Efficient Optimization Model for Buffer Size Problem in Production Line with Rework Path

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Abstract
Modern production systems in modern industries are constructed with one or more support lines, such as rework, parallel, assembly line, and so on, to satisfy their objectives. Producing a rejected part is a common issue in manufacturing processes. Rework path (RP) is one of the most common types of production paths used to repair rejected parts and return them to the main line for reprocessing. The ideal buffer size in front of each machine tool bay in both branches of the production system should be optimized to create an efficient production system, including the main production line with rework path (MPL-RP). The optimization of buffer size of the system leads to the improvement in their overall performance, management, and operations. The key criteria influencing the buffer size decision and hence the MPL-RP production rate are the characteristics of machine tools, particularly their uptimes and downtimes. An efficient optimization model (EOM) is conceived in this study to determine the buffer size between each two neighboring machine tools in MPL-RP at any given uptimes and downtimes. To determine the ideal buffer size of the MPL and its RP, EOM employs a genetic algorithm (GA) based optimization model. The suggested method can estimate the ideal buffer size in an acceptable period of time, as demonstrated in the numerical examples, for small and large production lines, included in this study. As a result, when compared to previous selection approaches, the buffer sizes can be selected in a short span of time. The suggested approach can assist manufacturing engineers in making decisions during the design of new MPL-RP systems, as well as be utilized to improve the working of existing MPL-RP systems.

Keywords
Production line, Rework path, Optimization, Buffer size, Productivity

1. Introduction
Production line, also known as a flow line, is a production system consists of a number of machines that are sequentially connected in series with buffers between each two neighboring machines (Shi and Gershwin 2009). Defective parts which must be rejected, are frequently produced during the production process in an industry. The reworking of the declined part is highly recommended to enhance the productivity of the process. This can be achieved through adopting main production line with rework path (MPL-RP), which is one of the most common types of production systems in modern plants. It is capable of repairing the defective parts through rework path in the system. The repaired parts will be returned to the main line after reprocessing through rework path. Adoption of this production

system with dedicated rework path results in better productivity, reduction in overall production cost, decrease in scrap and material wastage.

Design and development of a highly efficient manufacturing system requires the effective optimization of the main production line as well as its rework path. The buffer size of the bay machine tools located in the main production line and RPs is one of the most significant factors that affect the productivity of system. Primary objective of keeping buffers on the production line is to adjust the variations in production process, which makes it capable of performing a series of actions independently (Demir et al. 2014). Disruptions in the production process may be initiated due to the unforeseen occurrences such as machine breakdown and system errors. Increased operation independence achieved through buffer addition aids in lowering the impact of these disruptions. It also absorbs production variability induced by differences or stochasticity in machine tools/stage speeds in the production line. On the other hand, buffer installation necessitates additional capital cost, space, and inventories. As a result, selection of buffers is critical and should be done wisely. Implementation of optimal or near-optimal buffer sizes can ensure that products flow as smoothly as possible through the SPL and its RP. This leads to the improvement in overall performance, management, and operations. A well-optimized production line can increase the flexibility and throughput of the production system (Lee et al. 2008). Consequently, buffer size has posed a significant challenge in the manufacturing industry, necessitating the development of effective and efficient approaches for determining suitable buffer sizes.

The buffer size problem has been the subject of a large number of studies. For example, comprehensive reviews were conducted to study the buffer allocation challenges and its various approaches for analysis were conducted by (Demir et al. 2014, Lee et al. 2008, and Weiss et al. 2019). Many meta-heuristic methods, including simulated annealing, genetic algorithms, tabu search, and other techniques, have been used to analyze the buffer size problem (Qudeiri et al. 2008, Altiparmak et al. 2007, Demir et al. 2011, Vergara et al. 2009, Amiri et al. 2012, Zandieh et al. 2017, Kiesmüller et al. 2018, Kose and Kilincci 2015). To find the buffer sizes in open serial production lines (SPL), Kose and Kilincci (2015) developed a hybrid approach-based simulation. In order to obtain suitable buffer sizes, a hybrid approach employing a genetic algorithm and simulated annealing was used as a search tool. The average production rate was calculated using discrete event simulation modelling in their research.

ANN and simulated annealing were used to investigate buffer sizes in asynchronous assembly system combinations (Altiparmak et al. 2002). Artificial intelligence (AI), genetic algorithms (GA), and artificial neural networks (ANN) have all been used to look for optimal solutions to the buffer allocation problem (Demir et. al. 2014). To examine the buffer allocation problem (Zandieh et al. 2017). Zandieh et al. (2017) provided an combined simulation and optimization method. As meta-heuristic algorithms, GA and particle swarm optimization (PSO) were applied. The proposed model and solution techniques were adopted by Polar Saunier Duval Company in their water heater production line. Mathematical approaches were also developed for optimizing the buffer size in tandem production lines. Furthermore, Han and Park (2002) developed an analytical approach for optimizing buffer allocation, which in turn targets optimal throughput in a SPL with workstations, buffers, and quality inspection devices, but it is time intensive when the system grows complicated. Similarly, Usubamatov et al. (2013) provided an analytical method for evaluating the throughput of an automated production line consisting of serial as well as parallel machines with buffer storages installed. Analytical methods, on the other hand, have been found to be incompatible for solving challenging and complex computational problems.

The dynamic programming methodology was utilized by Hasama et al. (2011) to optimize the buffer size allocation for an assembly line. Good results were obtained through this methodology. But as the problem size becomes larger, it gets quite difficult and time intensive. The design of buffer in an automated transfer line was done using a numerical method by Prombanpong et al. (2013). The approach was intended to reduce the impact of line breakdowns on line efficiency (Prombanpong et al. 2013).

