A Problem Space Search Heuristics for The Integrated Fleet Sizing and Replenishment Planning Problem

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Abstract
In this study we propose solution metaheuristics for the integrated fleet sizing and replenishment planning problem based on the problem space search approach utilizing fix and optimize algorithm. The problem is about deciding both the composition of a fleet for distributing a single item and the replenishment planning based on a predetermined delivery frequency. This problem integrates two important logistical issues namely fleet sizing and replenishment planning. The objective is minimizing all relevant costs composed of vehicle ownership, inventory, and approximate routing costs. We suggest a heuristic solution approach based on the problem space search method from the literature. This heuristic rely on perturbing same of the data in the problems, and using the fix and optimize algoritm. We show the effectiveness of the proposed solution method on a set of fairly large size randomly generated problems.

Keywords
Fleet sizing, Replenishment planning, Problem space search, Fix and optimize.

1. Introduction
Logistic related costs constitute between 3% and 15% of the total cost of a product as shown in a recent study by BVL International (Handfield et al. (2013)). The effective utilization of vehicle fleet has a significant impact on logistics costs for the companies who own a fleet. Hence, bringing quality solutions to strategic fleet sizing and composition problem is a critical logistical decision. Strategic fleet sizing problem mainly focuses on optimizing the profitability of the companies by determining the most suitable fleet size and composition. Achieving better utilization of time and vehicle capacities in logistic systems not only depends on effective transportation, but also on inventory management, as noted in Rad et al. (2014). Inventory and transportation have atrade-off from the perspective of the supply chain management, therefore examining inventory control or transportation optimization separately may lead to suboptimal decisions. General tendencies are that from inventory point of view at customers, small and frequent replenishment are better while form transportation and fleet cost perspective opposite is true. Therefore, examining inventory control or transportation optimization separately may result in suboptimal decisions.

In this study, we introduce a new solution approach for the problem of integrated fleet sizing and replenishment planning under predetermined delivery frequencies studied in Dastjerd et al. (2019). This problem considers both inventory and transportation costs in distribution of an item in an integrated manner. There is a distributor transporting an item from a depot to a set of customers with regular replenishment frequencies and we need to decide the fleet composition and the assignment of customers both to a vehicle and to one of the given frequencies. This problem is a strategic one. We give a brief description and the model of the problem in section 3.We propose a new and effective metaheuristic solution approache for this problem. The metaheuristic suggested in this study is a problem space search (PSS) meta heuristic that involves a fix and optimize method. In the PSS we suggest, the problem data is perturbed to find a set of quality solution. Performance of the suggested heuristic methods is assessed by carrying a numerical study on a set of randomly generated problems with different characteristics.

This paper is organized as follows; in the following section, we investigate the relevant literature. In Section 3, we go over the description and mathematical model of the problem. The meta heuristic we propose to solve the problem is discussed in Section 4. Numerical analysis and performance evaluation of the suggested solution approach are described in Section 5. Section 6 includes our conclusion for the study.

2. Literature
In this section, we survey the solution approaches proposed for the problems similar to the fleet sizing problem we consider. We also examine the studies that uses FO and PSS heuristics. ,

One of the papers that suggests a solution approach for a problem under predetermined frequencies is Bertazzi et al. (1997). In this research, items are delivered from one origin to many destinations, same as in our problem. Continuous frequencies, on the other hand, are employed, in contrast to our problem where we consider given discrete frequencies. Another similarity to our study is that it considers transportation and inventory-related costs, but they do not consider the fixed replenishment cost per visit. The heuristics in this study are developed based on the notion of resolving a single link problem in the first phase, and then locally enhancing the solution in succeeding phases. In single link problems, each destination point is considered independently and a MIP is formulated. Then, a shortest path problem for this one-to-one connection is solved. The decisions to be made here are the frequency-truck assignment and the fraction of the load carried. An earlier study that considers distribution problem with candidate frequencies as well is M. G. Speranza et al. (1994). The integer and mixed integer formulations in M. G. Speranza & Ukovich (1994), allow for both divisible and indivisible demand scenarios. Moreover, transportation cost is assumed to be proportional to the number of the trips a vehicle makes. As the solution approach, they utilize some dominance rules devised in M. G. Speranza & Ukovich (1994) and use them in a two-step heuristic. Bertazzi et al. (2000) considers the problem of shipping several products from an origin to a single destination under a predetermined frequency set. The problem is solved both by an exact and a heuristic approach. The exact method is a problem specific branch and bound method, while the heuristic methods are based on shipment strategies and the famous EOQ formula. Bertazzi et al. (1999) investigates the problem of minimizing the sum of inventory and transportation costs in a different setting than our problem where there are multi-products, one origin, some intermediate nodes, and a destination assuming a predetermined delivery frequency set. They offer different heuristic methods. The first one is based on the idea of decomposing the sequences in links and optimizing each link separately. The second heuristic is developed upon the idea of applying the same approximate shipping percentages to all links in the network. The third and the fourth heuristics are discretized versions of the EOQ-based algorithms of Blumenfeld et al. (1985). The last heuristic is a dynamic programming-based technique in which the links are taken as stages and the set of shipping frequencies on the previous link is taken as states. As another study, Bertazzi et al. (2005) tackles a difficult production-distribution system, where several items are produced regularly and they are distributed to a set of retailers using a fleet of vehicles under VMI setting. They offer two hierarchical heuristics, in which the production sub problem is addressed first and then the distribution sub problem is solved based on the production solution. In M. Speranza et al. (1996), again, the authors study the problem of distributing various products from a single source to a multiple set of destinations under given delivery frequencies. A branch-and-bound algorithm is devised in this study.

