 0.012), ([0.4050, 0.5483], 0.018), \\ & ([0.3716, 0.5723], 0.02), ([0.4, 0.6], 0.008), ([0.3716, 0.5483], 0.012) \end{aligned} \right\}, \\
 & \left\{ \begin{aligned} & ([0.3680, 0.5030], 0.075), ([0.4012, 0.5378], 0.045), ([0.3259, 0.5030], 0.03), \\ & ([0.3680, 0.4704], 0.125), ([0.4012, 0.5030], 0.075), ([0.3259, 0.4704], 0.05), \\ & ([0.3259, 0.4315], 0.05), ([0.3552, 0.4614], 0.03), ([0.2885, 0.4315], 0.02), \\ & ([0.3366, 0.3812], 0.045), ([0.3669, 0.4076], 0.027), ([0.2980, 0.3812], 0.018), \\ & ([0.3366, 0.4373], 0.075), ([0.3669, 0.4676], 0.045), ([0.2980, 0.4373], 0.03), \\ & ([0.2980, 0.4012], 0.03), ([0.3249, 0.4290], 0.018), ([0.2639, 0.4012], 0.012), \\ & ([0.2551, 0.3812], 0.03), ([0.2781, 0.4076], 0.018), ([0.2259, 0.3812], 0.012), \\ & ([0.2551, 0.3565], 0.05), ([0.2781, 0.3812], 0.03), ([0.2259, 0.3565], 0.02), \\ & ([0.2259, 0.3270], 0.02), ([0.2462, 0.3497], 0.012), ([0.2, 0.3270], 0.008) \end{aligned} \right\} \\
 A_3 = & \left\{ \begin{aligned} & ([0.6077, 0.7104], 0.075), ([0.5723, 0.6730], 0.075), ([0.6331, 0.7344], 0.03), \\ & ([0.6, 0.7], 0.03), ([0.6331, 0.7648], 0.03), ([0.6, 0.7344], 0.03), \\ & ([0.6077, 0.7538], 0.1), ([0.5723, 0.7219], 0.1), ([0.6331, 0.7741], 0.04), \\ & ([0.6, 0.7449], 0.04), ([0.6331, 0.8], 0.04), ([0.6, 0.7741], 0.04), \\ & ([0.5710, 0.7104], 0.075), ([0.5324, 0.6730], 0.075), ([0.5988, 0.7344], 0.03), \\ & ([0.5627, 0.7], 0.03), ([0.5988, 0.7648], 0.03), ([0.5627, 0.7344], 0.03) \end{aligned} \right\}, \\
 & \left\{ \begin{aligned} & ([0.1320, 0.2656], 0.126), ([0.1320, 0.3], 0.054), ([0.1625, 0.2656], 0.168), \\ & ([0.1625, 0.3], 0.072), ([0.1320, 0.2352], 0.126), ([0.1320, 0.2656], 0.054), \\ & ([0.1, 0.2259], 0.084), ([0.1, 0.2551], 0.036), ([0.1231, 0.2259], 0.112), \\ & ([0.1231, 0.2551], 0.048), ([0.1, 0.2], 0.084), ([0.1, 0.2259], 0.036) \end{aligned} \right\} \\
 A_4 = & \left\{ \begin{aligned} & ([0.2260, 0.3958], 0.027), ([0.2610, 0.3958], 0.018), ([0.2260, 0.3618], 0.045), \\ & ([0.2610, 0.4650], 0.018), ([0.2944, 0.4650], 0.012), ([0.2610, 0.4349], 0.03), \\ & ([0.2260, 0.4280], 0.045), ([0.2610, 0.4280], 0.03), ([0.2260, 0.3958], 0.075), \\ & ([0.2616, 0.4319], 0.045), ([0.2950, 0.4319], 0.03), ([0.2616, 0.4], 0.075), \\ & ([0.2950, 0.4970], 0.03), ([0.3268, 0.4970], 0.02), ([0.2950, 0.4687], 0.05), \\ & ([0.2616, 0.4622], 0.075), ([0.2950, 0.4622], 0.05), ([0.2616, 0.4319], 0.125), \\ & ([0.3, 0.4719], 0.018), ([0.3316, 0.4719], 0.012), ([0.3, 0.4422], 0.03), \\ & ([0.3316, 0.5324], 0.012), ([0.3618, 0.5324], 0.008), ([0.3316, 0.5061], 0.02), \\ & ([0.3, 0.5], 0.03), ([0.3316, 0.5], 0.02), ([0.3, 0.4719], 0.05) \end{aligned} \right\}, \\
 & \left\{ \begin{aligned} & ([0.3565, 0.5], 0.18), ([0.3565, 0.5281], 0.108), ([0.3812, 0.5281], 0.072), \\ & ([0.3270, 0.4676], 0.06), ([0.3270, 0.4939], 0.036), ([0.3497, 0.4939], 0.024), \\ & ([0.3270, 0.5], 0.06), ([0.3270, 0.5281], 0.036), ([0.3497, 0.5281], 0.024), \\ & ([0.4, 0.4573], 0.06), ([0.4, 0.4830], 0.036), ([0.4277, 0.4830], 0.024), \\ & ([0.3669, 0.4676], 0.02), ([0.3669, 0.4939], 0.012), ([0.3923, 0.4939], 0.008), \\ & ([0.3669, 0.5], 0.02), ([0.3669, 0.5281], 0.012), ([0.3923, 0.5281], 0.008), \\ & ([0.3565, 0.4573], 0.06), ([0.3565, 0.4830], 0.036), ([0.3812, 0.4830], 0.024), \\ & ([0.3270, 0.4277], 0.02), ([0.3270, 0.4517], 0.012), ([0.3497, 0.4517], 0.008), \\ & ([0.3270, 0.4573], 0.02), ([0.3270, 0.4830], 0.012), ([0.3497, 0.4830], 0.008) \end{aligned} \right\}
 \end{aligned}$$

