Pricing and Inventory Planning for Perishable Products with Price, Stock, and Promotional Effort Sensitive Demand

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Abstract

This study models the problem of a retailer in making a joint pricing and inventory decision of a perishable product considering the price, stock, and promotional effort sensitive demand. The main objective is to determine the optimal price, replenishment schedule, and promotional effort cost to maximize total profit in a finite planning horizon. We show that the total profit generated during the planning horizon is a concave function of the price and the cost of promotional effort. It has been proven through an analytical approach that the optimal solution exists and is unique. We also develop an efficient algorithmic procedure to determine the optimal solutions of the proposed model. Numerical examples are provided to illustrate the proposed model's applicability. The optimal solution's sensitivity to key parameters has been investigated, and the results are discussed.

Keywords
Inventory, Pricing, Promotional effort, Perishable product and Optimization.

1. Introduction
Many retail companies face disruption in the era of digitalization and the COVID-19 pandemic. The increasingly fierce market competition followed by the rapid development of technology has caused many retailers to eventually close their stores or adopt new business models to survive. Online sales growth is also a challenge for brick-and-mortar retailers with high investment costs in both outlets and high inventory costs. Retailers should consider customer behavior in making operational decisions in stores, such as determining product selling prices and quantity/schedule of product orders from suppliers and managing strategic relationships with suppliers.

The management of perishable products inventories is a critical issue in a modern retail market. There are various items that deteriorate with time, including foods such as fruits and vegetables, meats, pharmaceuticals, chemicals, even fast fashion and high technology products are also included (Ahmadi et al. 2019). For a certain type of perishable product, its quality gradually deteriorates until it decays. It is typically found in fresh food products such as poultry, beef, fish, fruit, and vegetables. This product category is highly profitable and contributes significantly to sales revenue despite its short sales period.

Deterioration characterized as spoilage or decay that occurs during the normal storage period and, therefore, should be considered in inventory modelling (Mukhopadhyay 2004). Moreover, demand elasticity for perishable products is typically influenced by retailers' prices, promotional effort, and inventory levels of products. Retailers must calculate the optimal order quantity from suppliers while considering the possibility of products being decayed before the end of the sales period. However, the overstock and unsold products can deteriorate into food waste, which is costly to business owners. This is certainly a challenge for retailers in managing optimal inventories in stores. Managing perishable inventory effectively helps to reduce waste, increases the number of customers who can receive the products, and minimizes the total cost of ownership (Duong et al. 2015).

Harris (1913) introduced the economic order quantity model in the inventory management stream, followed by numerous studies conducted to enhance more effective inventory management policies. Meanwhile, pricing for perishable products is a difficult business practice to execute. A high price typically discourages buying, while a reasonable and affordable price encourages purchase. Numerous literatures have been published that integrates the concept of inventory control and pricing decision-making problems, or even extent with other policies.

To develop an inventory model for perishable products, the manner a demand function is constructed has a significant impact on inventory policy and its decision variables. For perishable product, Wu, et al. (2009) developed a joint inventory and pricing model with price-sensitive demand. Shaikh, et al. (2017) developed the inventory model with price and stock-dependent demand for fully backlogged shortages. Zhang, Chen, and Lee (2008) presented a single item with a finite horizon planning and the demand rate is influenced by price and promotion effort with the objective to maximize the total profit by combining promotion, pricing and inventory replenishment policies.

In addition, there has been extensive research on promotion and pricing strategies. Promotion could engage people, drives demand, identifies businesses, and makes items visible (Soni and Chauhan 2018). Businesses usually use promotions to tempt customers with price reductions, discounts, and gifts. Currently, firms also sell products through online banner ads, social medias, and blogs, among other tactics, to change demand patterns. Effective promotion and pricing decisions could induce an increased market demand. Zhang et al. (2008) developed joint planning of pricing, promotion, and inventory management decisions to maximize overall profit under a finite horizon and periodic review setting. They investigated the retailer problem with the promotional effort for a perishable product. Maihami and Karimi (2014) addressed the issue of non-instantaneous deterioration products sensitive to promotional efforts.