The studies that are directly on fleet sizing are also interesting to us. The study in Żak et al. (2011) considers a fleet sizing problem in a road freight transportation company with a heterogeneous fleet. There are two steps to the solution method in Żak et al. (2011). In the first, an innovative software named MEGROS generates a sample of Pareto-optimal solutions. This collection is examined and assessed in the second phase, using the Decision Maker's model of preferences. Sayarshad et al. (2009) suggests a new formulation and a solution procedure for optimizing the fleet size and freight car allocation. A Simulated Annealing technique is presented to solve the model to address this issue. To get out of a trap, the algorithm uses a neighborhood search inside solution space, acceptance probability, and inferior solutions. Another study that deals with a problem that is comparable to the one we are dealing with here, is Desrochers et al. (1991). In this work, the authors address the problem of simultaneously determining the fleet composition and the routing, where the customer demands are known and satisfied from a central depot. A novel savings heuristic based on consecutive route fusion is presented. The optimal fusion is chosen at each iteration by solving a weighted matching problem. This results in a less myopic criterion than the traditional savings heuristics.

Problem Space Search (PSS) is a local search metaheuristic that combines a fast problem specific heuristic into a search procedure. This method was first introduced in Storer et al. (1992). PSS is applied to the resource-constrained scheduling problem in Naphade et al. (1997). The results produced by PSS was comparable to the ones from the branch and bound method. In Leon et al. (1997), authors applied PSS metaheuristic to the flexible flow line scheduling problem and showed that the results prove the effectiveness of the solution method. Rail networks rescheduling problem is studied in Albrecht et al. (2013). They applied PSS metaheuristic and showed the PSS can rapidly generate a large number of alternative train timetables. In Storer et al. (1996), researchers used PSS to solve the number-partitioning problem. They claimed that improvements occurred in solution quality despite the greater computational time. A heuristic based on Lagrangian relaxation and PSS is developed in Jeet et al. (2007).They applied the generated metaheuristic to the generalized assignment problem. Some areas to which PSS has been applied are job shop scheduling, resource constrained project scheduling, number partitioning, and more recently, train timetabling.

In this paper, we embeded a Fix and Optimize (FO) heuristic in PSS. It is possible to follow the origin of FO to the Pochet et al. (2006) who described an improvement heuristic named as “exchange heuristic" to solve a multi-level capacitated lotsizing problem (CLSP). The need for MIP based heuristics arised from the fact that the state of art solvers were not able to produce high quality or optimal solutions to MIPs within acceptable computational time (Tanksale et al. (2020)). FO is based on dividing the main problem into smaller subproblems that are supposed to be easier to solve. One of the researches is Gintner et al. (2005) in which fix and optimize heuristic is used for bus scheduling problem. Federgruen et al. (2007) has applied the fix and optimize heuristic to multi-product, capacitated lot sizing problem. In Helber et al. (2010) the same heuristic is utilized for solving the multi-level capacitated lot sizing problem. One problem which has some similarities to our problem is the one studied in Dorneles et al. (2014). The problem considered in this research is a full integer problem with all variables defined as binaries except for one integer variable. Fix and optimize heuristic is applied to the problem and the efficiency of the algorithm is analyzed. It is stated that the proposed fix and optimize heuristic were able to find new best known solutions for seven instances including three optimal ones. FO is applied to different problem types, such as stochastic CLSP (Helber et al. (2013)), cooperative lot-sizing problem (Drechsel et al. (2011)), and CLSP with setup carryover (Chen (2015), Gören et al. (2015), Goren et al. (2012)).