Similarly, the fused values of each cluster resulted by WIVDHFWDG indicating as below:



$$A_1 = \left\{ \begin{array}{l} ([0.4373, 0.5681], 0.045), ([0.4676, 0.6], 0.03), ([0.4012, 0.5681], 0.045), \\ ([0.4373, 0.6], 0.03), ([0.4676, 0.5681], 0.018), ([0.5, 0.6], 0.012), \\ ([0.4290, 0.5681], 0.018), ([0.4676, 0.6], 0.012), ([0.4373, 0.5378], 0.027), \\ ([0.4676, 0.5681], 0.018), ([0.4012, 0.5378], 0.027), ([0.4373, 0.5681], 0.018), \\ ([0.4704, 0.6042], 0.03), ([0.5030, 0.6382], 0.02), ([0.4315, 0.6042], 0.03), \\ ([0.4704, 0.6042], 0.02), ([0.5030, 0.6042], 0.012), ([0.5378, 0.6382], 0.008), \\ ([0.4614, 0.6042], 0.012), ([0.5030, 0.6382], 0.008), ([0.4704, 0.5720], 0.018), \\ ([0.5030, 0.6042], 0.012), ([0.4315, 0.5720], 0.018), ([0.4704, 0.6042], 0.012), \\ ([0.4, 0.5681], 0.045), ([0.4277, 0.6], 0.03), ([0.3669, 0.5681], 0.045), \\ ([0.4, 0.6], 0.03), ([0.4277, 0.5681], 0.018), ([0.4573, 0.6], 0.012), \\ ([0.3923, 0.5681], 0.018), ([0.4277, 0.6], 0.012), ([0.4, 0.5378], 0.027), \\ ([0.4277, 0.5681], 0.018), ([0.3669, 0.5378], 0.027), ([0.4, 0.5681], 0.018), \\ ([0.4374, 0.6042], 0.03), ([0.4676, 0.6382], 0.02), ([0.4012, 0.6042], 0.03), \\ ([0.4373, 0.6382], 0.02), ([0.4676, 0.6042], 0.012), ([0.5, 0.6382], 0.008), \\ ([0.4290, 0.6042], 0.012), ([0.4676, 0.6382], 0.008), ([0.4373, 0.5720], 0.018), \\ ([0.4676, 0.6042], 0.012), ([0.4012, 0.5720], 0.018), ([0.4373, 0.6042], 0.012) \end{array} \right\},$$

$$\left\{ \begin{array}{l} ([0.2950, 0.3958], 0.12), ([0.2314, 0.3316], 0.28), ([0.2661, 0.3672], 0.03), \\ ([0.2, 0.3], 0.07), ([0.2610, 0.3958], 0.12), ([0.1943, 0.3316], 0.28), \\ ([0.2307, 0.3672], 0.03), ([0.1614, 0.3], 0.07) \end{array} \right\}$$

