Two-Dimensional Cutting Problem: Application of Heuristics to Troubleshooting in the Textile Sector

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Abstract

This paper aims to study a cutting problem inherent to the modelling process of a textile company and find a solution that minimizes fabric waste. Thus, several heuristics were adopted, whose results allowed inferring about the company's current condition, i.e., whether the current solution adopted is effective in minimizing fabric waste. The heuristics have been tested, analyzed, and compared using a pre-existing database. Following this hypothetico-deductive method, the initial heuristic and its improvements were subsequently applied to the real problem. Regarding the solutions, the 1st heuristic presented the worst solution, as expected, and the subsequent heuristics (2nd-5th) presented continuous improvements of approximately 5% compared to their predecessors. Based on the results, it was found that the created heuristics did not generate better solutions than the current one adopted by the company (H = 4.04m), i.e., its solution is robust and effective, considering the company's objective. However, the successive improvements made to the heuristics allowed generating of good solutions comparable to the ones given by the specialized software.

Keywords
2D Cutting Stock Problem, Regular, Irregular, Heuristics, Textile Sector

1. Introduction
The Portuguese textile company is headquartered in the district of Viseu and employs 165 co-workers. It offers a bespoke tailoring service that aims to provide personalized experiences.
The products are designed using four distinct process phases:
- Modelling - This first phase involves creating patterns for the pieces on the cutting plan (raw material) using CAD and CAM tools, to use the least amount of material for the largest number of pieces.
- Cutting - This sector is crucial to ensure product quality and accuracy in the measurements of each piece.
- Assembly - After cutting and labelling, the different subcomponents of the piece (such as collars, sleeves, and lining) are sent to their specific assembly lines.
- Finishing - In this phase, the suits undergo finishing operations such as ironing, and any defects are analyzed.

This work proposes an optimization for the modelling phase. In this phase the main objective is to solve the inherent cutting problem with the highest possible space utilization rate. For that, some approaches were developed to minimize the fabrics waste in the cutting phase. The approaches complexity increases with the gradual inclusion of some considerations inherent to the irregular cutting problem. The approaches performances were compared using some benchmark instances and real data given by the company.

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The aim of this paper is to verify the quality of the solution presented by the company. For that, the problem addressed in this work is briefly presented in this section (section 1). In section 2, is made a literature review of Cutting problems. The developed approaches are presented in section 3. To prove the efficiency of the approaches and compare them, some tests were performed using both benchmark instances and the results are presented in section 4. The related results are presented in section 5. The conclusions and relevant remarks are presented in section 6.

2. Literature Review

2.1 Cutting Problems

Cutting problems are combinatorial optimization problems that arise when a raw material, in this case, a large object, must be cut into smaller pieces, minimizing waste. Due to the importance of these problems and their complexity, several studies have been carried out, allowing companies to improve both the economic and environmental aspects by maximizing resource utilization and reducing waste.

The solution to these problems lies in finding the best possible layout of all the pieces completely embedded in the raw material and without overlap, to minimize material waste (Nascimento et al. 2022).

In recent years, several mathematical models and heuristic approaches have been proposed to solve different variations of the cutting problem. One approach is to use mixed-integer programming (MIP) models. The Floating-Cuts model proposed by Silva et al. (2014) is a flexible MIP model that can handle non-guillotine and guillotine rectangular cutting problems, which are commonly used in the metal and paper industries. Martin et al. (2015) extended this approach to three-dimensional cutting problems with constrained patterns, which are useful in applications such as furniture manufacturing. However, MIP models can be computationally intensive, especially for larger problems. To overcome this, heuristic and metaheuristic algorithms have been developed. For example, Parreño et al. (2016) used a combination of constructive heuristics and local search to solve a large cutting problem in the glass manufacturing industry. Similarly, Luo et al. (2018) proposed metaheuristic algorithms for a special cutting stock problem with multiple stocks in the transformer manufacturing industry. Another aspect to consider in cutting problems is the leftover material that cannot be used in subsequent cutting operations. The two-dimensional cutting stock problem with usable leftovers has been studied by do Nascimento et al. (2018), who proposed mathematical models and heuristic approaches to tackle the problem. These problems can be found with great frequency in industries such as metallurgy, furniture, paper, glass, as well as in the textile industry, which is the area of application of the study. This problem is NP-hard, meaning that it becomes computationally infeasible to solve for larger instances.