For any production line, the selection of appropriate buffer sizes is very important since it has significant impact on the system's throughput. Bulgak et al. (1988) used simulated annealing (SA) to figure out the best buffer sizes for a production system that included different production and inspection lines. Bulgak (2006) additionally optimized the inter-stage buffer allocation to improve the system's total productivity. Lutz et al. (1998) established a meta-heuristic methodology for determining buffer position and sizes for a specific production line based on tabu search. Tsadiras et al. (2013) illustrated ANN's prediction capabilities in production systems, and described how well they can be utilized to produce better and faster outcomes. The GA algorithm was used by Nahas et al. (2014) to maximize the throughput via identifying buffers and machine tools in assembly and disassembly manufacturing systems at the same time. They stated that efficient machines and substantial buffers increased the system's typical output rate; nevertheless, this required a significant financial expenditure. As a result, they developed a design model for assembly/disassembly networks based on combinatorial optimization, with buffers and machines as decision variables. Furthermore, Papadopoulos and Vidalis (2001) suggested a heuristic technique to solve the problem of buffer allocation in unreliable and/or unbalanced manufacturing lines. Yamamoto et al. 2008 used GA to design a decision support system for choosing buffer size for a flexible flow line with bypass lines for production systems with a supporting line. Qudeiri et al. (2008) used GA to improve the buffer size and workstation capacity of parallel SPL. Furthermore, Qudeiri et al. (2007) investigated the buffer size for a flexible production system with sub-lines, which they have modeled. Recently, Alkhalefah et al., (2021) utilized GA and ANN to develop a prediction system to quickly identify the optimal buffer sizes for SPL based on the uptime and downtime characteristics of the production system. Despite the fact that there have been many studies on buffer-size optimization, few studies have taken rework path into account. These researches also take a relatively long time for buffer size optimization.

This paper aims to develop an efficient optimization model (EOM) to identify the buffer size between each neighboring MPL-RP machine tools at any given uptimes and downtimes. A numerical example used in this study illustrates that the proposed method can determine the ideal buffer size in a fraction of the time it takes to do so in simulations. EOM can be used to help manufacturing engineers make decisions during the development stage of a new production system with a rework path, as well as to improve the productivity of running MPL-RP systems, which may suffer from variations in machine tool uptimes and downtimes throughout the production system.

The remaining of this work is arranged in the following manner. In part 2, we cover recent research on buffer-size optimization approaches and point out their limitations. Section 3 depicts the problem statement, whereas Section 4 describes the design of proposed approach. An illustrative scenario is presented in section 4. The key advantages and drawbacks of the proposed methodology are discussed in Section 5. Section 6 concludes the paper and suggests possible future research directions.

2. Modeling the production line with rework path

The structure of MPL-RP studied in this paper is shown in Figure 1.

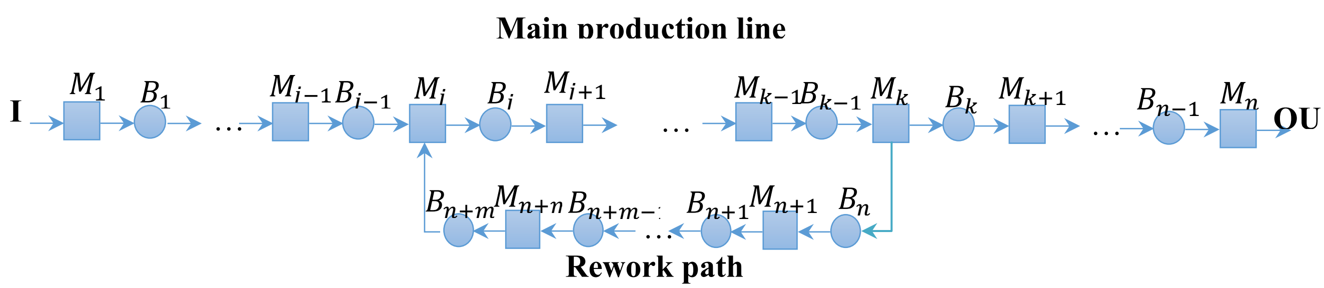

Figure 1. Structure of MPL-RP.

The main assumptions pertaining the MPL-RP components are same as those used by Jingshan (2004), and partially modified from and Alkhalefah et al., (2021), as follows.

I. The production system consists of a main production line and rework path. The main production line has n machine tools ($M_1, M_2, \ldots., M_n$) arranged serially $n-1$ buffers ($B_1, B_2, \ldots., B_{n-1}$) separating each consecutive pair of machine tools. In addition to the two machine tools shared by the main production line and rework path namely M_k and M_j, the rework path has m machine tools ($M_{n+1}, M_{n+2}, \ldots., M_{n+m}$) and $m+1$ buffers ($B_n, B_{n+1}, \ldots., B_{n+m}$). Machine tools M_k and M_j (where $1 < j < k < n$) are the starting and ending points of the rework path. The machine tools in rework path are also arranged serially, and the buffers in rework path separating each consecutive pair of machine tools including B_n and B_{n+m} separating the machine tool pairs (M_k, M_{n+1}) and (M_{n+m}, M_j) respectively. and each buffer separating each consecutive pair of machine tools.

II. Each machine tool in the production line and its rework path M_i, $i = 1, 2, \ldots, n+m$, has two states: up and down.

III. The uptime and the downtime of each machine M_i, $i = 1, 2, \ldots, n+m$, are random variables and given symbols p_i and r_i, respectively.

IV. Each buffer in the main production line and its rework path B_i, $i = 1, 2, \ldots, n+m$, is categorized by its capacity, $0 \le N_i < \infty$.

V. Machine tool M_i is starved at time t if buffer B_{i-1} is empty at time t. The first machine tool in the main production line, M_1 is never starved. Furthermore, machine M_j is starved if both B_j and B_{in+m} are empty.