$$A_2 = \left\{ \begin{array}{l} ([0.3, 0.4573], 0.125), ([0.3270, 0.4830], 0.05), ([0.3, 0.4277], 0.075), \\ ([0.3497, 0.4830], 0.075), ([0.3812, 0.5102], 0.03), ([0.3497, 0.4517], 0.045), \\ ([0.3270, 0.4830], 0.05), ([0.3565, 0.5102], 0.02), ([0.3270, 0.4517], 0.03), \\ ([0.3, 0.5378], 0.075), ([0.3270, 0.5681], 0.03), ([0.3, 0.5030], 0.045), \\ ([0.3497, 0.5281], 0.045), ([0.3812, 0.5578], 0.018), ([0.3497, 0.4939], 0.027), \\ ([0.3270, 0.5281], 0.03), ([0.3565, 0.5578], 0.012), ([0.3270, 0.4939], 0.018), \\ ([0.3366, 0.5378], 0.05), ([0.3669, 0.5681], 0.02), ([0.3366, 0.5030], 0.03), \\ ([0.3923, 0.5681], 0.03), ([0.4277, 0.6], 0.012), ([0.3923, 0.5313], 0.018), \\ ([0.3669, 0.5681], 0.02), ([0.4, 0.6], 0.008), ([0.3669, 0.5313], 0.012) \end{array} \right\},$$

$$\left\{ \begin{array}{l} ([0.3881, 0.5170], 0.075), ([0.4158, 0.5427], 0.045), ([0.3631, 0.5170], 0.03), \\ ([0.3881, 0.4898], 0.125), ([0.4158, 0.5170], 0.075), ([0.3631, 0.4898], 0.05), \\ ([0.3631, 0.4657], 0.05), ([0.3919, 0.4941], 0.03), ([0.3371, 0.4657], 0.02), \\ ([0.3419, 0.4719], 0.045), ([0.3716, 0.5], 0.027), ([0.3150, 0.4719], 0.018), \\ ([0.3419, 0.4422], 0.075), ([0.3716, 0.4719], 0.045), ([0.3150, 0.4422], 0.03), \\ ([0.3150, 0.4158], 0.03), ([0.3459, 0.4469], 0.018), ([0.2870, 0.4158], 0.012), \\ ([0.2616, 0.3958], 0.03), ([0.2950, 0.4280], 0.018), ([0.2314, 0.3958], 0.012), \\ ([0.2616, 0.3618], 0.05), ([0.2950, 0.3958], 0.03), ([0.2314, 0.3618], 0.02), \\ ([0.2314, 0.3316], 0.02), ([0.2661, 0.3672], 0.012), ([0.2, 0.3316], 0.008) \end{array} \right\}$$

$$A_3 = \left\{ \begin{array}{l} ([0.5950, 0.6957], 0.075), ([0.5681, 0.6684], 0.075), ([0.6284, 0.7286], 0.03), \\ ([0.6, 0.7], 0.03), ([0.6284, 0.7584], 0.03), ([0.6, 0.7286], 0.03), \\ ([0.5950, 0.7339], 0.1), ([0.5681, 0.7050], 0.1), ([0.6284, 0.7686], 0.04), \\ ([0.6, 0.7384], 0.04), ([0.6284, 0.8], 0.04), ([0.6, 0.7686], 0.04), \\ ([0.5531, 0.6957], 0.075), ([0.5281, 0.6684], 0.075), ([0.5842, 0.7286], 0.03), \\ ([0.5578, 0.7], 0.03), ([0.5842, 0.7584], 0.03), ([0.5578, 0.7286], 0.03) \end{array} \right\},$$

$$\left\{ \begin{array}{l} ([0.1414, 0.2714], 0.126), ([0.1414, 0.3], 0.054), ([0.1712, 0.2714], 0.168), \\ ([0.1712, 0.3], 0.072), ([0.1414, 0.2416], 0.126), ([0.1414, 0.2714], 0.054), \\ ([0.1, 0.2314], 0.084), ([0.1, 0.2616], 0.036), ([0.1312, 0.2314], 0.112), \\ ([0.1312, 0.2616], 0.048), ([0.1, 0.2], 0.084), ([0.1, 0.2314], 0.036) \end{array} \right\}$$