2.2 Structure and Definition

Cutting problems can be approached in various dimensions and involve large objects and small items, with homogeneous or heterogeneous dimensions. The resolution of these problems consists of placing all smaller items, in a specific quantity and grouped into one or more subsets, entirely within one or more larger objects, without overlap, maximizing the use of the large object.

The goal is to select the small items, group them, and assign each subset to a large object and a position within it so that all small items in the subset are inside the large object and the small items do not overlap.
According to Washer, Haußner, and Schumann (2011), the definition and characterization of cutting and packing (C&P) problems follow these criteria:

1. Dimensionality - It can be characterized by 1, 2, or 3 dimensions.
2. Type of allocation - Maximization of output: All large objects must be used, and a selection of small items are assigned to them, to maximize the large objects use with the small items. Minimization of input: All small items must be assigned to a selection of large objects, without selecting small items, minimizing the number of large objects used.
3. Variety of small items - All the same, weakly heterogeneous, strongly heterogeneous.
4. Shape of small items - Regular or irregular.
5. Variety of large objects - One large object with all fixed dimensions or with one or more variable dimensions; or several large objects with all fixed dimensions, all being: the same, weakly heterogeneous, strongly heterogeneous.
Regular cutting problems with more than one dimension can still be guillotine or non-guillotine.

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Table 1. Characterization of the problems addressed by different authors

Reference	Soluction Method		Type of Cut (2D)			Stages		
	Exact	Heuristic	Regular		Irregular	2	3	k
			Guillotined	Non-Guillotined				
Gilmore and Gomory (1961)	x		x					x
Teodoro (2003)		x	x					x
Gehring and Homberger (2011)		x			x			
Liao et al. (2016)		x			x			x
Oliveira (2018)		x		x				x
Parreño et al. (2020)		x	x					x
Lamas et al. (2020)		x		x				x
Theeb et al. (2021)		x			x			x
Zhang et al. (2021)		x	x					x
Luo et al. (2022)		x	x					x
Nascimento et al. (2022)	x	x	x				x	
Silva et al. (2022)	x		x	x				x

2.3 Regular Guillotine Cutting Problem

The linear programming model for these problems corresponds to an Open Dimensional Problem. In general, for regular 2D cutting problems, the types of cuts that exist can be defined as guillotine or non-guillotine. If the pieces are obtained through a sequence of end-to-end cuts of the initial plane, parallel to one of the sides of the plane, then we have guillotine cuts, otherwise we have non-guillotine cuts (Luo et al., 2018), as presented in Table 1.
Therefore, in regular guillotine cutting problems, the machine makes linear cuts that traverse the entire piece without changing direction during the cut (Al Theeb et al. 2021). Generally, the solution to these problems results in more waste due to the existing cutting limitation.

The dotted board model is a well-known heuristic approach to solving the 2-dimensional rectangular packing problem (2DRP), in which the board is represented as a grid of dots and pieces are cut along the grid lines. Al Theeb et al. (2021) proposed a new strategy to improve this model by introducing a new algorithm that considers the best-fit and worst-fit placements, in addition to the existing first-fit placement. The algorithm also includes a new mutation operation to improve the quality of solutions generated by the genetic algorithm used in the model.
Several other researchers have also studied the 2DRP problem and proposed various solution approaches. For instance, Zhang et al. (2021) developed a hybrid algorithm combining a genetic algorithm and a local search algorithm to obtain high-quality solutions. Lamas et al. (2020) presented a mathematical programming approach to the problem using a column generation algorithm. The available literature on guillotine cutting problems is vast, with researchers continuously proposing novel solution approaches and algorithms to enhance the efficiency and efficacy of the cutting process across diverse industries.

2.4 Regular Non-Guillotine Cutting Problem

According to Teodoro (2003), the guillotine cutting type may not be suitable for many real-world applications due to constraints related to the cutting equipment or the asymmetry of the size of the pieces to be cut.
Thus arises the two-dimensional non-guillotine cutting problem, where Oliveira (2018) mentions that the machine no longer makes a linear cut that crosses the entire piece, but rather a cut where the machine can change direction along the cut, allowing for greater variety, as it no longer follows a guillotine cut pattern, meaning there can be different cut-outs depending on the pieces that have relevant size differences.