VI. Machine tool M_i is blocked at time t if B_i is full at time t. The last machine tool in the main production line, M_n is never blocked. Machine M_k is blocked by the main line if it produces a good part and B_k is full, while machine M_n is blocked by the rework loop if it produces a defect part and B_n is full.

VII. The part leaving machine tool M_k (shared by the main production line and rework path) is defective with probability α (called rework rate), $(0 < \alpha < 1)$, and needs to by repaired by rework path. A defective part will be sent to first buffer in rework path, B_n, with a probability α if buffer B_n is not full. The good part is sent to B_k with a probability $(1 - \alpha)$, if B_k is not full.

VIII. The second machine tool shared by the main production line and rework path, M_j can take one part each cycle either from the B_{j-1} or B_{n+m}. The part in B_{n+m} has priority to be sent to M_j first if it is not empty.

3. Throughput evaluation of ASPS and DisASPS

To evaluate the performance of MPL-RP, the overlapping decomposition approach proposed by (Jingshan 2004), this approach decompose the MPL-RW into four serial production lines (SPLs) as shown in Figure (2). This decomposition approach includes overlapping because machine tools Mj and Mk are shared by more than one serial production line. After modifying the shared machine tools, the performance of these serial production line can be evaluated using the novel aggregation procedure presented by (Chiang et al. 2000).

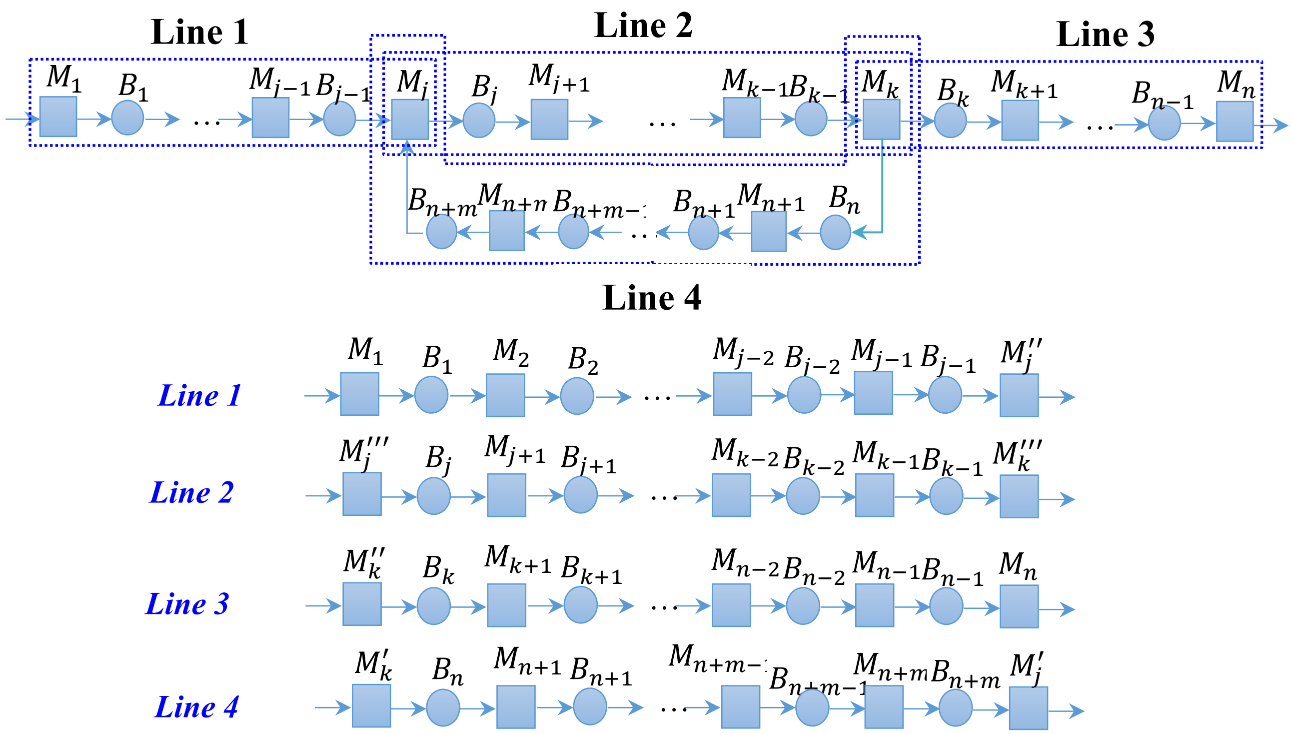

Figure 2: Overlapping Decomposition of MPL-RP into four SPL

First, consider the serial production line, line 1, machine tool, Mj is modified to $\boldsymbol{M_j''}$ with the uptimes and downtime parameters of $\boldsymbol{r_j''}$ and $\boldsymbol{p_j''}$ so as to account for the existence of other machine tools and buffers. These parameters can be calculated as follows.

$$r_j'' = r_j Prob\{M_j \text{ is straved by } B_{n+m}\}(1 - Prob\{M_j \text{ is blocked}\}),$$

$$p_j'' = p_j + r_j[1 - Prob\{M_j \text{ is straved by } B_{n+m}\}(1 - Prob\{M_j \text{ is blocked}\})], \quad (1)$$

Next, in the serial production line, *line 2*, machine tools, M_j and M_k are modified to M_j''' and $M_k'''$with the uptimes and downtime parameters of r_j''', p_j''' and r_k''', p_k''' respectively the parameters can be calculated as follows.

