

Determination of Burn-in Period Based on Reliability Target for Product Failure Time with Weibull Distribution

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Abstract

In today's market issues, competitions between industries are highly inevitable. Strategic planning must really be considered and one of these is by improving the reliability of the products. It was also known that burn-in process has a great impact towards having this kind of plan. In this paper, the researchers will have to determine the burn-in period based on the reliability target for product failure rates under Weibull distribution. In doing this, the methodology has been designed by analysing the properties of the functions of reliability. Conditional reliability was used to know the time T_0 which indicates as burn-in time that will best fit into the reliability target. The researchers incorporated the Weibull distribution to this computation to specify the kind of behavior of the failure rates. Moreover, numerical examples were also used to prove how this process works in a certain scenario.

Keywords

Reliability, Reliability Target, Burn-in Period, Weibull Distribution

1. Introduction

In an economy where the competition is high and every player wants to acquire as much market share as possible, satisfying the customer is an essential part. This is especially true in the technology industry. According to Deloitte Touche Tohmatsu organization (2013), technology companies around the world are confronted with growing competition from adjacent and overseas markets and the accelerating pace of various technologies.

Moreover, one of the methods of approaching the question of customer satisfaction is through improvements of the reliability of the product. There are many ways to improve the reliability of a product or component; one of them is to undergo burn-in process. According to Kuo and Kim (1999), burn-in is an accelerated life test that subjects units to higher than usual levels of stress like voltage, temperature and loading to speed up the deterioration of materials to collect failure information more quickly.

Although burn-in is effective in eliminating defective units before they reach the customers, it also constitutes costs. Over application of burn-in will cause higher reliability. Manufacturing companies must be able to determine the minimal burn-in period to also minimize the burn-in cost. This will not only cause them ample savings but keep them competitive in the market.

Studies made from previous papers about determining the burn-in period were conducted in different forms. Perlstein, Jarvis and Mazuzuchi (2001), presented an analysis of computing burn-in through the method called Bayesian calculation. This calculation was designed for a batch of products whose life distribution is modelled as a mixture of two exponential sub-populations. Same analysis approach was done in the research of Fang and Huang (2007), Bayesian analysis was the tool they used to effectively asses the deterioration based on experts' opinions and possibly few relevant data. In the discussion of Kim's research (1998), the optimal burn-in procedure was used to minimize total costs based on the assumption that some of the components are weak and deteriorate faster than the strong components. A cost function was developed by Wu, Chou and Huang (2006) to determine the optimal burn-in time and warranty length for the non-repairable products under the fully renewing combination free replacement and pro-rata warranty policy. Some findings tackled about the burn-in under different distributions such as Arrhenius-Lognormal distribution and mixed exponential distribution were done by Lee, Torng and Lin (2011). It described the lognormal lifetime of electronic products under different temperature levels.

On the perspective of Perlstein et al. (2001) the mixed exponential model is based on the assumption that products may come from one of two possible populations denoted weak and strong with a mixing parameter denoting the proportion of the weak population in the total.

Based from the review of the previous researches, it can be noticed that the determination of burn-in period focused mainly on minimizing the manufacturing cost. In this study, however, the researchers will determine the burn-in time required to achieve a reliability target, not to minimize the total cost. This reliability target is assumed to be the reliability level at which the customers will be satisfied.

Moreover, the researchers will also focus on the product failure time with different distribution from other papers, which is the Weibull distribution. This distribution is flexible enough to any kind of data because its parameter has properties that portray to decreasing failure rate, increasing failure rate and even constant failure rate.

Furthermore, this paper will also apply other analysis approach which is the properties of conditional reliability to compute the required burn-in time.