Dastjerd & Ertogral (2019) is the first paper, to the best of our knowledge, that considers fleet sizing and replenishment planning problem under candidate delivery frequencies, which considers ownership, routing, and inventory related costs. They suggest a FO type heuristic approach for this problem. Here we suggest an enhanced solution approach for the same problem. Our suggested heuristics are PSS type that employs FO solution procedure.

3. Problem Description and Formulation

In this section, we give a description and a mathematical formulation of our problem. The problem description and the formulation is adopted from Dastjerd & Ertogral (2019) and they are included below for the sake of completeness. A single product with deterministic demand is available at a depot, and the product is demanded at multiple destinations with a constant rate. We aim at minimizing the sum of transportation and inventory cost of shipping the product. The problem is about deciding how often and how much to replenish inventory of each customer along with the determination of vehicle fleet size and composition. A set of heterogeneous vehicles is used to ship a single product to the customers, and the vehicles vary in terms of carrying capacity, cost per kilometer, and ownership costs. Replenishments are carried out based on a set of given frequencies, which are, in general, defined based on the number of weeks between deliveries and the day of delivery. Thus, inventory of a customer can be replenished weekly, biweekly, thrice or quarto-weekly, on any of the five weekdays. In addition to weekly frequencies, we also consider daily frequency. In total, we have 21 different possible frequencies. In terms of labeling frequencies, frequency 1 corresponds to the daily frequency, while remaining frequencies from 2 to 6 correspond to weekly frequencies for five weekdays. Similarly, frequencies 7–11, 12–16 and 17–21 correspond to biweekly, thrice- weekly and quarto-weekly frequencies, respectively, for each of the five weekdays. Some of the frequencies coincide periodically, and it is important to keep track of the coinciding frequencies in order to both preventing multiple counting of deadheading costs and representing capacity constraints correctly on coinciding days. If we consider the frequencies that include Mondays, the coinciding frequencies would be 2, 7, 12, 17, plus the daily frequency. We make few assumptions on some operational issues in our problem as follows; inventory of each customer must be replenished using a single vehicle and a single frequency, limited number of customers can be visited on a specific day and we considered geographic location in grouping customers on a route. An important aspect of our model is that it takes the routing cost into account in an approximate fashion as the product of the number of customers visited on a route and the average cost of travel between customers. The notations and the MIP model is as described below:

Sets:

I : Set of customers
V : Set of vehicles
F : Set of frequencies, F= {1, 2, ..., 21}
F_j : Set of coinciding frequencies, $\forall j = 1, \dots, n$
D : Set of days of the week, D = {1, 2, …, 5}
H : Set of weeks per year, H = {1, 2, … , 52}

Parameters:

N : Number of coinciding frequency sets
m : Number of customers
r_v : Approximate routing cost between two customers (fixed per kilometer cost of each vehicle)
g : Dead heading cost
a_v : Annual ownership cost of vehicle type v
λ_{if} : Annual demand of customer i using frequency f
h : Annual inventory holding cost per unit of a product
k_{if} : Fixed cost of replenishing customer i using frequency f
s_{max}: Maximum number of customers that can be visited during the day
c_v : Capacity of vehicle type v
M : A big number

p_f : Total number of annual replenishments for frequency f
t_{ik} : Incidence matrix of customers i and k (customers i and k can be in the same route if t_{ik}=2, and cannot be in the same route when t_{ik}=1)

$$Min \ \sum_{v\in V} gp_1 L_{v1} + \sum_{d\in D}\sum_{h\in H}\sum_{v\in V} gR_{dvh} + \sum_{v\in V}\sum_{f\in F} r_v \cdot p_f \cdot C_{vf} + \sum_{i\in I}\sum_{v\in V}\sum_{f\in F}\lambda_{if} \cdot h \cdot X_{ivf} \cdot (\tfrac{1}{2}) + \sum_{i\in I}\sum_{v\in V}\sum_{f\in F} k_{if} X_{ivf} \quad (3.1)$$