One classical approach for solving the problem is column generation, proposed by Gilmore and Gomory (1961), which involves generating new cutting patterns as they are needed, while maintaining a selected set of columns that form a basis for the problem. Teodoro (2003) proposed a column generation approach, which has been widely used in the literature. The method consists of generating columns of variables that represent cutting patterns and solving a restricted master problem to select the best columns to include in the final solution.

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Another popular approach to the non-guillotine two-dimensional cutting problem is metaheuristics. Oliveira (2018) proposed a specialized metaheuristic approach that combines tabu search, simulated annealing, and path relinking to obtain high-quality solutions. Other metaheuristic approaches that have been applied to the problem include genetic algorithms, particle swarm optimization, and ant colony optimization.
The literature surrounding non-guillotine two-dimensional cutting problems is expansive and continuously improving.

2.5 Irregular Cutting Problem

As mentioned in 2.2, concerning the shapes of small items, they can be regular or irregular. As expected, solving these problems results in a higher space utilization rate. However, it requires more computational effort.
Al Theeb et al. (2021) noted that to date, there is no exact mathematical programming model-based methods for solving irregular C&P problems. Currently, the most common model used for this problem is the MIP model.
This type of material cutting is a well-known issue and the literature provides several optimization techniques to solve it, including exact methods, heuristic algorithms, and metaheuristic algorithms (Bortfeldt & Wäscher 2014). Exact methods such as integer programming and dynamic programming provide optimal solutions, but are computationally expensive (Gehring & Homberger 2011). Heuristic algorithms such as greedy algorithms and constructive heuristics have been widely used to solve the problem, providing suboptimal solutions in a reasonable amount of time (Gehring & Homberger 2011). Metaheuristic algorithms, such as simulated annealing, genetic algorithms, and tabu search, provide a good balance between solution quality and computational efficiency (Bortfeldt & Wäscher 2014).

In recent years, new approaches have been proposed to tackle the irregular cutting problem. Xiaoping Liao et al. (2016) proposed a visual nesting system based on the rubber band packing algorithm to optimize the irregular cutting-stock problem. The proposed method consists of a new geometric representation and an iterative algorithm that optimizes the packing efficiency. The results showed that this approach surpassed other heuristic algorithms in both solution quality and computational efficiency, making it a significant contribution to the existing literature.

Yan-xin Xu's (2021) proposes an efficient heuristic approach to the problem of irregular material cutting in shipbuilding. The approach is based on two main steps: a cutting heuristic that uses a greedy algorithm to establish the best cutting sequence, and a packing heuristic that uses a local search algorithm to allocate the cutting pieces on the steel plate. This approach's results showed promising results in terms of processing time and solution quality.
In conclusion, the literature provides a range of optimization techniques to solve the problem of irregular material cutting, with heuristic and metaheuristic approaches being widely used.

3. Methodology

3.1 Example and Assumptions

Initially, it was assumed that the rolls (cutting plans) have an infinite height (H), due to their large height compared to the size of the pieces. Before approaching the initial heuristic, it is important to perform a feasibility range of the solutions identified by the heuristics to verify if the results proposed by them are reliable. Thus, for the lower limit (1), a blind expression to the format of the pieces was used, taking only their area into account.

$$(1) \quad \left[\frac{\sum_{i=1}^{N} A_i}{W}; +\infty\right[$$

It should be noted that this lower limit (1) may not be optimal since, in this case, the format of the pieces is not being considered and therefore the total area value of the pieces may be less than the optimal area value of the roll for the considered problem (2). This happens because the pieces shapes cause some unused spaces between them.

$$(2) \quad \frac{\sum_{i=1}^{N} A_i}{W} \leq H^* \Leftrightarrow \sum_{i=1}^{N} A_i \leq H^* \times W$$

For the upper limit (3), the initial heuristic was considered with the limitation of only 1 piece per level.

$$(3) \quad \left]-\infty; \sum_{i=1}^{N} h_i\right]$$

Therefore, the intersection of the two sets yields the feasibility range (4) of the results.

(4) $[\frac{\sum_{i=1}^{N} A_i}{W}; +\infty[\cap]-\infty; \sum_{i=1}^{N} h_i] = \left[\frac{\sum_{i=1}^{N} A_i}{W}; \sum_{i=1}^{N} h_i\right]$

An important analysis measure is the utilization rate (5).