Table 1. Summary of the relevant articles for perishable products

Article	Demand Type				Planning Horizon
	Price Dependent	Stock Dependent	Promotional Effort Dependent	Others	
Ouyang et al. (2006)				Constant	Infinite
Wu et al. (2006)		√			Infinite
Ouyang et al. (2008)		√			Infinite
Zhang et al. (2008)	√		√		Finite
Wu et al. (2009)	√				Infinite
Yang et al. (2009)	√				Infinite
Chang et al. (2010)		√			Infinite
Soni and Patel (2012)	√				Infinite
Dye (2013)				Constant	Infinite
Maihami and Karimi (2014)	√		√		Infinite
Chang et al. (2015)	√				Infinite
Shaikh et al. (2017)	√	√			Infinite
Maihami et al. (2017)	√		√		Infinite
This Paper	√	√	√		Finite

The above studies combined with other notable articles in the related stream are summarized in Table 1 to simplify comparisons with the current model. According to the literature we explored, the development of EOQ model on perishable products with price, stock, and promotional effort-sensitive demand in a finite planning horizon is not conducted so far. So, this paper aims to develop a joint inventory and pricing planning model taking a deterministic form of the price, stock, and promotional effort cost dependent demand rate. Analytical methods are used to develop and solve the model. The profit function is generated by taking into account the retailer's acquisition cost, setup cost, holding cost, selling price, and promotional effort cost. The condition for the profit function's concavity is established and a numerical example is used to illustrate the model's solution procedure.

2. Assumptions and Notations
The following assumptions and notations are used in the model.

2.1 Assumptions
1. The proposed model is developed for a single-item replenishment policy in the retailer.
2. The lead time is negligible.

3. Shortages are not allowed.
4. The time horizon is finite.
5. Market demand at time t is assumed as $D(t;\ p; u) = a - bp + \delta u + \beta I(t)$, where market scale parameter $a > 0$, price elasticity parameter $b > 0$, promotional effort elasticity parameter $\delta > 0$, and stock elasticity parameter $\beta > 0$ such that the demand is always positive.
6. There is no additional replenishment to replace the deteriorated products.

2.2 Notations

D	Market demand
p	Unit retail price
u	Promotional effort of the retailer in a time cycle
τ	The promotional effort cost efficiency coefficient ($\tau > 0$)
$I(t)$	Inventory level of the retailer at time t
θ	Deterioration rate ($0 \le \theta \le 1$)
Q	Order quantity
Q_d	Quantity deteriorated in a cycle
c	Unit purchase price
c_d	Unit cost of deterioration
h	Unit holding cost
S	Ordering cost
H	Length of the finite time horizon
T	Length of replenishment cycle ($T > 0$)
n	Number of replenishments under a finite horizon planning ($n = H/T$)
TP	Total profit

3. Model Formulation

In our proposed model, we consider a retailer and a supplier for an EOQ model where the demand sensitive to the selling price, stock, and promotional effort. This research aims to develop a joint pricing and inventory planning problem for a perishable product during a finite time horizon. Based on the above assumptions, the retailer orders and receives Q units from the supplier at the beginning of time $t = 0$. The reduction of the inventory is due to the effect of demand and deterioration in the interval $[0, T]$, at time $t = 0$ the inventory level reaches zero.

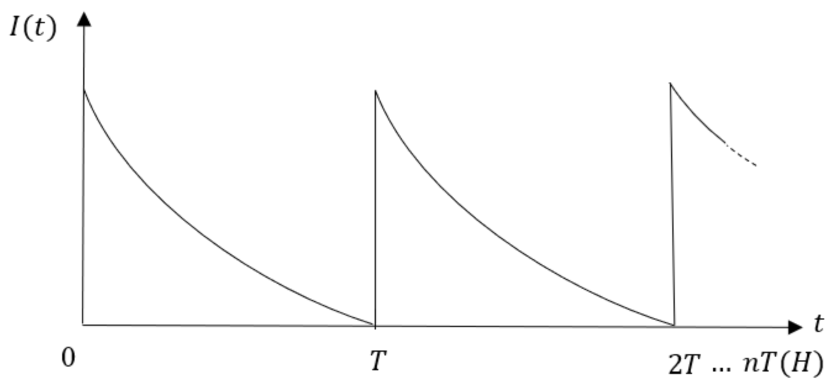

Figure 1. Inventory Levels of the Retailer

- The retailer gets the first replenishment at time $t = 0$. When at the point in time $t \in [0, T]$, the rate of change in the retailer's inventory level integrates the demand and deterioration rates (Tavakoli and Taleizadeh, 2017). Thus, the inventory changes with respect to time, $I(t)$, are satisfied by the following equation

$$\frac{dI(t)}{dt} = -\theta I(t) - D(t; p; u), \qquad 0 \le t \le T$$

$$D(t;\ u;\ p) = a - bp + \delta u + \beta I(t)$$

$$\frac{dI(t)}{dt} = -\theta I(t) - (a - bp + \delta u + \beta I(t)),$$

$$\frac{dI(t)}{dt} = -[a - bp + \delta u + (\beta + \theta) I(t)] \qquad 0 \le t \le T \tag{1}$$