$$r_j''' = r_j\big(1 - Prob\{M_j \text{ is straved by } B_{j-1}\}\,.Prob\{M_j \text{ is straved by } B_{n+m}\}\big),$$
$$p_j''' = p_j + r_j Prob\{M_j \text{ is straved by } B_{j-1}\}\,.Prob\{M_j \text{ is straved by } B_{n+m}\}, \quad (2)$$
$$r_k''' = r_k[1 - \alpha Prob\{M_k \text{ is blocked by } B_n\} - (1-\alpha)\, Prob\{M_k \text{ is blocked by } B_k\}],$$
$$p_k''' = p_k + r_k[\alpha Prob\{M_k \text{ is blocked by } B_n\} + (1-\alpha)\, Prob\{M_k \text{ is blocked by } B_k\}], \quad (3)$$

Similarly, in the serial production line, *line 3*, machine tool, M_k is modified to M_k'' with the uptimes and downtime parameters of r_k'', p_k'' and the parameters can be calculated as follows.

$$r_k'' = r_k(1-\alpha)(1 - Prob\{M_k \text{ is straved}\}),$$
$$p_k' = p_k + r_k[1 - (1-\alpha)(1 - Prob\{M_k \text{ is straved}\})], \quad (4)$$

Finally, in the serial production line, line 4, machine tools, Mk and Mj are modified to M_k' and $M_j'$with the uptimes and downtime parameters of r_k', p_k' and r_j', p_j' respectively the parameters can be calculated as follows.

$$r_k' = r_k\alpha(1 - Prob\{M_k \text{ is straved}\}),$$
$$p_k' = p_k + r_k[1 - \alpha(1 - Prob\{M_k \text{ is straved}\})], \quad (5)$$

$$r_j' = r_j\big(1 - Prob\{M_j \text{ is blocked}\}\big),$$
$$p_j' = p_k + r_j rob\{M_j \text{ is blocked}\} \quad (6)$$

The probabilities in Equations (1 to 6) can be calculated as follows (Jingshan 2004);

$$Prob\{M_i \text{ is blocked}\} = Q\big(p_{i+1}^b, r_{i+1}^b, p_i^f, r_i^f, N_i\big)$$
$$Prob\{M_i \text{ is straved}\} = Q\big(p_{i-1}^f, r_{i-1}^f, p_i^b, r_i^b, N_i\big) \quad (7)$$

where $Q(p_a, r_a, p_b, r_b, N)$ is defined as follows (Chiang et al. 2000);

$$Q(p_a, r_a, p_b, r_b, N) = \begin{cases} \frac{(1-e_a)(1-\emptyset)}{1-\emptyset e^{-\beta N}}, & if\ \frac{p_a}{r_a} \neq \frac{p_b}{r_b} \\ \frac{p_a(p_a+p_b)(r_a+r_b)}{(p_a+r_a)[(p_a+p_b)(r_a+r_b)+p_b r_a(p_a+p_b+r_a+r_b)N]}, & if\ \frac{p_a}{r_a} = \frac{p_b}{r_b} \end{cases} \quad (8)$$

And

$$e_i = \frac{r_i}{p_i+r_i}, i = a, b,$$

$$\emptyset = \frac{e_a(1-e_b)}{e_b(1-e_a)}, \quad (9)$$

$$\beta = \frac{e_a(p_a + p_b + r_a + r_b)(p_a r_b - p_b r_a)}{(p_a+p_b)(r_a+r_b)}.$$

There have been a number of approximation approaches utilized to evaluate the SPL based on aggregation and decomposition (Chiang et al. 2000, Gershwin1987, Dallery et al. 1989, Jacobs et al. 1995). This research study follows the aggregation procedure reported in (Chiang et al. 2000, and Alkhalefah et al., 2021) to evaluate the SPL. This aggregation procedure is described as follows.
First, the serial production line is assumed to have M machines as shown in Figure 1 and defined by assumptions (I-VI).

The first two machine tools (M_1 and M_2) are aggregated into a single machine, M_2^f, with the following uptime and downtime parameters:

$$p_2^f = p_2 + r_2 Q(p_1, r_1, p_2, r_2, N_1) \tag{10a}$$

$$r_2^f = r_2 - r_2 Q(p_1, r_1, p_2, r_2, N_1) \tag{10b}$$

Where $Q(p_1, r_1, p_2, r_2, N_1)$ is the probability that the machine tool M_2 is starved and can be calculated using Equation (8)
Next, continue aggregation in forward direction (forward aggregation) as presented in Alkhalefah et al., (2021), until the following criteria is satisfied

$$\frac{r_n^f}{p_n^f} = \frac{r_1^b}{p_1^b} \tag{11}$$

Formally, this process is represented as follows:

$$r_i^f(s+1) = r_i - r_i Q\left(p_{i-1}^f(s+1), r_{i-1}^f(s+1), p_i^b(s+1), r_i^b(s+1), N_{i-1}\right), i = 2, \dots, n$$

$$p_i^f(s+1) = p_i + r_i Q\left(p_{i-1}^f(s+1), r_{i-1}^f(s+1), p_i^b(s+1), r_i^b(s+1), N_{i-1}\right), i = 2, \dots, n$$

$$r_i^b(s+1) = r_i - r_i Q\left(p_{i+1}^b(s+1), r_{i+1}^b(s+1), p_i^f(s), r_i^f(s), N_i\right), i = 1, \dots, n-1 \tag{12}$$

$$p_i^b(s+1) = p_i + r_i Q\left(p_{i+1}^b(s+1), r_{i+1}^b(s+1), p_i^f(s), r_i^f(s), N_i\right), i = 1, \dots, n-1$$

with the following initial conditions

$$p_i^f(0) = p_i, \quad r_i^f(0) = r_i, \qquad \forall\, i = 2, \dots, n-1,$$

and boundary conditions

$$p_1^f(s) = p_1, \qquad r_1^f(s) = r_1,$$

$$p_n^b(s) = p_n, \qquad r_n^b(s) = r_n,$$

$$\forall\, s = 0, 1, 2, \dots$$

where function $Q(p_a, r_a, p_b, r_b, N)$ is defined in (2)
Finally, production rate for the defined SPL can be approximated as follows:

$$\widehat{PR}_{SPL}(p_1, r_1, \dots, p_n, r_n, N_1, \dots, N_{n-1}) = \frac{r_n^f}{p_n^f + r_n^f} = \frac{r_1^b}{p_1^b + r_1^b}. \tag{13}$$