Subject to

$$\sum_{v\in V}\sum_{f\in F} X_{ivf} = 1 \qquad \forall\, i\in I \qquad (3.2)$$

$$\sum_{i\in I} X_{ivf} = C_{vf} \qquad \forall\, v\in V, \forall f\in F \qquad (3.3)$$

$$ML_{vf} \geq C_{vf} \qquad \forall v\in V, f\in F \qquad (3.2)$$

$$L_{vf} \leq C_{vf} \qquad \forall v\in V, f\in F \qquad (3.3)$$

$$M\, V_v \geq \sum_{i\in I}\sum_{f\in F} X_{ivf} \qquad \forall\, v\in V \qquad (3.4)$$

$$\sum_{i\in I}\lambda_{if} X_{ivf} \leq c_v \qquad \forall v\in V, \forall f\in F \qquad (3.5)$$

$$\sum_{i\in I}\sum_{f\in F_j}\lambda_{if} X_{ivf} \leq c_v \qquad \forall\, v\in V, \forall j = 1,\dots,n \qquad (3.6)$$

$$\sum_{f\in F_j} C_{vf} \leq s_{max} \qquad \forall\, v\in V, \forall j = 1,\dots,n \qquad (3.7)$$

$$\sum_{f\in F_j} X_{ivf} + \sum_{f\in F_j} X_{kvf} \leq t_{ik} \qquad \forall v\in V, i\in I, k\in I, \forall j = 1,\dots,n \qquad (3.8)$$

$$R_{dvh} \geq L_{vf} - L_{v1} \qquad \forall v\in V,\ d\in D, f\in Fj, h\in H \qquad (3.9)$$

$$X_{ivf} \in \{0,1\} \qquad \forall\, i\in I, \forall v\in V, \forall f\in F \qquad (3.10)$$

$$L_{vf} \in \{0,1\} \qquad \forall v\in V, \forall f\in F \qquad (3.11)$$

$$C_{vf} \in Z_{\geq 0} \qquad \forall v\in V, \forall f\in F \qquad (3.12$$

$$V_v \in \{0,1\} \qquad \forall v\in V \qquad (3.13)$$

$$R_{dvh} \in \{0,1\} \qquad d\in D, v\in V, h\in H \qquad (3.14)$$

In this model, we ensure demand satisfaction by (3.2). Number of customers assigned to a vehicle-frequency combination is calculated by (3.3). Constraints (3.4) and (3.5) calculate the input for constraint (3.11) which eliminates unnecessary deadheading cost. By (3.6) we decide whether a vehicle type is used or not. Capacity limit is reflected to the model by constraints (3.7)and (3.8). Number of customers to be visited on a route in a day is considered in (3.9). Constraint (3.10)is used to show geographically clustered customers. Constraints (3.12)-(3.16)show the decision variable domains. For further details about the model and its constraints one can refer to Dastjerd & Ertogral (2019).

4. Problem Space Search Metaheuristic

A novel meta heuristic technique known as Problem Space Search (PSS) was first introduced in 1992 (Storer et al. (1992)). As stated in Naphade et al. (1997), PSS is a local search metaheuristic which embeds a fast problem-specific heuristic within a probabilistic search procedure. In PSS, the neighborhood in which the search is done is defined as a pair denoted as (h, p). Here, h is the problem-specific heuristic and p represents the problem data. The solution obtained is simply the result of applying the heuristic to the problem, or s = h (p). By perturbing either the heuristic or the problem data (or both), a neighborhood of solutions is generated. This neighborhood can then be searched using local search techniques.

The basic and most important idea in PSS is that by perturbing the data and reapplying a problem specific base heuristic, the sequence of decisions within the heuristic is changed, which leads to a different solution. When assessing these non-identical solutions, however, the original problem data is used to calculate the value of the objective. One important assumption in PSS is that the base heuristic is well suited to the problem, which means that it produces quality solutions.

As a result, even if the original problem data is slightly changed, it is acceptable to anticipate that the use of the base heuristic would yield good results in terms of the original data.

The insertion of a randomly generated vector is the most frequent approach for perturbing the original data. The magnitude of each perturbation is one of the most critical considerations in PSS. If the perturbation size is too small, the data may not have enough variety; on the other hand, with a value that is too large, the solution quality obtained from the heuristic will diverge by making almost entirely random decisions.

Here, we suggest two different versions of PSS. Both PSS heuristics utilize a Fix and Optimize (FO) heuristic we previously introduced for the problem in Dastjerd & Ertogral (2019) as the base heuristic. We first explain the FO heuristic used in our solution approach and then explain the two PSS heuristics.

4.1 Fix and optimize heuristic

The FO approach is a two-phase heuristic that works by breaking down a problem into smaller subproblems which are solved iteratively. In the first phase, we divide the problem into subproblems/subgroups of variables and then solve the problem by setting variables in only one subproblem as integers, and taking the remaining variables as either fixed to the values found in the previous iterations or as linear variables. The solution for binary variables found in an iteration is fixed and used in the next iterations. This procedure is repeated until all variable values are found. The exact steps of firt phase is given below.