Equation (1) has a boundary condition of $I(T) = 0$ and the solution is as follows

$$I(t) = \frac{a-bp+\delta u}{\beta+\theta}\left(e^{(\beta+\theta)(T-t)} - 1\right), 0 \le t \le T \tag{2}$$

and we could solve the Equation (1) and obtain the order quantity as

$$Q = I(0) = \frac{a-bp+\delta u}{\beta+\theta}\left(e^{(\beta+\theta)T} - 1\right). \tag{3}$$

Thus, we can get the total purchase cost in $[0, H]$

$$\frac{nc(a-bp+\delta u)}{\beta+\theta}\left(e^{(\beta+\theta)T} - 1\right) \tag{4}$$

and the total of ordering costs during the period $[0, H]$ is nS. (5)

- The total inventory held in $[0, H]$ is defined as

$$n\int_0^T I(t)dt = \frac{n(a-bp+\delta u)}{\beta+\theta}\left(\frac{e^{\theta T}-1}{\beta+\theta} - T\right) \tag{6}$$

and the total holding cost is generated by

$$\frac{nh(a-bp+\delta u)}{\beta+\theta}\left(\frac{e^{(\beta+\theta)T}-1}{\beta+\theta} - T\right). \tag{7}$$

- The number of products deteriorated in a cycle is formulated as

$$Q_d = \theta\int_0^T I(t)\,dt = \frac{\theta(a-bp+\delta u)}{\beta+\theta}\left(\frac{e^{(\beta+\theta)T}-1}{\beta+\theta} - T\right) \tag{8}$$

and the total deterioration cost in $[0, H]$ is

$$nc_d Q_d = \frac{nc_d\theta(a-bp+\delta u)}{\beta+\theta}\left(\frac{e^{(\beta+\theta)T}-1}{\beta+\theta} - T\right). \tag{9}$$

Thus, we have the total quantity sold in each cycle as follow

$$Q - Q_d = \frac{a-bp+\delta u}{\beta+\theta}\left(\frac{\beta}{\beta+\theta}\left(e^{(\beta+\theta)T} - 1\right) + \theta T\right). \tag{10}$$

- The total promotional effort cost in $[0, H]$ is $\frac{n}{2}\tau u^2$. (11)

The quadratic function is commonly utilized in operations research to represent the characteristics of the retailer's promotional effort, e.g., by Bai et al. (2016), Wu (2011), and Mukhopadhyay et al. (2009).

The total cost in the inventory system is equal to the sum of total holding cost, ordering cost, deterioration cost, and the promotional effort cost. Since the total revenue is $np(Q - Q_d)$, therefore, the total profit is given by

$$TP = \frac{np(a-bp+\delta u)}{\beta+\theta}\left[\frac{\beta}{\beta+\theta}\left(e^{(\beta+\theta)H/n} - 1\right) + (\theta H/n)\right] - \frac{nc(a-bp+\delta u)}{\beta+\theta}\left(e^{(\beta+\theta)T} - 1\right) - \frac{n(h+c_d\theta)(a-bp+\delta u)}{\beta+\theta}\left(\frac{e^{(\beta+\theta)H/n}-1}{\beta+\theta} - H/n\right) - nS - \frac{n}{2}\tau u^2. \tag{12}$$

while $T = H/n$, therefore the profit of the supply chain per unit of time is given by

$$TP = \frac{np(a-bp+\delta u)}{\beta+\theta}\left[\frac{\beta}{\beta+\theta}\left(e^{(\beta+\theta)H/n} - 1\right) + (\theta H/n)\right] - \frac{nc(a-bp+\delta u)}{\beta+\theta}\left(e^{(\beta+\theta)H/n} - 1\right) - \frac{n(h+c_d\theta)(a-bp+\delta u)}{\beta+\theta}\left(\frac{e^{(\beta+\theta)H/n}-1}{\beta+\theta} - H/n\right) - nS - \frac{n}{2}\tau u^2.$$

For a small x value, Taylor series could be used to approximate the exponential function as $e^x \approx 1 + x + \frac{x^2}{2!}$. To utilize the approximation, the value of x should satisfy $(\beta + \theta)H/n < 1$. Therefore, it indicates that $\theta H/n < 1$ and $\beta H/\,n < 1$ where $\beta < n/H$ as the upper bound of parameter β.