Based on the performance of the four serial production lines of MPL-RP, under the assumption (I to VIII), the production rate of MPL-RP can be determined as follows (Jingshan 2004);

$$\widehat{PR}_{MPL-RP}(M_1, M_2, \dots, M_n, M_{n+1}, \dots, N_{n+m}, B_1, B_2 \dots, B_{n-1}, B_n, \dots, B_{n+m}) = \widehat{PR}_{Line\ 3} \tag{14}$$

where $\widehat{PR}_{Line\ 3}$ is the production rate of serial production line (Line 3), and

$$\widehat{PR}_{Line\ 1} = \widehat{PR}_{Line\ 3},$$

$$\widehat{PR}_{Line\ 2} = \widehat{PR}_{Line\ 1} + \widehat{PR}_{Line\ 4}$$

4. Optimization Approach

The optimization approach aims to estimate the optimal values of the buffer size and for every machine in the production lines and estimate the productivity of the production lines. In this paper, the optimization model is developed based on one of the famous meta-heuristic optimization algorithms, which is called a genetic algorithm (GA). The GA performs complex computational operations in order to find the optimal solution to the desired problem. The GA was inspired from the concept of biological gens enhancement in the nature of the evaluation theory. The concept is defined as a method of learning from nature and search for an enhanced elements/gene which is incorporated into new genes in order to reach genome evolution (Yang 2001).

All steps of the artificial genetic algorithm were borrowed from the evaluation theory. The GA has a population that refers to a group of individuals. Every individual consists of a group of jointed genes, and every gene refers to a specific feature. In particular, in our optimization model, every individual is a solution for the buffer size estimation problem. The proposed GA algorithm is performed in five phases (initial population, fitness function, selection, crossover, mutation). Firstly, the algorithm starts to generate random solutions (individuals) for the buffer size problem, and this step is called the initial population. The second step is the fitness calculation for every individual. The fitness function determines the ability of an individual to compete with other individuals by giving a fitness score to each individual. This fitness score is calculated from the objective function by knowing its effect to maximize production line productivity. Thirdly, the algorithm selects the individuals with the highest fitness and lets them pass their genes to the next population. Fourthly, the crossover is performed on the selected individuals and performs exchanging in their genes to carry out enhanced individuals. The exchanging process is called offspring, performed between every two individuals called parents, and the generated offspring individuals called the child. Fifthly, in order to get maintain diversity with the population and avoid convergence in local optimum, the mutation is applied on the child individuals with a low random probability. That is carried out by flipped some bits value in the child individuals' genes. Finally, the algorithm is repeated until it reaches the optimal value then it will be terminated with the optimal buffer size.

In Figure 3, the algorithm steps are illustrated. The solution of objective function is defined by a group of individuals which are defined as an array of N elements and each N element refers to a machine tool in the SPL, as shown in equation 15. Every value of elements in Equation 15 refers to buffer size of every machine tool in the SPL. The I value refers to selected individuals for crossover process and the J value refers to number of selected individuals for mutation process. The S values refers to the reset of individuals, which will be fed to the next population and be selected based on elitist Theory.

$$Individual = [N_1, N_2, N_{3,} N_4 \dots, N_{n-1}] \tag{15}$$

In Figure 4, the class diagram of the designed genetic algorithm framework is shown. The proposed design makes the framework able to solve many optimization problems without any restriction. Moreover, the proposed architecture can maintain the chromosome structure. The solve() method describe the implementation of the flowchart in Figure 3.

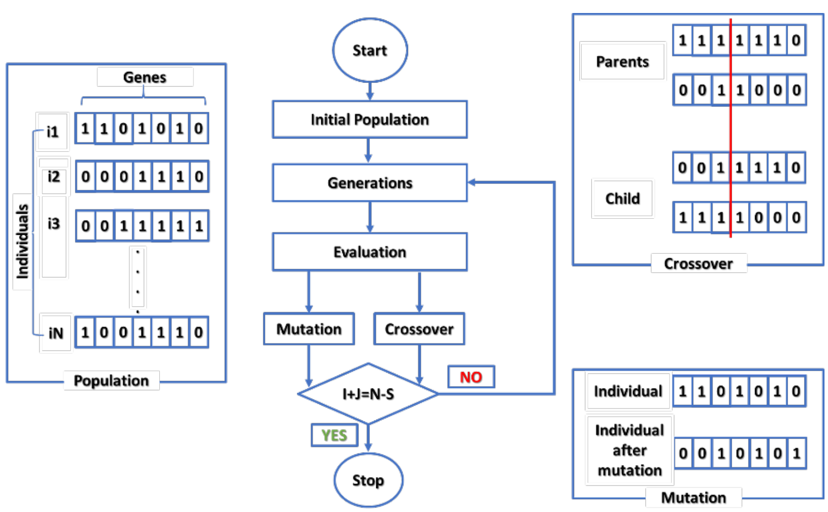

Figure 3. GA outline.