(5) $\tau = \frac{A_{tp}}{A_u} = \frac{\sum_{i=1}^{N} A_i}{H_u \times W} = \frac{\frac{\sum_{i=1}^{N} A_i}{W}}{H_u}$

Due to the irregular initial pieces of the problem, as happens in most real situations in the textile sector, it was necessary to transform them into rectangles (regularized), as shown in Figure 1.

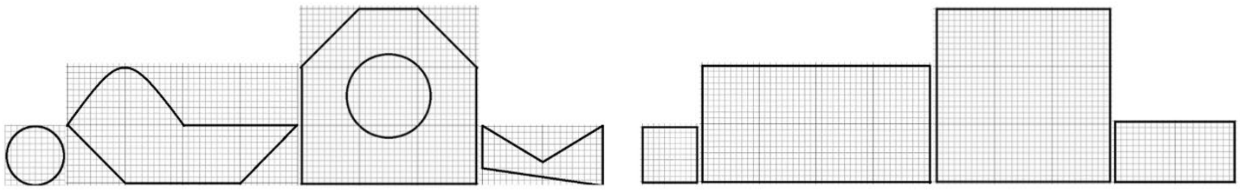

Figure 1. Irregular to regular transformation

For regular pieces, their width (w_i) and height (h_i) were considered. For irregular pieces, approximations were considered through grids consisting of squares, being the necessary parameter for the heuristics their area (A_{irreg_i}), as shown in Figure 2.

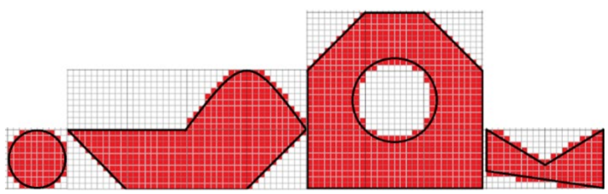

Figure 2. Irregular Approximation

With these approximations, two distinct equations were used to calculate regular (6) and irregular (7) utilization rates.

(6) $\tau_{reg} = \frac{\frac{\sum_{i=1}^{N} A reg_i}{W}}{H_u}$

(7) $\tau_{real} \approx \tau_{irreg} = \frac{\frac{\sum_{i=1}^{N} A irreg_i}{W}}{H_u}$

The regular utilization rate (6) considers that the pieces were approximated by rectangles, whereas the irregular utilization rate (7) considers the irregular painted approximation.
These approximations generate an error (8) per piece.

(8) $\xi_i = A_{ap_i} - A_{r_i}$

Noticing that the irregular approximation tends to zero the smaller the used scale (9).

(9) $\lim_{l_{\blacksquare} \to 0^+} \xi_i = 0^+$

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It is important to point out that $\xi_i \geq 0$ since the shape of the pieces was always overestimated in the approximations, to ensure that the solutions are both acceptable and valid in real-world contexts. Although using approximations that underestimate the piece shape could yield better solutions, such solutions may not be feasible in a real-world context.

3.3 1st Heuristic – First-Fit Decreasing-Height (FFDH) - Guillotined

On this heuristic, the pieces are sorted in decreasing order of height (h_i), with the larger width (w_i) tiebreaker. The highest piece is selected and placed in the bottom left corner, representing the pivot of the level, determining its height, as presented in Figure 3.

Figure 3. Pieces example

After the pivot of the level has been chosen, pieces are placed on its right side until no more pieces can fit, noticing that the pieces to be placed cannot exceed either the level height (height of the pivot piece) or the remaining width of the roll. In this way a layer is formed. After this, the new list of remaining pieces is updated, and another pivot piece is selected for a subsequent layer, which is placed on top of the pivot piece of the previous layer. This process is repeated until all pieces have been selected and placed on the roll (Figure 4).