The major objective of this study is to determine the burn-in period necessary to achieve a reliability target for product failure time with Weibull distribution. Moreover, this study has the following specific objectives:

- 1) To establish the difference between Reliability and Conditional Reliability
- 2) To determine and prove when the burn-in can be beneficial
- 3) To transform the time-to-failure into Weibull distribution
- 4) To estimate the parameters of the Weibull distribution

2. Methods

2.1 Conceptual Framework

Figure 1 shows the conceptual framework of this paper

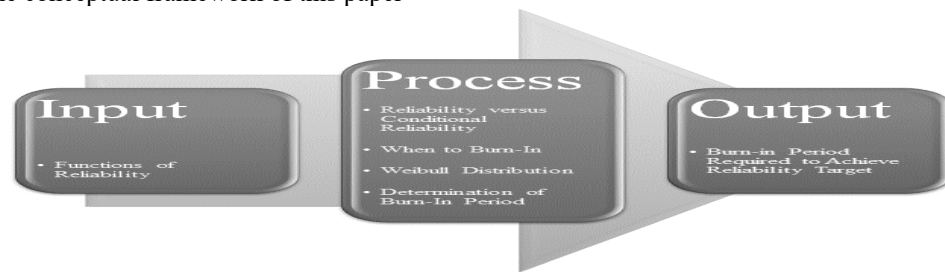


Figure 1: Conceptual Framework

2.1.1 Functions of Reliability

The mathematical equations of the functions of reliability was used to know how the elements can affect the burn-in time that follows a Weibull distribution.

2.1.2 Reliability versus Conditional Reliability

In this process, the derivation on how the reliability of a product after it has undergone burn-in test were presented using the conditional reliability.

2.1.3 When to Burn-In

The next step is to determine when burn-in can be applicable. This was done by giving different scenarios of failure rates namely, DFR, CFR, and IFR. At the end of this process, the researchers will be able to know when the burn-in can be beneficial or not.

2.1.4 Weibull Distribution

Since this study focused on a Weibull distribution, this process will show how burn-in will work if this kind of distribution will be used by incorporating the properties of Weibull to conditional reliability. The parameters were also estimated in this section.

2.1.5 Determination of Burn-In Period Based on Reliability Target

In this process, the data were transformed into a Weibull distribution using the Median Ranks method. This method was used because of its simplicity and because the sample data is less than 20 observations.

Once the data was fit into a line, the researchers estimated the shape and scale parameter using ReliaSoft Weibull ++.

2.1.6 Burn-In Period for Product Failure Time with Weibull Distribution

The researchers arrived at burn-in period necessary to achieve the reliability target for product failure time with Weibull distribution. This showed how the product failure time can affect the burn-in period. Consequently, it can predict the best span of burn-in time of a product that may still lead to be continuously operating at its desired reliability level.

2.2 Research Design

This research is of descriptive type. It aims to describe a situation when the burn-in process will be beneficial by providing comprehensive mathematical proof and examples. This study will also prove how effective to know the time to burn-in because it may also lead to product failure if it exceeded the required span.

Consequently, the researchers set numerous assumptions so there will be no need for actual field data. The equations were then unified and modified according to the behaviour of the aforementioned distribution.

After all the equations were modified and unified, the researchers provided a numerical example to see how it works in definite scenario

2.3 Respondents of the Study

Since this study is theoretical in nature, the researchers did not need any respondents. However, the researchers used various models from proven principles in the area of reliability under Weibull distribution. Some of these models are from Kececioglu and Ebeling.

2.4 Research Locale

This research study focused on providing a decision model and proof when and why burn-in is beneficial. Moreover, the necessary burn-in period for a certain reliability target was determined. The researchers did not do any surveys but needed ample amount of references that were provided by Chung Yuan Christian University (CYCU).

CYCU was founded in 1953 and has been ranked as the integrated and most prestigious institution of higher education among private universities by Ministry of Education since 1995.

2.5 Instrumentation

The researchers used reliability software to prove that the model is consistent and working to set of numerical samples. The said software can help analyze if the data gathered fits with the relationship formed by the group as well as if there are errors that must be tested and validated and to visually show the characteristics of each parameters used.

2.6 Data Gathering Procedure

Since the flow of the paper will be more on proving theories, the researchers gathered all the related equations and adjusted them based on the given distribution accordingly.

With regards to the case study, the authors used the examples given in the past studies to see if the unified model is still applicable to those scenarios.

3. Results and Discussion

3.1 Nomenclature

The following Table 1 presents the list of the variables used in this study and their definition.