Using this result into Eq. (12), the approximated profit function is obtained as follows

$$TP = np(a - bp + \delta u)\left[\frac{\beta(H/n)^2}{2} + H/n\right] - nc(a - bp + \delta u)\left(\frac{(\beta + \theta)(H/n)^2}{2} + H/n\right) - \frac{n(h + c_d\theta)(a - bp + \delta u)}{2}(H/n)^2 - nS - \frac{n}{2}\tau u^2. \tag{13}$$

Theorem 1

There exists a unique value of selling price p^* that maximizes profit function TP for any given values of n and u.

In the inventory system, for any given n and u, the retailer sets optimal retail price that maximizes their profit over period $[0, H]$. The optimal selling price p^* can be obtained by assigning a value of zero to the first-order derivative of the function TP defined by Equation (13) with respect to p and we get

$$p = \frac{a+\delta u}{2b} + \frac{1}{2X_1}\left(cX_2 + \frac{(h+c_d\theta)X_3}{2}\right). \tag{14}$$

To simplify the equation, we denote $X_1 = \left(\frac{\beta(H/n)^2}{2} + \frac{H}{n}\right)$, $X_2 = \left(\frac{(\beta+\theta)(H/n)^2}{2} + H/n\right)$, $X_3 = (H/n)^2$.

Proof Refer to Appendix 1.

Theorem 2
There is a unique value of optimal promotional effort cost u^* that maximizes profit function TP for any given values of n and p.
For any given values of n and p, the profit function TP obtained by Equation (12) is concave with respect to the promotional effort cost u. Hence, it achieves its global maximum at point u^* when the reaction functions of the retailer satisfy $\frac{dTP}{du} = 0$, and we have:

$$u = \frac{\delta}{\tau}\left(pX_1 - cX_2 - \frac{(h+c_d\theta)X_3}{2}\right) \tag{15}$$

Proof Refer to Appendix 2.

To verify analytically the optimality of solutions in (14) and (15), the determinate of the Hessian matrix is calculated as $\det H = \left(\frac{d^2TP}{dp^2}\right)\left(\frac{d^2TP}{du^2}\right) - \left(\frac{d^2TP}{dpdu}\right)^2 > 0$ and obtained as $\det H = n^2X_1(2b\tau - \delta^2X_1)$. Given parameter $a > 0$ and $b > 0$, so that we have positive demand, while the deterioration rate θ and stock dependent consumption rate parameter β have small value, when $2b\tau > \delta^2X_1$ then it is can be verified that $\det H = n^2X_1(2b\tau - \delta^2X_1) > 0$. It shows that the Hessian matrix associated with TP of Equation (12) is a strictly concave function and negative definite at p^* and u^*, and could satisfy the optimality conditions for the profit function.

Theorem 3
For any given n, there are unique values of the optimal solutions p^* and u^* that maximizes the total profit.
Proof The optimal solution is can be obtained thru an iterative algorithm.
Using previous theorems, an algorithm is developed to obtain the optimal solutions as follows:
(1) In a range of n values, calculate the values of p^* and u^* from Equation (13) and (14) set them equal to zero
(2) Generate the optimal value of n, such that $TP((n^*-1), p^*, u^*) \le TP(n^*, p^*, u^*) \ge TP((n^*+1), p^*, u^*)$.

4. Numerical Examples and Sensitivity Analysis

Example 1 To illustrate the problem solution in the numerical study, we consider the parameter values as follows: a = 200 units, b = 4, β = 0.08, c = \$10 per unit, θ= 0.02, δ = 5, τ = 30, c_d = \$2 per unit, S = \$50 per order, h = \$2 per unit per week, and the planning horizon H is set as 12 weeks. With these values, the optimal solution is obtained as the optimal ordering frequency $n^* = 22$, the optimal price p^* = \$32.96, the promotional effort cost u^* = \$2.09 and the corresponding profit is \$19132.