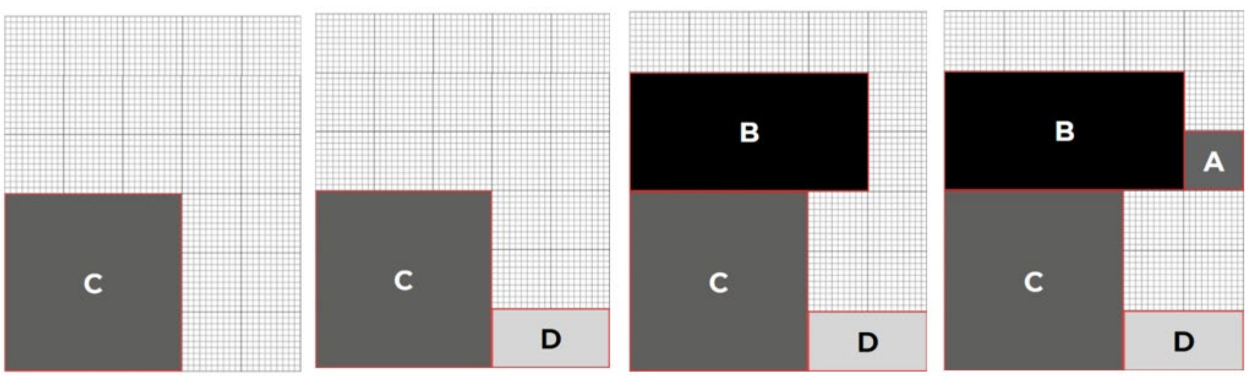

Figure 4. 1st heuristic implementation example

3.4 Exact Model - Level Cutting Problem

The linear programming model is an Open Dimensional Problem, which is regularly guillotined into 2D levels. It aims to determine the best way to cut a roll of material to minimize the required height. This model is equivalent to FFDH.

(10) $Min\ \sum_{i=1}^{N} h_i\, y_i$

(11) $s.a.: \sum_{i=1}^{j-1} x_{ij} + y_j = 1;\ \forall\, j \in N$

(12) $\sum_{j=i+1}^{n} x_{ij}\, w_j \leq (W - w_i)\, y_i;\ \forall\, i \in N-1$

$x_{ij} \in \{0,1\};\ \ y_i \in \{0,1\}\,;\ \forall\, i,j \in N, j < i$

The objective (10) is to minimize the height of the roll used, as stated. The first constraint (11) ensures that each piece is allocated only once. The second constraint (12) ensures that the width of the pieces allocated at a level does not exceed the total width of the roll. The remaining constraints define the variables domain.

3.5- 2nd Heuristic - First-Fit Decreasing-Height with Rotation (FFDHR) – Guillotined

This heuristic follows the same logic as the one presented in section 3.3 but introducing the possibility of rotating the pieces by 90 degrees to enhance the previously obtained solution. This second heuristic was developed to address the limitations of the first one. It is worth noting that rotations of 180 and 270 degrees are not considered, as for this regular problem (rectangles), these rotations (Figure 5) are equivalent to 0 and 90 degrees, respectively.

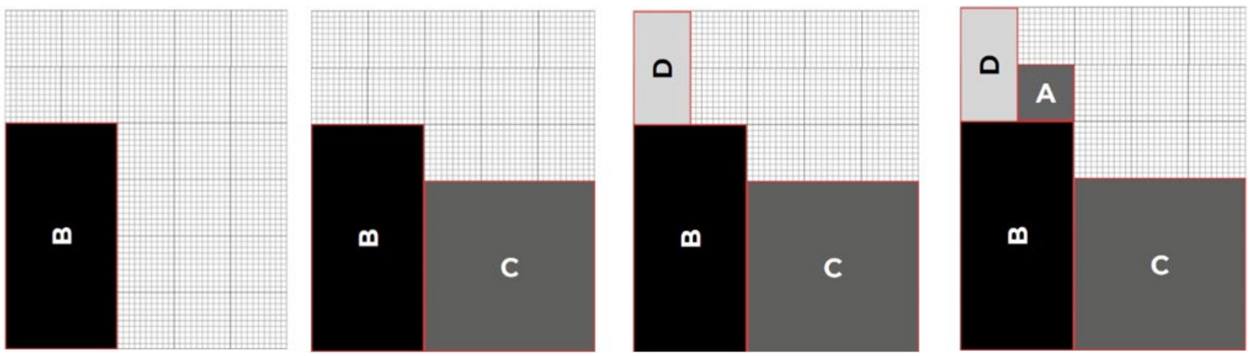

Figure 5. 2nd heuristic implementation example

3.6- 3rd Heuristic - Recursive First-Fit Decreasing-Height with Rotation (RFFDHR) – Guillotined

This heuristic follows the same logic as the one presented in section 3.5; however, it differs from it by resetting when a new piece is placed, using a recursive function. The new roll has a new height (H) corresponding to the height of the placed piece (h_i) and a width (W) corresponding to the remaining width (w_r). This enables the creation of layers within layers, i.e., pieces on top of others at the same level (Figure 6).