Table 1

Variable	Definition
$R(t)$	Reliability function
T	Random variable failure time
t	Period
Pr	Probability
T_0	Burn-in period
$R(t T_0)$	Conditional Reliability
$R(T_0)$	Reliability at the beginning of burn-in period
$R(T_0 + t)$	Reliability at the end of the burn-in period
β	Shape or slope parameter
η	Scale parameter
F_i	Failure estimate for i rank
i	Rank number
N	No. of observations

3.2 Reliability vs. Conditional Reliability

In this study, conditional reliability will be considered as the reliability of a component or system given that it has undergone burn-in for time T_0 . Mathematically, it can be computed and derived as follows: (Figures 2 – 3)

$$R(t|T_0) = \Pr\{T > T_0 + t | T > T_0\}$$

$$R(t|T_0) = \frac{\Pr\{T > T_0 + t\}}{\Pr\{T > T_0\}}$$

$$R(t|T_0) = \frac{R(T_0 + t)}{R(T_0)}$$

where $R(T_0)$ represents the reliability at the beginning of the interval T_0 which is illustrated at Figure 2 and $R(T_0 + t)$ represents the reliability at the end of the burn-in period which can be seen in Figure 3.

The random variable failure time T must be greater than the period T_0 because the conditional reliability covers only the period after the burn-in.

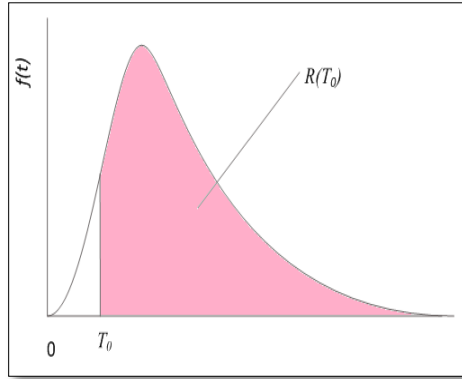


Figure 2: $R(T_0)$

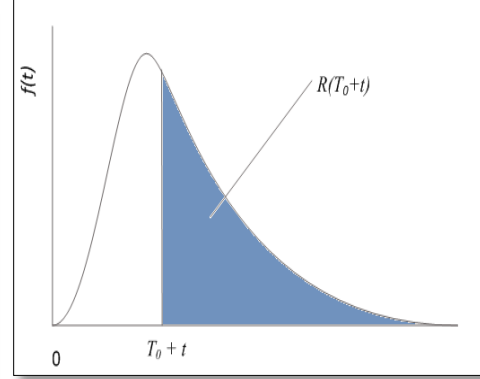


Figure 3: $R(T_0 + t)$

3.3 When to Burn-in?

In determining the burn-in period based on the reliability target, the decision whether to do burn-in or not must be made first. Hypothetically, the failure rate has three common behaviors as time passes by. According to Cheng (2013), these three are decreasing failure rate for infant mortality, constant failure rate for useful life and increasing failure rate for wear-out period.

According to Ebeling (2012), for the reliability to improve as a function of T_0 , the slope of $R(t|T_0)$ with respect to T_0 must be positive. To know the slope, the derivative of $R(t|T_0)$ must be determined. Below is the computation of that derivative.

$$R(t|T_0) = \frac{R(T_0 + t)}{R(T_0)}$$

$$R(t|T_0) = e^{-\int_{T_0}^{t+T_0} \lambda(t') dt'}$$

Since

$$\frac{d(e^u)}{dx} = e^u \frac{du}{dx}$$

Differentiating the function $R(t|T_0)$ we have:

$$\frac{dR(t|T_0)}{dT_0} = \left[e^{-\int_{T_0}^{t+T_0} \lambda(t') dt'} \right] \times \frac{d}{dT_0} \left[\int_{T_0}^{t+T_0} \lambda(t') dt' \right]$$

Getting $\frac{d}{dT_0}$, we have:

$$\frac{d}{dT_0} = \left[\int_{T_0}^{t+T_0} \lambda(t') dt' \right]$$

$$\frac{d}{dT_0} = \lambda(T_0) - \lambda(t + T_0)$$

Substituting this to the original equation, we can get the slope.

$$\frac{dR(t|T_0)}{dT_0} = \left[e^{-\int_{T_0}^{t+T_0} \lambda(t') dt'} \right] \times [\lambda(T_0) - \lambda(t + T_0)]$$

$$\frac{dR(t|T_0)}{dT_0} = R(t|T_0) \times [\lambda(T_0) - \lambda(t + T_0)]$$

Analyzing the slope, $\lambda(T_0)$ must be greater than $\lambda(t + T_0)$ for it to have a positive value. This means that the failure rate must be decreasing as the time goes by to obtain a higher reliability. Otherwise, it will not be beneficial.