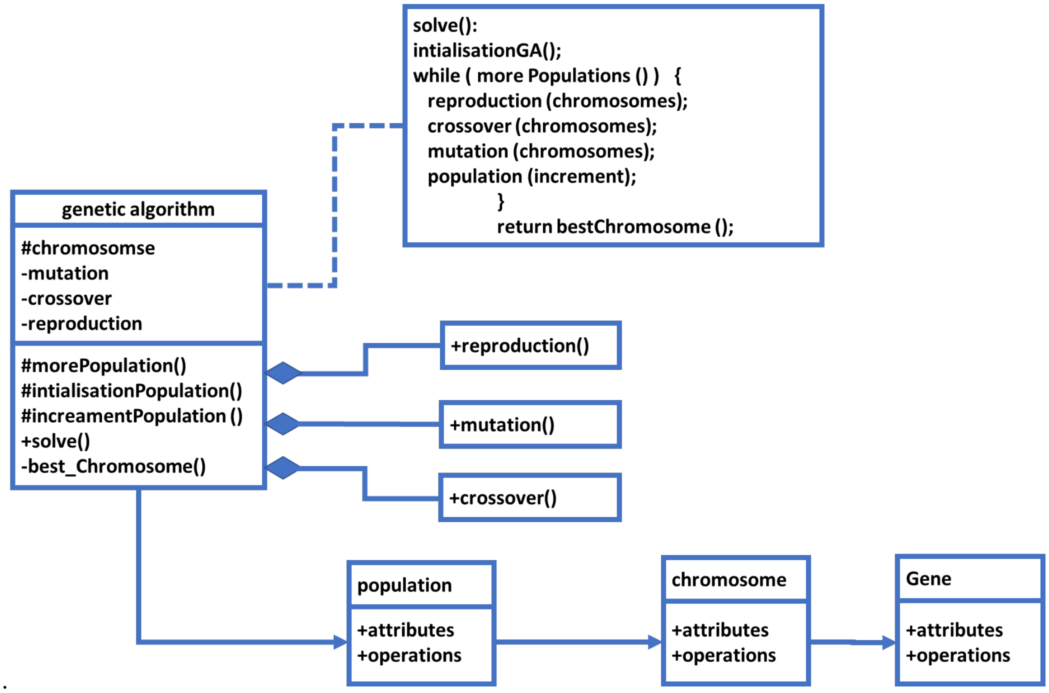

Figure 4. Class diagram of the genetic algorithm

5. Numerical Results and Discussion

5.1 Small Production Line with Rework Path

In this section, the designed methodology is validated by two different cases of small production lines with rework path. The full production line consists of four sub production lines. Each sub production line consists of four different machine tools. Every machine tool has its uptimes (p_i) and downtimes (r_i) parameters, as shown in Table 1. The algorithm was validated based on two different cases, which are configured as follows; the first case characterized by a complete random uptime parameters (p_i) and down parameters (r_i) for the whole machines of the four sub lines, and the second case was set by the same uptime parameters (p_i) and downtime parameters (r_i) values for four sub production lines, where the characteristics of the first production lines (uptime parameters (p_i) and down parameters (r_i) are set randomly and repeated for the other three sub lines. The variety in the selection of validation cases is intended to show the quality of the developed approach, which can obtain accurate estimation of the optimal buffer size for the production line for two different cases and to estimate the productivity for each sub line and in every case.

Table 1: Uptimes and downtimes parameters for two different SPLS

Example	Line	Machine	Pi	Ri
1	1, 1, 1, 1, 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 4	1, 2, 3, 4, 4, 5, 6, 7, 7, 8, 9, 10, 7, 11, 12, 4	0.015, 0.002, 0.05, 0.015, 0.015, 0.02, 0.05, 0.015, 0.015, 0.1, 0.2, 0.08, 0.015, 0.35, 0.3, 0.015	0.085, 0.08, 0.45, 0.085, 0.085, 0.08, 0.45, 0.085, 0.085, 0.4, 0.8, 0.35, 0.085, 0.7, 0.65, 0.085,
2	1, 1, 1, 1, 2, 2, 2,2, 3, 3, 3, 3, 4, 4, 4, 4	1, 2, 3, 4, 4, 5, 6, 7, 7, 8, 9, 10, 7, 11, 12, 4,	0.09, 0.12, 0.062, 0.21, 0.21, 0.062, 0.12, 0.09, 0.09, 0.12, 0.062, 0.21, 0.09, 0.12, 0.062, 0.21,	0.54, 0.62, 0.72, 0.81, 0.81,0.72, 0.62, 0.54, 0.54, 0.62,0.72, 0.81, 0.54, 0.62, 0.72,0.81,

The proposed genetic algorithm solves the optimization problem and the parameters are defined as follows; population size of 150 individuals, crossover rate of 0.8, and mutation rate of 0.05. After some iterations, the Pareto front, Figs. 5-6, show an example the optimized results of the genetic algorithm for multi-objective function, machine tool buffer size and lines throughput.

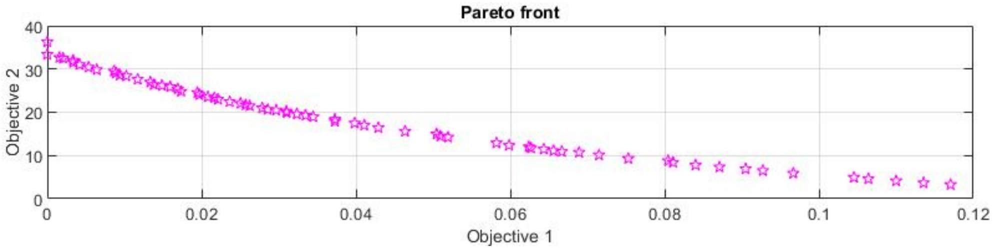

Figure 5. Pareto front of optimal values for optimization model of SPL with 4 machine tools for line 2