In Table 3, we investigate the changes of parameter deterioration rate θ and the promotional effort cost efficiency coefficient τ on the optimal decisions, the total promotional effort cost Tu^*, and total profit TP. The basic settings of these parameters are in the previous Example 1; thus, we compare the sensitivity of the parameter changes on the optimal decision. For instance, from the solution obtained in Example 2, it shows that when the deterioration rate of product is high $\theta = 0.4$, the promotional effort cost and the selling price decrease. For a highly perishable product, the retailer needs higher frequency of replenishment and larger order quantity to fulfill demand which is generate higher ordering and inventory cost. In the Example 3, when the efficiency coefficient of promotional effort higher at $\tau = 60$, the retailer's optimal promotional effort cost and the selling price decrease. Furthermore, in the Example 4, with a less efficient promotional effort conducted at $\tau = 10$, the optimal promotional effort cost and selling price increase in the system. In addition, for a less efficient promotional effort, the optimal replenishment frequency decreases and the optimal ordering quantity increases that requires less ordering cost and more inventory cost at the retailer.

Table 2. The optimal solution is a global maximum at $n = 22$

n	u^*	p^*	Q	TP
16	3,79	40,28	45	16814
17	3,35	38,44	46	17796
18	2,99	36,92	46	18395
19	2,69	35,65	46	18746
20	2,45	34,58	46	18934
21	2,24	33,66	45	19015
22	**2,07**	**32,88**	**44**	**19024**
23	1,92	32,20	43	18986
24	1,79	31,61	42	18917
25	1,68	31,09	41	18827

Table 3. Impact of θ and τ changes on the optimal decisions, total promotional effort cost, and total profit over horizon planning H

Example	θ	τ	n^*	Q^*	p^*	u^*	Tu^*	TP^*
2	0.4	30	23	46	32,83	1,872	1210	17806
3	0.02	60	22	43	32,21	1,004	665	18422
4	0.02	10	20	55	38,37	8,512	7246	22092

In Figure 2 and Figure 3, we illustrate the optimal solution for various values of u. The distinct values of u are considered in numerical study to investigate the behavior of total profit function. We examine the impact of changes in the value of promotional effort cost u on optimal solution. Similar to Example 1, an identical set of input data is used. The findings indicate that increasing promotional effort leads to an increase in order quantity, whereas increasing the cost of promotional effort up to the feasible limit results in an increase in total profit. The results shows that as u increases, the optimal selling price p^* and the ordering quantity Q^* increase whereas the optimal ordering frequency n^* decrease which is quite rational. Furthermore, as the promotional effort cost increases, there is an increase in the total profit, but at some point, the next trend of total gain decreases as the promotional effort cost increases. In contrast, the total profit increases to a certain extent before declining subsequently. It shows a unique value of promotional effort cost u that is optimal for the inventory system. With an increased demand rate, the retailer clearly able to increase the order quantity and the promotional effort cost. Consequently, the increased demand could enhance the total profit, which is entirely reasonable.