3.4 Weibull Distribution

Based from Reliability HotWire, Weibull distribution is one of the most commonly used distribution in life data analysis due to its versatility and relative simplicity. Among the Weibull distributions, the two-parameter is the most widely used. Below is the formula to compute for the reliability of a product failure rate that follows a Weibull distribution.

$$R(t) = e^{-\left(\frac{t}{\eta}\right)^\beta}$$

To get the reliability of the product after burn-in, the researchers used the conditional reliability which was previously determined.

$$R(t|T_0) = \frac{R(T_0 + t)}{R(T_0)}$$

$$R(t|T_0) = \frac{e^{-\left(\frac{t}{\eta}\right)^\beta}}{e^{-\left(\frac{T_0}{\eta}\right)^\beta}}$$

$$R(t|T_0) = \frac{e^{-\left(\frac{T_0+t}{\eta}\right)^\beta}}{e^{-\left(\frac{T_0}{\eta}\right)^\beta}}$$

$$R(t|T_0) = e^{-\left(\frac{T_0+t}{\eta}\right)^\beta + \left(\frac{T_0}{\eta}\right)^\beta}$$

The value of β differs from the shape or slope of the distribution. For the exponential and constant failure rate, $\beta = 1$. For increasing failure rate in time, the shape parameter must be greater than 1. And for the decreasing failure rate, the β must be less than 1.

3.5 Determination of Burn-in Period Based on the Reliability Target

A toy company that produces jack-in-the-box spring housing wants to know the reliability of their product at 65,000 cycles. To know this, 10 units were assembled and tested until their spring housings failed and were arranged in ascending order and given below. (Table 2)

Table 2

Sample	Cycles-to-failure
1	220,349
2	231,876
3	242,874
4	256,985
5	278,875
6	354,875
7	484,987
8	720,772
9	1,457,086
10	2,987,097

Since the number of samples is less than 20, the Median Rank method was used to estimate the proportion of the number of failures to the population. Hence, the Benard's equation was used to compute for the median ranks.

$$F_i = \frac{(i - 0.3)}{N + 0.4}$$

The results of the computations were given Table 3 below.

Table 3

Design Cycles	Rank	Median Ranks
220,349	1	0.067307692
231,876	2	0.163461538
242,874	3	0.259615385
256,985	4	0.355769231
278,875	5	0.451923077
354,875	6	0.548076923
484,987	7	0.644230769
720,772	8	0.740384615
1,457,086	9	0.836538462
2,987,097	10	0.932692308

The computed median ranks are estimates of the unreliability function of $F(t)$. To transform the data into a Weibull distribution, the researchers used the equation below.

$$F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^\beta}$$

$$1 - F(t) = e^{-\left(\frac{t}{\eta}\right)^\beta}$$

$$\frac{1}{1 - F(t)} = e^{\left(\frac{t}{\eta}\right)^\beta}$$

Taking two times the natural logarithms of both sides, gives an equation of a straight line:

$$\ln \ln \frac{1}{1 - F(t)} = \beta \ln t - \beta \ln \eta$$

To get the values of these variables, the researchers used MS Excel. The Table 4 is shown below.

Table 4

Design Cycles	Rank	Median Ranks	1/(1-Median Ranks)	ln(ln(1/(1-median rank)))	ln(Cycles)
220,349	1	0.067307692	1.072164948	-2.663843085	12.3029679
231,876	2	0.163461538	1.195402299	-1.72326315	12.353958
242,874	3	0.259615385	1.350649351	-1.202023115	12.4002981
256,985	4	0.355769231	1.552238806	-0.821666515	12.456773
278,875	5	0.451923077	1.824561404	-0.508595394	12.5385189
354,875	6	0.548076923	2.212765957	-0.230365445	12.7795209
484,987	7	0.644230769	2.810810811	0.032924962	13.0918774
720,772	8	0.740384615	3.851851852	0.299032932	13.4880781
1,457,086	9	0.836538462	6.117647059	0.593977217	14.1919491
2,987,097	10	0.932692308	14.85714286	0.992688929	14.9098126