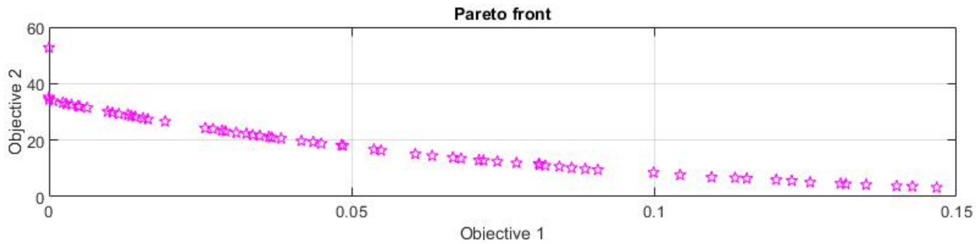

Figure 6. Pareto front of optimal values for optimization model of SPL with 4 machine tools for line 3

The optimal buffer sizes of the small production lines with the given parameters in Table 1 is carried out by the proposed genetic algorithm. The optimal buffer size for every machine in the 4 lines of the production line is shown in Table 2. Also, the productivity of every line is showed in Table 2. From the results, it not so difficult to see that the presented demonstrated cases shows the ability of the developed algorithm to estimate the optimal buffer size of production lines in different working scenarios with different uptimes and downtimes of the machine tools. Also, the total productivity of the production lines in-case number 2 is higher than the in-case number 1.

Table 2: Optimal buffer size and productivity

Example	Pi	Ri	Ni	Productivity
1	0.015, 0.002, 0.05, 0.015, 0.015, 0.02, 0.05, 0.015, 0.015, 0.1, 0.2, 0.08, 0.015, 0.35, 0.3, 0.015	0.085, 0.08, 0.45, 0.085, 0.085, 0.08, 0.45, 0.085, 0.085, 0.4, 0.8, 0.35,] 0.085, 0.7, 0.65, 0.085,	7, 12, 16, 1, 7, 20, 14, 1, 45, 44, 32, 1 1, 1, 1, 1	0.760051, 0.679634, **0.780102,** 0.427078
2	0.09, 0.12, 0.062, 0.21, 0.21, 0.062, 0.12, 0.09, 0.09, 0.12, 0.062, 0.21, 0.09, 0.12, 0.062, 0.21,	0.54, 0.62, 0.72, 0.81, 0.81, 0.72, 0.62,0.54, 0.54, 0.62, 0.72, 0.81, 0.54, 0.62, 0.72,0.81,	13, 14, 9, 1, 2, 3, 3, 1, 7, 11, 13, 1, 1, 1, 1, 1	0.793632, 0.712843, **0.792047,** 0.652992

5.2 Large Production Line with Rework Path

The proposed method is validated by another two different cases of large production lines. The productions consist of four sub production line, each line with 11 machine tools. Table 3 show the uptimes and down times parameter of every machines. The case studies were set as follows;

Case 1: set all machine in random uptimes (p_i)and downtime (r_i) parameters.

Case 2: set all machine in the same uptimes (p_i)and downtime (r_i) parameters.

The variety in selection of validation cases is intended to demonstrate the feasibility of the developed method in order to estimate the optimal buffer size and throughput of the production lines. These case studies show the capability of the designed method to find the optimal buffer size in large production lines with different working characteristics.

Table 3: Uptimes and downtimes parameters for two different SPLS

Example	Line	Machine	Pi	Ri
1	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 1, 2, 3, 4	0.06, 0.07, 0.03, 0.02, 0.08, 0.06, 0.04, 0.1, 0.2, 0.08, 0.2, 0.2, 0.05, 0.35, 0.052, 0.179,0.08, 0.044, 0.067, 0.077, 0.049, 0.2, 0.2, 0.4, 0.078, 0.066, 0.483,0.07, 0.09, 0.122, 0.039, 0.186, 0.073, 0.2, 0.383, 0.476, 0.2	0.75, 0.74, 0.88, 0.86, 0.81, 0.8, 0.85, 0.4, 0.8, 0.35, 0.83, 0.83, 0.2, 0.82, 0.468, 0.84, 0.98, 0.87, 0.97, 0.73, 0.89, 0.83, 0.83, 0.78, 0.39, 0.88, 0.75, 0.98, 0.487, 0.347, 0.997, 0.313, 0.81, 0.83, 0.98, 0.413, 0.83
2	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10,11, 1, 2, 3, 4	0.2, 0.22, 0.25, 0.1, 0.15, 0.17, 0.23, 0.24, 0.2, 0.18, 0.2, 0.2, 0.22, 0.25, 0.1, 0.15, 0.17, 0.23, 0.24, 0.2, 0.18, 0.2, 0.2, 0.22, 0.25, 0.1, 0.15, 0.17, 0.23, 0.24, 0.2, 0.18, 0.2, 0.2, 0.22, 0.25, 0.2	0.83, 0.86, 0.85, 0.94, 0.93, 0.95, 0.86, 0.84, 0.9, 0.95, 0.83, 0.83, 0.86, 0.85, 0.94, 0.93, 0.95, 0.86, 0.84, 0.9, 0.95, 0.83, 0.83, 0.86, 0.85, 0.94, 0.93, 0.95, 0.86, 0.84, 0.9, 0.95, 0.83, 0.83, 0.86, 0.85, 0.83

Similar to the small production lines case studies in Section 5.1, the optimization algorithm is utilized to estimate the optimal buffer size of the machine tools in the large production line considering the given uptimes and downtimes parameters. The GA parameters is set similar to the problem of the small production line case studies in Section 5.1, please see above. Figures 7-8 show the Pareto front for algorithm of multi-objective function, machine tool buffer size and machine productivity rate.

