Solution Method for a Max-min-plus Scaling Scheduling Problem by Formula Manipulation

Kyohei Sagawa  
Faculty of Information & Management Systems Engineering  
Nagaoka University of Technology  
Nagaoka, Niigata, Japan  
s203357@stn.nagaokaut.ac.jp

Hiroyuki Goto  
Department of Industrial & Systems Engineering  
Hosei University  
Koganei, Tokyo, Japan  
goto-h@hosei.ac.jp

Yoichi Shimakawa  
Department of Computer Science & Technology  
Salesian Polytechnic  
Tokyo, Japan  
simakawa@salesio-sp.ac.jp

Abstract

A discrete event system (DES) changes the internal states at discrete time instants according to event occurrences. Regular systems have a structure of parallelization and synchronization of multiple events. In formulating a scheduling problem in a DES context, the earliest event occurrence time is described by the ‘max’ and addition operations. These operations correspond to the synchronization and time elapse of events, which are respectively represented by addition and multiplication in max-plus algebra. The time instant of earliest event occurrence is associated with the earliest completion time in the program evaluation and review technique (PERT) context. The max-plus linear (MPL) representation described in max-plus algebra is beneficial in dealing with formulation of scheduling problems in PERT.

On the other hand, the max-min-plus scaling system (MMPS), an extended formalization of max-plus algebra, provides a richer descriptive capability than that of MPL systems have. This involves the ‘min’ operation, in addition to the ‘max’ and addition operations. The ‘min’ operation represents selection of events applied to scheduling problems. In contrast to the MPL representation, however, it has been hard to obtain an explicit solution in the max-min-plus (MMP) equation, where a scheduling problem on MMPS can be confined to an MMP equation.

In this study, we focus on a scheduling problem with a directed acyclic graph (DAG)-structured network. The aim is to develop a framework for solving MMP equations. Since a DAG is acyclic, an MMP equation associated with the earliest completion time has constantly a solution. Thus, an MMP equation could be solved directly through formula manipulation using a computer; some algebraic expressions might aid representing efficiently the relevant operations along with associated parameters. With the constructed framework, MMP equations can be solved in a straightforward manner, even for large-scale systems for which the resulting formula would be intractable in dealing with it manually.

Keywords  
max-plus algebra, max-min-plus scaling system, scheduling, program evaluation and review technique

Biographies
Kyohei Sagawa is currently a student in the Faculty of Information and Management Systems Engineering, Nagaoka University of Technology, Japan. His research interests include operations research and complex systems.

Hiroyuki Goto is a Full Professor in the Department of Industrial and Systems Engineering at Hosei University, Japan. He received his B.Sc. and M.Sc. degrees from The University of Tokyo in 1995 and 1997, respectively. He received his D.E. degree from Tokyo Metropolitan Institute of Technology in 2004. His research and teaching interests include operations research, geographic information science, and high-performance computing.

Yoichi Shimakawa is a Professor and Director of the Department of Computer Science and Technology, Salesian Polytechnic Japan. He received his B.S. and M.Sc. degrees from Chuo University in 1990 and 1996. In 1998, he joined the staff as a research assistant on the research project “Integrated Geographic Information Systems” at Chuo University. He received his D.E. degree from Chuo University. He received paper awards from the Operations Research Society of Japan (ORSJ) in 2002. He is a member of ORSJ.