The values of the double natural logarithmic of 1/(1-median mark) represent the values of y while the values of the natural logarithmic of the cycles represent the values of x. These values were, then, plotted to fit a line to the data. (Figure 4)

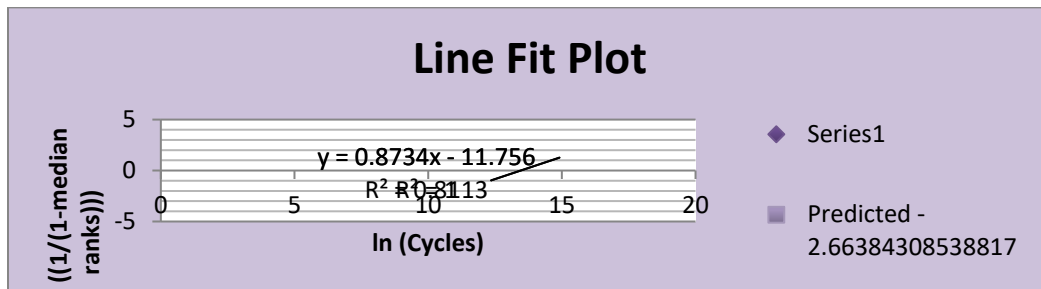


Figure 4: Line Fit Plot

Furthermore, the slope and shape parameters were estimated using the ReliaSoft Weibull ++. The results are shown Figure 5 below.

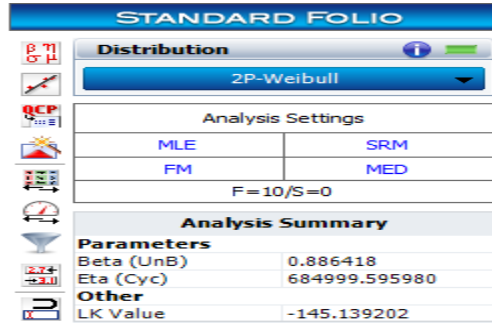


Figure 5: ReliaSoft Weibull++ Results

Based from the table above, the value of β and η are 0.8864 and 685,000 cycles respectively. Since the value of β is less than 1, it means that burn-in will be beneficial once it is applied. Thus, the computation for the minimum burn-in period is as follows.

Assuming that the toy manufacturer has a reliability target of 0.90 and their spring housings have a design life of 65,000 cycles. The researchers computed first the reliability at $t = 65,000$ cycles.

$$R(t) = e^{-\left(\frac{t}{\eta}\right)^\beta}$$

$$R(t) = e^{-\left(\frac{65,000}{685,000}\right)^{0.8864}}$$

$$R(t) = \mathbf{0.8834}$$

Clearly, the current reliability of the company is less than their reliability target. Assuming that the company plans to have burn-in to achieve the reliability target of 0.90, however, the burn-in cost is expensive and the management wants to spend fewer resources as possible. To get the necessary burn-in period, the computation is shown below.

$$R(t|T_0) = e^{-\left(\frac{T_0+t}{\eta}\right)^\beta + \left(\frac{T_0}{\eta}\right)^\beta}$$

$$0.90 = e^{-\left(\frac{T_0+65,000}{685,000}\right)^{0.8864} + \left(\frac{T_0}{685,000}\right)^{0.8864}}$$

$$\ln 0.90 = \ln e^{-\left(\frac{T_0+65,000}{685,000}\right)^{0.8864} + \left(\frac{T_0}{685,000}\right)^{0.8864}}$$

$$\ln 0.90 = -\left(\frac{T_0 + 65,000}{685,000}\right)^{0.8864} + \left(\frac{T_0}{685,000}\right)^{0.8864}$$

$$T_0 = \mathbf{63,897.51 \text{ cycles}}$$

Based from the results of the computations, the minimal burn-in period to achieve the reliability target of 0.90 is 63,897.51 cycles. Furthermore, the burn-in period required for different reliability targets were computed and shown in Table 5 below.

Table 5

Reliability Target	Burn-in Period Required
0.90	63,897.51 cycles
0.905	120,519 cycles
0.91	218,386.36 cycles
0.915	391,555.95 cycles
0.92	707,345.04 cycles

Furthermore, it can be seen in the Table 5 above that as the reliability target increases, the burn-in time or period gets longer.