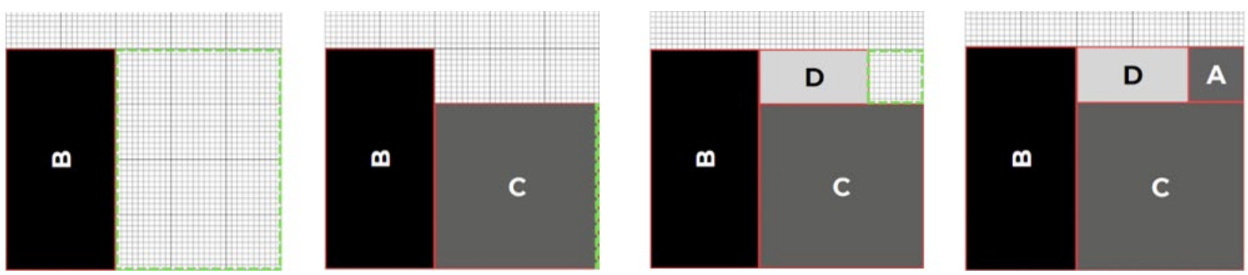

Figure 6. 3rd heuristic implementation example

3.7- 4th Heuristic – Lowest Gap Fill with Rotation (LGFR) – Non-Guillotined

This heuristic considers rotation, where pieces are ordered by decreasing order of area (A_i) with the tiebreaker being the height (h_i) also in decreasing order. Due to this heuristic not working with layers, the pieces must be placed in the PPCs (possible placement points) of the span that are updated throughout the iterations. The PPC with the smallest h is always used. When a piece is placed in a PPC, it is replaced by two PPCs (13), with the initial PPC being (0;0).

(13) $PPC\ (X; Y) \rightarrow PPC\ (X + w_i\ ; Y) + PPC\ (X'\ ; Y + h_i\ )$

Given that X' represents a variable that will be changed during the placement of pieces on the roll, as it represents the farthest right X possible (without exceeding the roll or overlapping any pieces), as shown in Figure 7.

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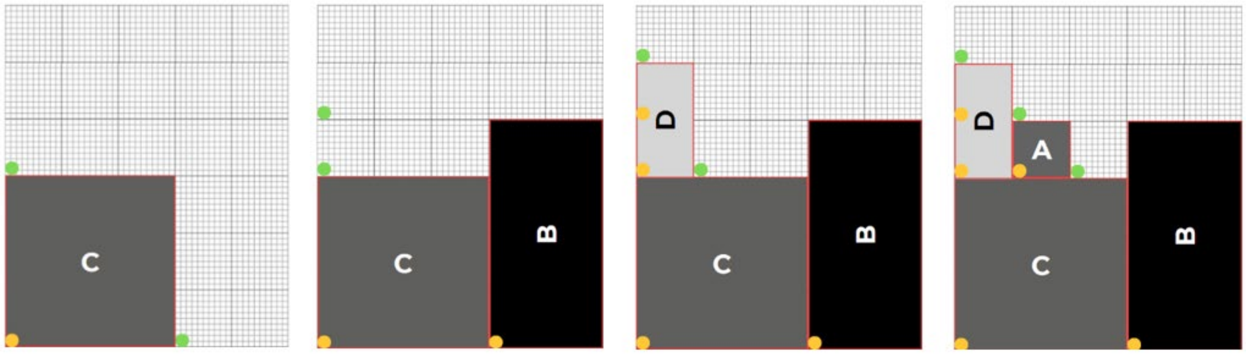

Figure 7. 4th heuristic implementation example

3.8- 5th Heuristic - Irregular Improved Lowest Gap Fill with Rotation (IILGFR) – Irregular

This heuristic, unlike the rest, works with irregular pieces, although it cannot handle curves. Therefore, the irregular approximation was considered. The algorithm starts by sorting the pieces in descending order of area. After that, the pieces are placed one by one in the defined order on the roll in the first space they fit. For this, the pieces and the roll must be written in a matrix form, which will be compared at different points in each iteration to verify the possibility of placing the piece in each set of cells on the roll, as presented in Figure 8.