FO heuristic –Phase I

1. Divide the customers into P sub groups with equal number of customers, or as equal as possible. Divide the decision variables in to P sub groups corresponding to P customer groups. Let A_i be the set of binary variables in ith sub group.
2. Set the iteration counter i to 1.
3. If i>1, Fix the binary variables in A_j to the values of variables in iteration i-1 for j = 1. .i-1.
4. Set the variables in A_i as binary variables.
5. Relax the binary variables in A_j for j = i + 1. .P as linear variables.
6. Solve the complete model
7. Set i = i + 1. If i <= P go to step 3. If i > P, STOP.

In the second phase, we try to improve the solution obtained in the first phase. At each iteration in the second phase, we set variables of one subproblem as binaries and reoptimize them as we keep all other variables fixed to the solution found in the previous iteration. If there is any change in the solution we restart the iteraitons of the second phase with the new solution. We stop the second phase if there is no change in the solution. The reason for trying to reoptimize the variables of each subproblem is that when they are decided in the first phase the remaining variables are taken as linear but now these ramining varibales have integer values and we may want to change the solution to the current subproblem. The steps for the second phase is given below.

FO heuristic –Phase II

1. Take the solution from Phase I as the current solution.
2. Set the iteration counter i to 1.
3. If i=1 Fix the values of variables in A_j for j=i+1..P to the values found in the current solution.
4. İf i>1 Fix the values of variables in A_j for j=1..i-1 and j=i+1..P to the values found in iteration i-1
5. Define the variables in A_i as binary variables. (Note that all other variables in other subgroups are fixed)
6. Solve the complete model and set i = i + 1.
7. Check the objective value, if the objective value is better than best solution so far and i < P, update the best solution as the current solution (improved solution) and go to step 2. If no improvement occurred in the current iteration and i < P go to step 4. If no improvement occurred in the current iteration and i > P STOP and RETURN the solution.

4.2 PSS with cost data perturbation

In the PSS heuristic we sugeest, all the cost components in the problem, i.e., routing costs, ownership costs, setup costs and holding costs, are perturbed using a randomly generated vector. Then phase I of FO is applied to the problem for each of the generated instances. The best solution in terms of the original data among the solutions produced is chosen. Then the phase II of FO is applied on the best solution from phase I in order to obtain the final solution. The notations and steps for PSS is as follows:

Notations:

n: Number of random instances to be generated
$Data_i$: Generated random instance i
R: Random range vector for random cost component generation.
C: Original cost components vector.
RC_i: Random cost components vector for instance i
FOS_i : FO solution for instance i
S: Current best solution
CurrentObj : Current best objective value.
Obj_i: Objective value of instance i.
α: Perturbation factor

PSS with cost purturbation:

1. Set instance counter i=1 , *currentObj*=infinity.
2. Generate a range vector R for cost vector C (to be used to generate RC_i's) using R = $C +/- \alpha * C$
3. Generate $Data_i$ of instance i by randomly producing RC_i in R
3. Solve the instance i by applying phase I of FO using $Data_i$.
4. Calculate obj_i using the FOS_i and C.
5. If obj_i < $CurrentObj$. set $CurrentObj \leftarrow Obj_i$, and $S \leftarrow FOS_i$
6. Set i=i+1 and Repeat step 2 through 5 until i>n.
7. Select the best solution out of n solutions (i.e. *currentObj and S*).
8. Try Improve $CurrentObj$ of the best solution by applying Phase II of FO on the best solution found, using C as the data vector. Return the final solution.

5. Numerical Analysis

In this section, we introduce the instances solved and the results from the PSS based solution approach. We tested our approach on four different scenarios corresponding to four problem characteristics: Normal demand with no customer clusters, 50% increased demand with no customer clusters, normal demand with four customer clusters, and 50% increased demand with four customer clusters. In the clustered scenarios, two customers in different clusters cannot be served by the same vehicle. To obtain versatility in problem parameters, we consider 24 different parameter settings regarding setup cost (k), ownership cost (A), routing cost (R) and capacity (cap) of the vehicles as described in the Table 1

Table 1. Parameter setting.

Parameter Setting					**No**
				Cap	1
			R=20%	Cap	2
		A=20%		Cap	3
				Cap	4
			R=40%	Cap	5
	h=300			Cap	6
				Cap	7
			R=20%	Cap	8
k=50		A=40%		Cap	9
				Cap	10
			R=40%	Cap	11
				Cap	12
				Cap	13
			R=20%	Cap	14
	h=600	A=20%		Cap	15
				Cap	16
			R=40%	Cap	17
				Cap	18

A=40%	R=20%	Cap	19
		Cap	20
		Cap	21
	R=40%	Cap	22
		Cap	23
		Cap	24

All problems have 40 customers, and the annual demand of the customers are generated using uniform distribution in the range [80,120].