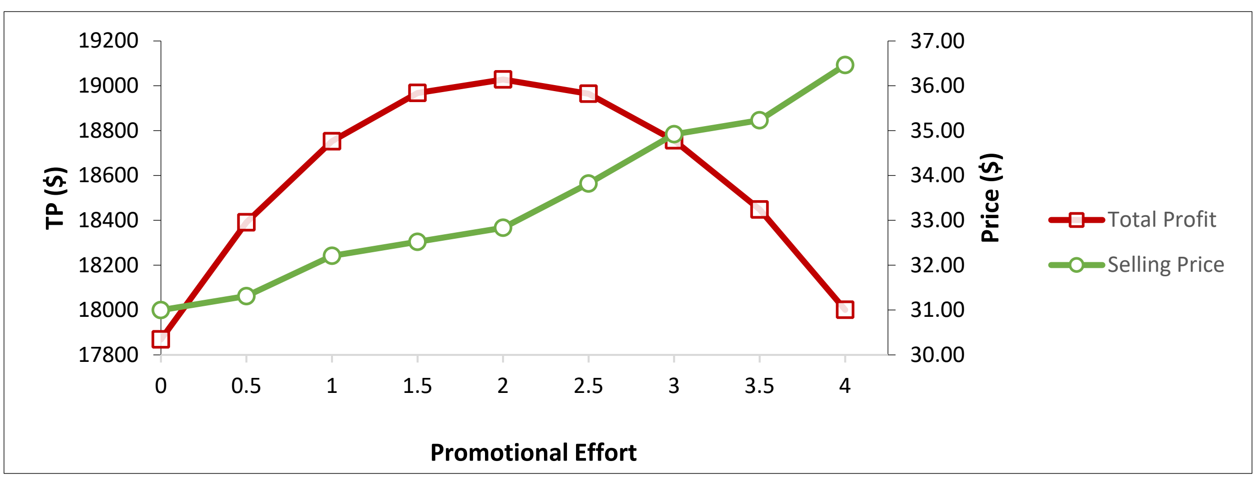

Figure 2. Impact of promotional effort on selling price and retailer profit

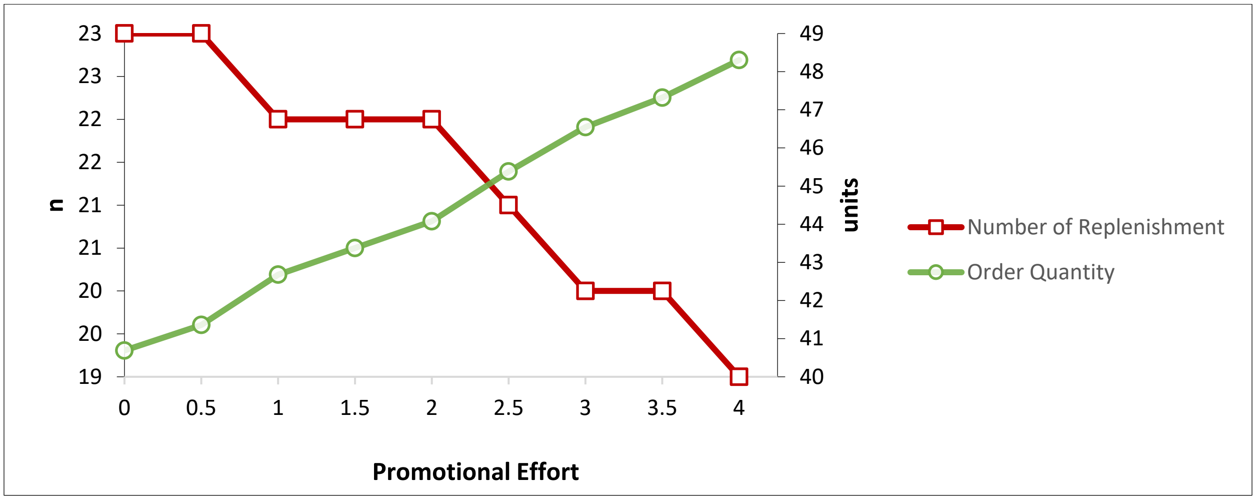

Figure 3. Impact of promotional effort on replenishment frequency and order quantity

Furthermore, a sensitivity analysis was carried out to investigate the effect of the parameters a, b, β, and δ on the optimal decisions in the retailer inventory system. Changes in parameter values by +50%, +25%, -25%, and -50% are applied to obtain the solutions shown in Table 4.

Table 4. Sensitivity Analysis of Demand Parameter

Parameter	% changes	n^*	Q^*	p^*	u^*	Tu^*	TP^*	% change Tu^*	% change TP^*
a	50%	20	79	47,93	3,817	4371	52965	209,20	178,41
	25%	21	61	40,32	2,892	2635	33923	86,44	78,31
	-25%	22	29	26,24	1,453	697	8235	-50,69	-56,71
	-50%	23	15	18,96	0,745	191	1523	-86,46	-91,99
b	50%	23	36	23,13	1,115	429	8697	-69,67	-54,29
	25%	23	39	26,72	1,433	708	12694	-49,88	-33,27
	-25%	20	52	44,24	3,439	3549	30312	151,07	59,33
	-50%	16	73	72,90	7,993	15335	56126	984,85	195,02
β	50%	22	44	33,04	2,107	1466	19239	3,69	1,13
	25%	22	44	32,96	2,088	1439	19132	1,83	0,57
	-25%	22	44	32,80	2,051	1388	18916	-1,80	-0,57
	-50%	22	44	32,72	2,032	1363	18807	-3,57	-1,14
δ	50%	20	51	36,81	4,017	4842	20797	242,54	9,32
	25%	21	47	34,53	2,911	2670	19774	88,88	3,94
	-25%	22	43	32,29	1,511	754	18496	-46,67	-2,78
	-50%	23	41	31,29	0,919	292	18136	-79,37	-4,67

Based on the analysis of these results, several conclusions can be presented as follows.