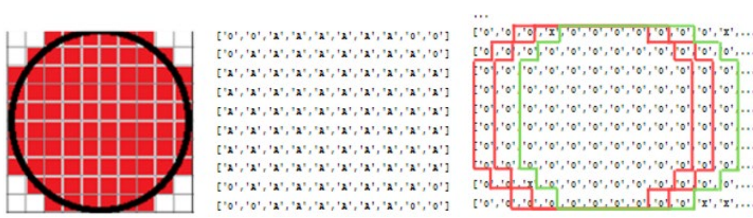

Figure 8. 5th heuristic implementation example

As this heuristic is irregular, eight possibilities for the position of each piece were considered, as shown in Figure 9.

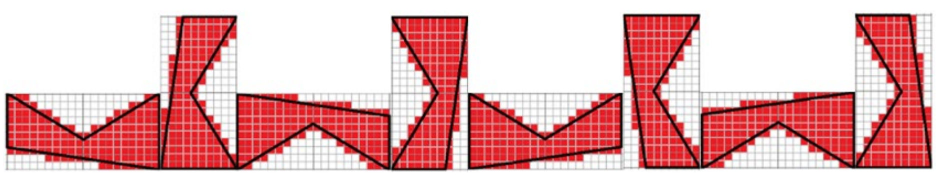

Figure 9. Irregular pieces possible positions (0º, 90º, 180º, and 270º, plus inverted piece at 0º, 90º, 180º, and 270º)

4. Heuristics Validation Using Benchmark Instances

The heuristics were implemented in Python and coded on Geany. The exact model was coded and executed in CPLEX Studio IDE, version 22.1.0. These programs were executed in a computer with an Intel Core i7-11370H @3.30 GHz processor and 16 GB of RAM. It was also used specialized software regarding this problem, Cutting Optimization Pro 6, allowed the authors to make comparisons with the solutions obtained using the developed heuristics.

In order to test, analyze, and compare the performance of the proposed heuristics, they were applied to the first 50 benchmark instances presented by Clautiaux et al. (2018) and available at EURO, resulting in Table 2, which shows the number of rolls used by the solution of each heuristic, as well as $\frac{\sum_{i=1}^{N} A_i}{W \times H}$ rounded up to the nearest integer in order to identify cases where we are sure that the heuristics reached an optimum value, $\frac{\sum_{i=1}^{N} A_i}{W \times H} \leq N^*$.

Note that the IILGFR heuristic was excluded from this analysis since it is designed for irregular problems, has a long execution time per interaction (>1 hour) and requires pre-processing to transform the pieces into matrices.

Table 2. Instances results

Nº	FFDH	FFDHR	RFFDHR	LGFR	$\frac{\sum_{i=1}^{N} A_i}{W \times H}$	Nº	FFDH	FFDHR	RFFDHR	LGFR	$\frac{\sum_{i=1}^{N} A_i}{W \times H}$
1	8	8	7	7	6	26	17	17	14	14	13
2	27	27	22	22	20	27	27	26	21	22	20
3	26	25	21	21	19	28	6	6	5	5	5
4	5	5	4	4	4	29	18	18	14	15	13
5	34	33	26	27	25	30	21	20	16	17	15
6	23	22	18	19	17	31	8	8	6	7	6
7	7	7	6	6	6	32	12	11	9	10	9
8	12	12	10	10	9	33	26	25	20	21	19
9	27	27	22	23	20	34	8	8	7	6	6
10	9	9	7	7	6	35	11	11	9	9	8
11	12	12	9	11	9	36	32	31	25	26	24
12	21	21	17	17	16	37	6	6	5	5	5
13	15	15	13	12	11	38	25	23	19	20	18
14	25	23	19	20	18	39	27	26	20	21	19
15	20	20	16	17	15	40	7	7	6	6	5
16	6	6	5	5	5	41	11	11	9	10	8
17	13	12	10	11	9	42	21	21	17	18	16
18	23	22	18	19	17	43	6	6	5	5	5
19	9	8	7	7	7	44	19	18	14	15	13
20	33	32	26	27	24	45	40	39	31	32	29
21	50	49	39	39	36	46	10	10	8	9	7
22	7	7	6	6	5	47	16	16	13	13	12
23	23	23	18	19	17	48	24	23	19	20	18
24	26	25	20	22	19	49	6	6	5	6	5
25	14	13	11	10	10	50	12	12	10	10	9

After obtaining the results of each instance a comparison was made between the results of each heuristic to see how many times each heuristic presented better results than the others, as shown in the Table 3.