$$A_4 = \left\{ \begin{array}{l} ([0.1933, 0.3812], 0.027), ([0.2107, 0.3812], 0.018), ([0.1933, 0.3565], 0.045), \\ ([0.2107, 0.4305], 0.018), ([0.2297, 0.4305], 0.012), ([0.2107, 0.4026], 0.03), \\ ([0.1933, 0.4076], 0.045), ([0.2107, 0.4076], 0.03), ([0.1933, 0.3812], 0.075), \\ ([0.2551, 0.4676], 0.045), ([0.2781, 0.4676], 0.03), ([0.2551, 0.4373], 0.075), \\ ([0.2781, 0.4830], 0.03), ([0.3031, 0.4830], 0.02), ([0.2780, 0.4517], 0.05), \\ ([0.2551, 0.4573], 0.075), ([0.2781, 0.4573], 0.05), ([0.2551, 0.4277], 0.125), \\ ([0.3, 0.4676], 0.018), ([0.3270, 0.4676], 0.012), ([0.3, 0.4373], 0.03), \\ ([0.3270, 0.5281], 0.012), ([0.3565, 0.5281], 0.008), ([0.3270, 0.4939], 0.02), \\ ([0.3, 0.5], 0.03), ([0.3270, 0.5], 0.02), ([0.3, 0.4676], 0.05) \\ ([0.3618, 0.5], 0.18), ([0.3618, 0.5324], 0.108), ([0.3958, 0.5324], 0.072), \\ ([0.3316, 0.4719], 0.06), ([0.3316, 0.5061], 0.036), ([0.3672, 0.5061], 0.024), \\ ([0.3316, 0.5], 0.06), ([0.3316, 0.5324], 0.036), ([0.3672, 0.5324], 0.024), \\ ([0.4, 0.5], 0.06), ([0.4, 0.5324], 0.036), ([0.4319, 0.5324], 0.024), \\ ([0.3716, 0.4719], 0.02), ([0.3716, 0.5061], 0.012), ([0.4050, 0.5061], 0.008), \\ ([0.3716, 0.5], 0.02), ([0.3716, 0.5324], 0.012), ([0.4050, 0.5324], 0.008), \\ ([0.3618, 0.4622], 0.06), ([0.3618, 0.4970], 0.036), ([0.3958, 0.4970]), \\ ([0.3316, 0.4319], 0.02), ([0.3316, 0.4687], 0.012), ([0.3672, 0.4687], 0.00), \\ ([0.3316, 0.4622], 0.02), ([0.3316, 0.4970], 0.012), ([0.3672, 0.4970], 0.008) \end{array} \right\}$$

These are transformed to crisp numbers by using two methods weighted averaging and geometric averaging. As can be seen in Table 7 and 8, the ranking result is  $A_3 > A_2 > A_4 > A_1$ . Hence, the cluster  $A_3$  is the most potential customer group. The negative numbers (e.g. -0.023, see Table 7) show the level of uncertain of decision makers is higher that of certain and vice versa when they give their judgment.

Table 7. The score values of each alternative aggregated by using WIVDHFVA

	$A_1$	$A_2$	$A_3$	$A_4$
$S(d)$	-0.0233	0.0014	0.0171	-0.0025

Table 8. The score values of each alternative aggerated by using WIVDHFVG

	$A_1$	$A_2$	$A_3$	$A_4$
$S(d)$	-0.0251	0.0005	0.0162	-0.0032

## 6. Conclusion

Segmenting customer and identifying target customers are a complex procedure because integrating opinions of group rather than that of individual. Inviting experts in various fields to give their judgements in the same object. However, experts may give no judgment or show degree of certain or uncertain in their judgments. Additionally, the judgments of the key person are considered more impact than that of others. These factors are ignored in the existing studies.

The study proposes the two-phased framework for segment and ranking segment customers based on RFM model by using the Fuzzy C-means clustering algorithm and weighted interval-valued dual hesitant fuzzy numbers. Highlights of this study are to allow to aggregate degree of certain and uncertain of decision makers when giving the judgements. The role of each expert is expressed under the weights. Higher weight shows the more important role.

The final result shows that the framework is effective for segmenting and ranking group of customer with the same priority for R, F, M, various fields and different weights of experts. This method still works if these priorities of R, F, M are changed based on type of business or suggestions of experts.

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## **Biographies**

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**Ha Chi Xuan Chi** is a lecturer in School of Industrial Engineering and Management at International University – VNU HCMC- VietNam. She received her PhD from School of Industrial Management at the National Taiwan University of Science and Technology, Taiwan in 2014. Her main research interest is ranking fuzzy number support for making decision, Multi-criteria decision making, VRP and Business Analytic.