4. Conclusion

Determining the required burn-in period is essential in improving the reliability if the outgoing product reliability cannot meet the reliability target. Not only it helps in achieving the reliability goal, it also minimizes the cost of burn-in through limiting the burn-in period to the necessary time.

In this study, the researchers first established the difference between the reliability and conditional reliability. Reliability is the probability that the product will perform its specific functions for a certain period of time after undergoing some predefined period of time and check operational and its functions.

Moreover, the researchers were able to show when the predefined period of time i.e. burn-in time in this study will be beneficial or not by getting the derivative of the conditional probability. Since the derivative represented the slope or the change of reliability over time t after the burn-in period T_0 , its value should be positive, the failure rate must be a decreasing function with respect to time.

For the constant and increasing failure rate function, the slope of the derivative of $R(t|T_0)$ is equal to zero and less than one respectively. Therefore, having a burn-in test for products that have constant or increasing failure rate will not improve reliability. The case is different for increasing failure rate because it will have a negative effect on the reliability.

After this, the researchers provided a numerical example to determine the required burn-in time to achieve certain reliability targets. The Median Ranks method was used to estimate the proportion of failure to the population. These values were then transformed into a straight line by getting the double natural logarithmic of the equation.

Furthermore, the Weibull shape and scale parameters were estimated. This was done using the ReliaSoft Weibull ++.

These values were then substituted to the conditional probability equation to get the required burn-in period to achieve certain reliability targets.

5. Recommendations

To further improve this study, the researchers give the following recommendations:

1. Use different methods in estimating the Weibull parameters and see if there are any significant differences.
2. Use a bigger sample size or better yet apply this method to an actual case study.
3. Determine the burn-in time for other distribution such as Log Normal.
4. Consider censored data and adjust the data based on the censoring.
5. Use of industrial burn-in data to track product failure rate before burn-in and to compare product field reliability with reliability target after burn-in test.

- Determine until what period of burn-in can still be cost-effective for the company.