The perturbation factor alpha used in PSS is set to 10% (α_1) and 20% (α_2). Using the given perturbation factor we randomly generated and solved n=50 instances by calling phase I of the FO heuristic. Additionally, we applied FO directly to the original data so that we can see what improvement that the PSS we suggest brings over simple FO apporach. The results for the PSS and simple FO heuristics are presented below in Table 2. The initials, Sc, stands for scenarios. Tables 2 shows the deviation of objective function value from either the optimal solution or the best lower bound found by solving the model directly using CPLEX with a time limit of three hours run time. The gaps marked with star (*) show the percentage deviation from the best lower bounds.

In Table 3, the solution durations for FO, PSS are tabulated in seconds. The average vehicles used in the solutions across the problems in each scenario is given in Table 4. L and S as the headings of the columns stand for large and small vehicle vehickles.

In general, PSS heuristic perform better in the scenarios with no customer clusters. As stated in Table 2, the gaps for the scenarios with clustered customers are higher than the cases without clusters. As we can see in Table 3, the solution times for the clustered scenarios are also higher in comparison to the ones without clustering. The explanations for this increased gap and solution times of clustered cases is as follows; clustering puts limitation on the customers who can be on the same route, which results in higher number of used vehicles as we can see on table 4. Among the cost elements, the highest one is the ownership cost and the extra vehicle usage resulting from clustering increases objective function value significantly. Additionally, the fleet composition generation takes longer because of the higher number of choices due to increased number of vehicles needed in the solutions. Demand increase also affects optimality gaps in the same way but not as significantly as the clustering.

Performance of the PSS with $\alpha = 20\%$ seems to better than solutions obtained with $\alpha = 10\%$. As stated in previous sections, some sub problems are easier to solve than the others due to the variable structure which decreases the solution times.

Comparing gaps of FO to PSS, we can seen that the neighborhood generation through perturbation was effective in terms of improving the solution quality. As we mentioned above, PSS generates neighbors not very dispersed from the original data in the problem space, which makes it possible for the heuristic to find the near optimal solutions. Solution times for PSS is reasonable given the improvement they bring.

In terms of vehicle usage, Tables 4 gives the average number of large and small vehicles used in FO and PSS. Demand increase and clustering results in vehicle usage increase, which explains the increase in gaps from optimal or bounds. As it is demonstrated in tables, PSS decrease the total average of used vehicles due to the neigbor generation strategy which makes it possible to make better choices of vehicle combinations. Overall, PSS decreases the number of vehicles used and leads to lower total cost.

Table 2. Gaps from optimal/ best bounds for FO and PSS-I .

Problem No.	Sc1			Sc2			Sc3			Sc4		
	FO	**PSS(α_1)**	**PSS(α_2)**	**FO**	**PSS(α_1)**	**PSS(α_2)**	**FO**	**PSS(α_1)**	**PSS(α_2)**	**FO**	**PSS(α_1)**	**PSS(α_2)**
1	14.70%*	9.95%*	9.95%*	3.76%*	3.76%*	3.76%*	17.61%	10.93%	11.22%	15.62%*	15.62%*	15.62%*
2	4.57%	3.53%	0.51%	7.46%*	7.46%*	7.46%*	1.34%*	1.64%*	1.64%*	21.17%*	6.94%*	6.94%*
3	1.65%*	1.41%*	1.22%*	10.29%*	10.26%*	10.26%*	2.06%	1.44%	1.36%	12.93%*	12.28%*	12.28%*
4	9.90%	1.55%	9.90%	16.58%*	4.77%*	4.77%*	17.05%	11.55%	11.64%	12.00%*	12.00%*	12.00%*
5	4.15%	3.07%	1.35%	2.18%*	2.18%*	2.18%*	1.43%	1.29%	1.12%	5.60%*	5.60%*	5.60%*