(i) The replenishment frequency decision n^* is highly affected by the changes in market scale parameter a. Meanwhile, the price b and promotional effort elasticity δ parameters have more impact on the optimal replenishment frequency decision than the stock elasticity β parameter does.

(ii) The optimal promotional effort cost u^* increases with an increase of the market scale a and promotional effort elasticity δ parameter values, while decreases with an increase of the price sensitivity b parameter value. It is highly sensitive by the changes of price elasticity b parameter, and less sensitive to changes in the value of stock elasticity β parameter.

(iii) The optimal selling price p^* is highly sensitive to changes in the value of parameter b, with the optimal selling price increasing as the price elasticity of the perishable product decreases. Correspondingly, the optimal selling price solution is more sensitive to changes in the market scale a parameter than changes in the stock elasticity β and promotional effort sensitivity δ parameters. An increased value of price elasticity b parameter implies a decreased market demand that obtains a lower total profit. With a decreasing demand, the retailer should reduce the order quantity and order more frequently. In addition, the retailer could be interested in decreasing the promotional effort cost and the selling price to improve sales and total profit.

(iv) The optimal total profit TP^* increases with an increase in the value of market scale a parameter and decreases with an increase in the value of price elasticity b parameter. It is less sensitive to changes in the stock elasticity β and promotional effort sensitivity δ parameter value. It is reasonable since the increase in market scale a implies an increase in market demand and consequently results in increased order quantity and less frequent replenishments. Increased order quantity results in more promotional effort cost invested in improving the product sales efficiently. Moreover, with an increased market demand scale a and a decreased price elasticity b, this condition allows the retailer to increase the selling price to obtain higher total profit.

5. Conclusions

This paper investigated the inventory model for a joint pricing, inventory, and promotional effort decision planning of a perishable product under a finite horizon setting. The demand is characterized as dependent on price and available stock in the retailer, which is further influenced by promotional effort. Furthermore, numerical examples, sensitivity analysis, and managerial implications are provided by varying the values of critical parameters to illustrate the model. Besides identifying the importance of pricing and inventory level, the numerical results also demonstrated the importance of promotional effort in operational decision-making at the retailer. The solution shows that investing in an optimal promotional effort cost can increase overall profit. It explains that in order to generate maximum profit, the retailer should promote the perishable product in a certain way so that the items can be efficiently promoted and sold during the selling period before being deteriorated. For future works, it is possible to extend this model to include a multi-item EOQ model, a trade credit policy, and the expansion of the entities involved in the distribution channel. Moreover, it would be interesting to investigate the model in the context of a supply chain.

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Appendix 1

Differentiating the profit function TP in Equation (12) twice partially with respect to p gives

$$\frac{dTP}{dp} = n(a-2bp+\delta u)X_1 + nbcX_2 + \frac{nb(h+c_d\theta)X_3}{2},$$

$$\frac{d^2TP}{dp^2} = -2nb\left(\frac{\beta\left(\frac{H}{n}\right)^2}{2} + \frac{H}{n}\right) < 0.$$

This implies that $\pi(n,p,u)$ is concave with respect to p. Now, for any given n, $\frac{dTP}{dp} = 0$ gives

$$p = \frac{a+\delta u}{2b} + \frac{1}{2X_1}\left(cX_2 + \frac{(h+c_d\theta)X_3}{2}\right).$$

Appendix 2

Differentiating (12) twice partially with respect to p, given $\alpha > 0$, $\delta > 0$, and $\beta > 0$, we get

$$\frac{dTP}{du} = n\delta pX_1 - n\delta cX_2 - \frac{n\delta(h+c_d\theta)X_3}{2} - n\tau u,$$

$$\frac{d^2TP}{du^2} = -n\tau < 0.$$

This implies that TP is concave with respect to p. Now, for any given n, $\frac{dTP}{dp} = 0$ gives

$$p = \frac{a+\delta u}{2b} + \frac{1}{2X_1}\left(cX_2 + \frac{(h+c_d\theta)X_3}{2}\right).$$

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