References

- Brinkley, B. *The Effects of Maintenance on Reliability*. Retrieved January 15, 2014, from (2008). http://www.reliabilityweb.com/art08/effects_of_maintenance_on_reliability.htm
- Chowdhury, R. *TECH 65800: BURN-IN / STRESS TESTING FOR RELIABILITY* [Document]. Retrieved January 15, 2014, from (2006). www.personal.kent.edu/~Research-Paper_by_Cagatay%20Bozturk.doc
- C&D Technologies. *A Burn-In Issue*. Retrieved January 15, 2014, from http://www.digikey.tw/Web%20Export/Supplier%20Content/MurataPower_811/PDF/Murata_CD-Burn-In.pdf?redirected=1
- Cheng, T. *A Critical Discussion on Bath-tub Curve*. Retrieved December 17, 2013, from (2001) <http://bm.nsysu.edu.tw/tutorial/iylu/conference%20paper/B035.pdf>, 2001
- Deloitte AG. *Technology industry - sector without a status quo*. Retrieved from (2014). http://www.deloitte.com/view/en_ch/ch/b857ad9fed2fb110VgnVCM100000ba42f00aRCRD.htm
- Ebeling, C.E. *An Introduction to Reliability and Maintainability Engineering 2nd Edition*. (2012).
- Fang, C. & Huang, Y. *A Bayesian decision analysis in determining the optimal policy for pricing, production, and warranty of repairable products*. Retrieved from (2007). http://ac.els-cdn.com/S0957417407003788/1-s2.0-S0957417407003788-main.pdf?_tid=ccbda466-7ea7-11e3-bee4-00000aab0f27&acdnat=1389874671_0483531b4602f0034209c09700-c2d79d
- ITEM Software, Inc. *Reliability Prediction Basics*. Retrieved January 3, 2014, from (2007). <http://www.reliabilityeducation.com/ReliabilityPredictionBasics.pdf>
- Kececioglu, D. *Reliability Engineering Handbook*. (1991).
- Klutke, G., Kiessler, P. C., & Wortman M. A. *A Critical Look at the Bathtub Curve*. Retrieved December 17, 2013, from (2003) <http://ieeexplore.ieee.org/xpl/articleDetails.jsp?tp=&arnumber=1179819&queryText%3DA+Critical+Look+at+the+Bathtub+Curve+by+klutke>
- Kuo, W. & Kim, T. *An Overview of Manufacturing Yield and Reliability Modeling for Semiconductor Products*. Retrieved September 2, 2013, from (1999) <http://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=01179819>
- Kim, K.N. *Optimal burn-in for minimizing cost and multiobjectives*. Retrieved from http://ac.els-cdn.com/S0026271498000316/1-s2.0-S0026271498000316-main.pdf?_tid=3f799344-7e9d-11e3-9c06-00000aab0f27&acdnat=1389870139_e1173760b32ee745df-95796292004a80 (1998).
- Lee, P., Torng, C. & Lin, Y. *Determination of the optimal accelerated burn-in time under Arrhenius-Lognormal distribution assumption*. Retrieved from http://ac.els-cdn.com/S0307904X11000795/1-s2.0-S0307904X11000795-main.pdf?_tid=b3ab5348-7e9c-11e3-ac29-00000aab0f27&acdnat=1389869904_32c2546415ff6e32fae6dd474-a03b4fc (2011).
- M. Noroña (personal communication, September 8, 2013)
- Perlstein, D., Jarvis, W. & Mazzuchi, T. *Bayesian calculation of cost optimal burn-in test durations for mixed exponential populations*. Retrieved from http://ac.els-cdn.com/S0951832001000254/1-s2.0-S0951832001000254-main.pdf?_tid=679c2102-7e9d-11e3-8235-00000aab0f27&acdnat=1389870206_ecde6645f2070425dc056c31257-b6cfd (2001).
- ReliaSoft. *Guidelines for Burn-in Justification and Burn-in Time Determination*. Retrieved January 15, 2014, from http://reliasoft.com/newsletter/v7i2/burn_in.htm
- ReliaSoft. *How Long Should You Burn In a System?* [eMagazine]. Retrieved January 15, 2014, from (2006). <http://www.weibull.com/hotwire/issue69/re basics69.htm>
- Reliability Hotwire, Issue 14, Published on April 2002*
- Tso, M. *Reliability and Survival*. Retrieved January 2, 2014, from <http://www.maths.manchester.ac.uk/~mkt/334%20Reliability/Notes08/RelS1.pdf>
- Tarr, M. *Improving reliability by screening*. Retrieved January 15, 2014, from http://www.ami.ac.uk/courses/topics/0188_irbs
- U.S Food and Drug Administration. *Reliability of Manufactured Products*. Retrieved October 29, 2013, from (2010). <http://www.fda.gov/ICECI/Inspections/InspectionGuides/-InspectionTechnicalGuides/ucm072912.htm>

- Vigrass, W. *Calculation of Semiconductor Failure Rates*. Retrieved October 8, from 2013, http://www.intersil.com/content/dam/Intersil/quality/rel/calculation_of_semiconductor_failure_rates.pdf
- An introduction to the Weibull Distribution. Retrieved January 14, 2014, from <http://www.weibull.nl/weibullstatistics.htm> Bath tub curve [Image]. Retrieved from http://upload.wikimedia.org/wikipedia/commons-/6/6e/Bathtub_curve.jpg
- Wu, C., Chou, C. & Huang, C. *Optimal burn-in time and warranty length under fully renewing combination free replacement and pro-rata warranty*. Retrieved from [http://ac.els-cdn.com/S0951832006001359/1-s2.0-S0951832006001359-main.pdf?_tid=122718ac-7eb7-11e3-bfcd-\(2006\).00000aab0f27&acdnat=1389881230_15bd193c494742d708-bc4eaeec2490d2](http://ac.els-cdn.com/S0951832006001359/1-s2.0-S0951832006001359-main.pdf?_tid=122718ac-7eb7-11e3-bfcd-(2006).00000aab0f27&acdnat=1389881230_15bd193c494742d708-bc4eaeec2490d2)
- Yun, W., Lee, Y. & Ferreira, L. *Optimal burn-in time under cumulative free replacement warranty*. Retrieved from (2002). http://ac.els-cdn.com/S0951832002000492/1-s2.0-S0951832002000492-main.pdf?_tid=c724e5e2-7e9c-11e3-8234-00000aab0f27&acdnat=1389869937_a14a02ddc305ef9a244988f370134ca2

Biography

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