6	1.90%*	1.30%*	1.40%*	10.06%*	10.04%*	10.04%*	1.47%	0.87%	1.18%	7.67%*	7.66%*	7.66%*
7	13.66%	4.00%	12.49%	14.72%*	6.56%*	6.56%*	13.91%	13.96%	7.53%	11.50%*	11.50%*	11.50%*
8	2.47%*	1.62%*	1.24%*	8.38%*	6.27%*	6.27%*	11.71%	1.01%	1.01%	20.78%*	10.04%*	10.04%*
9	0.98%	0.98%	1.02%	10.19%*	10.17%*	10.17%*	2.20%*	2.04%*	1.90%*	6.83%*	6.83%*	6.83%*
10	12.98%	13.15%	4.13%	14.49%*	14.49%*	14.49%*	13.51%	7.26%	7.26%	11.19%*	11.19%*	11.19%*
11	2.34%	1.60%	1.32%	7.75%*	6.17%*	6.17%*	9.88%	1.01%	1.01%	27.42%*	10.78%*	10.78%*
12	1.35%*	1.28%*	1.11%*	10.00%*	9.97%*	9.97%*	1.98%*	1.32%*	1.28%*	7.10%*	7.10%*	7.10%*
13	12.93%	9.46%	9.46%	15.70%*	4.16%*	4.16%*	15.69%	9.99%	9.99%	12.67%*	12.63%*	12.63%*
14	1.18%	0.22%	0.26%	10.31%*	6.62%*	6.62%*	13.82%	0.31%	0.60%	20.35%*	7.11%*	7.11%*
15	0.06%	0.23%	0.46%	0.00%	0.00%	0.00%	0.94%	0.76%	0.76%	5.70%*	5.69%*	5.69%*
16	13.88%	9.94%	9.94%	16.07%*	5.01%*	5.01%*	15.79%	11.96%	11.33%	12.25%*	12.25%*	12.25%*
17	1.79%	1.02%	1.07%	9.96%*	6.76%*	6.76%*	0.60%	0.60%	0.60%	31.70%*	7.12%*	7.12%*
18	0.06%	0.46%	0.46%	0.00%	0.46%	0.46%	1.25%	0.76%	0.70%	21.33%*	21.32%*	21.09%*
19	12.60%	13.05%	4.54%	14.01%*	15.10%*	15.10%*	13.48%	7.66%	7.66%	12.15%*	12.11%*	12.11%*
20	1.46%	0.81%	1.02%	7.68%*	7.68%*	7.68%*	0.31%	0.60%	0.29%	29.99%*	21.24%*	11.75%*
21	0.29%	0.23%	0.46%	0.00%	0.46%	0.00%	1.76%	0.76%	0.76%	3.78%*	3.77%*	3.77%*
22	11.95%	12.24%	12.46%	14.68%*	15.87%*	15.87%*	7.29%	7.29%	7.29%	11.80%*	11.80%*	7.60%*
23	1.50%	0.81%	0.87%	7.29%*	7.29%*	7.29%*	13.33%	0.32%	0.60%	21.74%*	21.74%*	12.59%*
24	0.29%	0.23%	0.23%	0.00%	0.46%	0.46%	1.41%	0.76%	0.76%	3.20%*	3.20%*	2.99%*
Ave	**5.36%**	**3.84%**	**3.62%**	**8.82%**	**6.75%**	**6.73%**	**7.49%**	**4.00%**	**3.73%**	**14.44%**	**10.73%**	**9.76%**

Table 3. CPU times in seconds for FO and PSS-I.