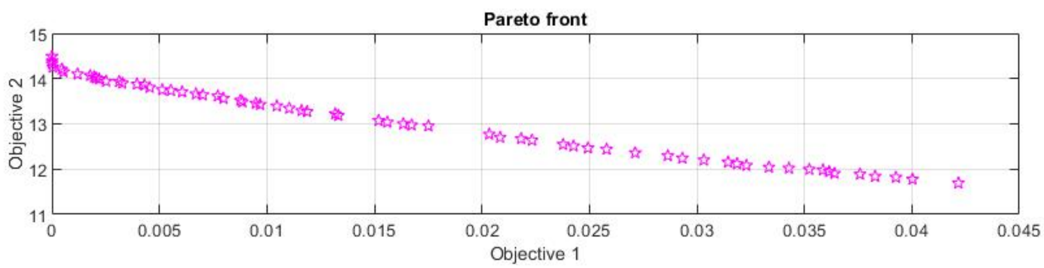

Figure 7. Pareto front of optimal values for optimization model of SPL with 4 machine tools for line 2

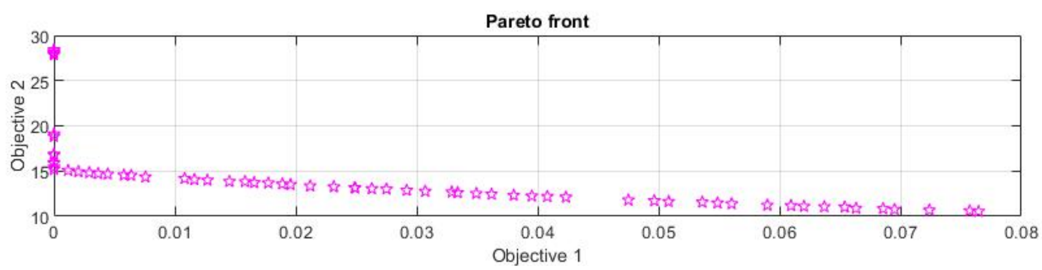

Figure 8. Pareto front of optimal values for optimization model of SPL with 4 machine tools for line 3

The GA is applied and the optimal buffer sizes of the large production lines is determined. In particular, the optimal buffer size for every machine in the 4 lines of the production line and the productivity of every line are showed in Table 4. From the results, the demonstrated cases show the ability of the developed algorithm to estimate the optimal buffer size of large production lines in different scenarios, particularly different uptimes and downtimes parameters. Also, the total productivity of the production lines in case number 1 is higher than the in-case number 2.

Table 4: Optimal buffer size and productivity

Ex.	Pi	Ri	Ni	Prod.
1	0.06, 0.07, 0.03, 0.02, 0.08, 0.06, 0.04, 0.1, 0.2, 0.08, 0.2, 0.2, 0.05, 0.35, 0.052, 0.179,0.08, 0.044, 0.067, 0.077, 0.049, 0.2, 0.2, 0.4, 0.078, 0.066, 0.483,0.07,	0.75, 0.74, 0.88, 0.86, 0.81, 0.8, 0.85, 0.4, 0.8, 0.35, 0.83, 0.83, 0.2, 0.82, 0.468, 0.84, 0.98, 0.87, 0.97, 0.73, 0.89, 0.83,	3, 1, 10, 6, 4, 7, 13, 19, 12, 12, 18, 7, 17, 30, 10, 9, 25, 26, 34, 29, 19, 21, 1, 4, 11, 4, 6, 7,	0.7181, 0.7687, **0.7028,** 0.5781

	0.09, 0.122, 0.039, 0.186, 0.073, 0.2, 0.383, 0.476, 0.2	0.83, 0.78, 0.39, 0.88, 0.75, 0.98, 0.487, 0.347, 0.997, 0.313, 0.81, 0.83, 0.98, 0.413, 0.83	9, 7, 9, 8, 1, 2, 2, 2, 1	
2	0.2, 0.22, 0.25, 0.1, 0.15, 0.17, 0.23, 0.24, 0.2, 0.18, 0.2, 0.2, 0.22, 0.25, 0.1, 0.15, 0.17, 0.23, 0.24, 0.2, 0.18, 0.2, 0.2, 0.22, 0.25, 0.1, 0.15, 0.17, 0.23, 0.24, 0.2, 0.18, 0.2, 0.2, 0.22, 0.25, 0.2	0.83, 0.86, 0.85, 0.94, 0.93, 0.95, 0.86, 0.84, 0.9, 0.95, 0.83, 0.83, 0.86, 0.85, 0.94, 0.93, 0.95, 0.86, 0.84, 0.9, 0.95, 0.83, 0.83, 0.86, 0.85, 0.94, 0.93, 0.95, 0.86, 0.84, 0.9, 0.95, 0.83, 0.83, 0.86, 0.85, 0.83	1, 2, 2, 2, 2, 2, 4, 2, 4, 4, 6, 13, 39, 10, 16, 19, 17, 12, 8, 23, 12, 1, 2, 10, 7, 13, 10, 5, 7, 5, 7, 4, 1, 1, 1, 1, 1	0.6612, 0.696, **0.6094**, 0.3629

6. Conclusion

This paper has presented the implementation of an accurate and generic optimization algorithm to estimate the optimal values of the buffer size and productivity of production lines with a rework path. The results show that the algorithm carried out the prediction/optimization values at any given uptimes and downtimes values and can perform the desired task with large and small production lines without restriction to a specific number of machines. The algorithm was tested in two case studies of parameter settings with small and large production lines. The results of the small production line show that case study 1 (same values of uptime and downtime parameters) gave a higher productivity than those obtained in case 2 (random selection of uptime and downtime parameters). However, in the large production line, the algorithm identified buffer sizes in case of random selection of uptime and downtime that gave higher productivity than those for the case where same values of uptime and downtime for the sub lines are considered. The results demonstrate that the proposed approach is capable to estimate the buffer size of SPL with a rework path. It is worth emphasizing that the proposed algorithm can be integrated with other software libraries because it is quite fast approach and can help the user make the rapid decisions with regard to the buffer size for the design of production lines. For future work, the authors aim to deploy this algorithm on an embedded system board and integrated it as an internet of things node within an Industry 4.0 framework.

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