Problem No.	**Sc1**			**Sc2**			**Sc3**			**Sc4**		
	FO	**PSS(α_1)**	**PSS(α_2)**	**FO**	**PSS(α_1)**	**PSS(α_2)**	**FO**	**PSS(α_1)**	**PSS(α_2)**	**FO**	**PSS(α_1)**	**PSS(α_2)**
1	51.72	1769.83	1783.86	197.05	11571.87	11571.87	37.47	1515.56	1494.37	414.43	21807.53	20648.54
2	29.47	650.32	701.75	121.73	4783.66	4783.66	30.97	958.34	897.6	160.29	5728.46	5728.46
3	20.25	571.85	556.16	107.06	3931.8	3931.8	46.91	736.47	685.56	116.99	4870.6	4870.6
4	52.98	1791.47	1729.82	233.39	13684.88	13684.88	42.17	1477.43	1557.75	793.41	22857.51	22857.51
5	37.41	659.57	643.45	125.98	5198.52	5011.31	33.14	1012.97	975.56	135.27	5841.61	6133.95
6	29.11	597.93	547.27	99.28	4095.44	4422.85	40.96	737.77	734.38	113.79	4984.75	4712.05
7	58.19	1784.2	1774.74	393.43	13047.93	13047.93	42.97	1615.65	1653.04	516.21	20428.12	20428.12
8	33.39	774.46	793.74	135.98	5439.69	5467.95	70.78	1155.26	1069.05	139.19	5866.64	6399.86
9	24.41	525.73	566.09	114.72	4083.09	4284.69	35.01	804.72	795.65	109.54	5380.35	5061.36
10	60.16	1755.54	1801.46	356.10	13954.52	13954.52	42.00	1545.85	1592.93	706.34	22066.71	22066.71
11	34.83	755.35	800.87	116.12	5789.67	5386.92	48.61	1135.29	1038.25	154.52	6328.42	6338.41
12	24.17	539.51	544.15	112.40	4010.21	4519.12	35.67	808.25	771.41	119.32	4977.58	4968.16
13	51.84	1774.1	1729.09	136.84	6492.48	6492.48	40.83	1579.85	1483.76	197.06	13109.53	13109.53
14	31.97	679.75	620.71	87.44	3217.56	3247.01	47.99	815.37	720.94	141.20	4135.01	4374.48
15	26.48	481.84	476.71	50.33	1641.18	1817.66	28.23	604.43	612.14	79.90	2930.74	2869.2
16	51.69	1742.37	1737.94	148.25	5983.38	5983.38	49.42	1525.99	1551.61	410.25	14065.42	14065.42
17	33.30	706.75	670.98	96.63	5983.38	3402.11	28.36	796.35	725.26	95.55	4470.91	4243.99
18	33.81	469.18	476.57	50.27	1683.79	1846.04	23.73	597.05	617.35	77.48	2964.55	2914.87
19	77.34	1670.1	1739.87	138.65	7009.22	7009.22	38.39	1591.87	1511.96	383.53	15728.56	15728.56
20	25.11	732.38	701.21	81.67	3489.03	3643.82	24.74	819.16	755.39	143.31	5235.93	5032.55
21	25.78	490.35	472.4	48.64	1692.2	1709.71	35.04	602.25	615.5	67.05	3096.08	3040.56
22	71.11	1689.82	1697.06	161.18	7491.51	7491.51	45.23	1533.98	1448.73	303.83	14880.84	13613.42
23	34.73	712.73	723.34	78.43	3608.35	3568.19	29.27	796.42	789.74	103.62	4827.56	4738.29
24	26.47	481.37	503.33	51.37	1752.55	1919.38	32.24	583.54	607.78	79.12	3084.7	3091.77
Ave	**39.40**	**991.94**	**991.36**	**135.12**	**5818.16**	**5758.25**	**38.76**	**1056.24**	**1029.4**	**231.72**	**9152.84**	**9043.18**

Table 4. Average vehicle numbers from FO and PSS heuristics.

	Sc1						**Sc2**						**Sc3**						**Sc4**					
	FO		**PSS-I(α_1)**		**PSS-I(α_2)**		**FO**		**PSS-I(α_1)**		**PSS-I(α_2)**		**FO**		**PSS-I(α_1)**		**PSS-I(α_2)**		**FO**		**PSS-I(α_1)**		**PSS-I(α_2)**	
	L	**S**	**L**	**S**	**L**	**S**	**L**	**S**	**L**	**S**	**L**	**S**	**L**	**S**	**L**	**S**	**L**	**S**	**L**	**S**	**L**	**S**	**L**	**S**
Ave	0.50	1.79	0.54	1.58	0.75	1.33	0.46	1.83	0.79	1.29	0.79	1.33	0.46	2.33	0.54	2.13	0.67	2	0.92	2.17	1.21	1.54	1.38	1.38

6. Conclusion

In this paper, a new solution technique is suggested for the problem of integrated fleet sizing and replenishment planning with a given candidate delivery frequencies is investigated. There is a set of customers with deterministic demand, which need to be replenished based on the set of given frequencies. The frequencies are based on the weeks between the deliveries and a daily replenishment is also considered. A fleet of heterogeneous vehicles are used to ship the demands to destinations. The problem under study is a strategic one, hence, the routing costs are taken as approximate values instead of detailed exact routing. Here, our main purpose is to determine the triple customer-vehicle-frequency assignment with the most possible least cost.

The problem is a NP-hard and in case of large instances the ready to use packages such as CPLEX are unable to produce optimal solutions to the problem in reasonable solution times. Here, in this study, we introduced a PSS meta heuristic and showed the effectiveness of the heuristics on a set of randomly generated problems with different characteristics.
As future research directions one can investigate integrating different heuristic in PSS instead of FO approach. Another possible direction of research is modeling the problem under more general replenishment frequency patterns instead of regular patterns based on weeks as we assume in the current study. One can also looking on to expanding the problem definition by including seasonal demand scenarios and option of renting vehicles in high demand seasons.

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