Table 3. Heuristics results comparation

Heuristic	FFDH	FFDHR	RFFDHR	LGFR
FFDH	-	0	0	0
FFDHR	21	-	0	0
RFFDHR	50	50	-	27
LGFR	49	49	3	-

In addition, it was determined how many times each heuristic reached the optimal result with certainty (Table 4).

Table 4. Number of optimal solutions reached

Heuristic	FFDH	FFDHR	RFFDHR	LGFR
Nº	0	0	11	9

From the obtained results, it is possible to infer that the heuristics that presented better results were obtained by RFFDHR and LGFR heuristics, since they were never surpassed by the FFDH nor FFDHR heuristics, also they achieved the optimum in at least 11 and 9 instances (22%, 18%), respectively.
However, the biggest surprise regarding the data was the performance of the RFFDHR heuristic, as on average it had the best performance, even though it was designed for the guillotine problem, unlike LGFR.

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5. Real Problem Results and Discussion

5.1 Company's Actual Solution

The company cuts the elements of the suits in cutting plans containing 2 jackets each, which corresponds to 50 pieces to be cut on the roll. Currently, a software is used that minimizes fabric waste, requiring a roll height of 4.04 m. The rolls used have a constant width of 1.50 m. The solution given by the software is presented in Figure 10.

Figure 10. Company solution layout

5.2 Numerical Results

By applying the 5 heuristics, the Exact model, and the Software COP6, the results in Table 5 were obtained.

Table 5. Case study results

	FFDH	Exact Model - LCP	FFDHR	RFFDHR	Software – COP6	LGFR	IILGFR
H	5,42m	5,10m	4,93m	4,68m	4,31m	4,68m	4,29m
τ_{reg}	79%	84%	87%	92%	98%	92%	-
τ_{irreg}	63%	67%	70%	73%	80%	73%	80%

All heuristics, except for the IILGFR, took seconds to provide the solution. The IILGFR heuristic took approximately 1 hour to generate the solution. For the exact method, coded in CPLEX, it was necessary to establish a time-stopping condition, 2 minutes, since it spent a considerable amount of time analyzing alternative solutions.

5.3 Graphical Results

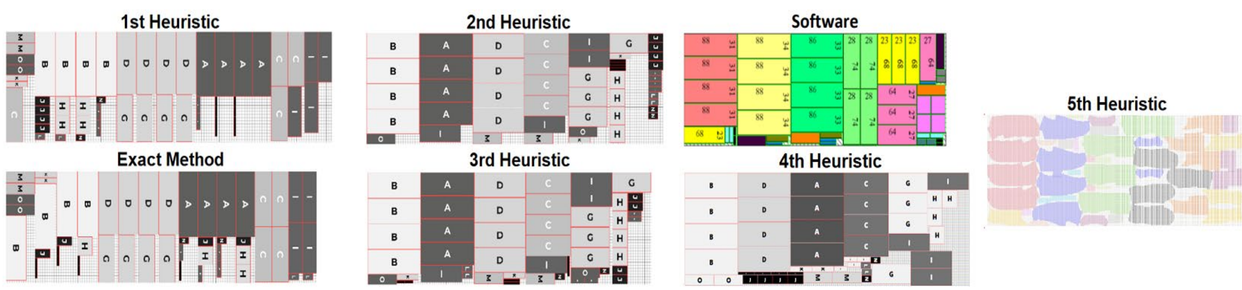

Figure 11. Solutions layout

5.4 Validation

Table 6. Parameter intervals

<table>
<tr><td>Regular Viability Range H</td><td>Irregular Viability Range H</td><td>Regular Usage Rate</td><td colspan="2">Irregular Usage Rate</td></tr>
<tr><td rowspan="2">[429; 1266] cm</td><td rowspan="2">[344; 1266] cm</td><td rowspan="2">[34;100] %</td><td>Exact method, COP6, Heuristics 1,2,3 and 4</td><td>Heuristic 5 and Enterprise Software</td></tr>
<tr><td>[27; 80] %</td><td>[27; 100] %</td></tr>
</table>

As seen in Table 6, all results presented in Table 5 and Figure 11 are viable and reliable, as they fall within the intervals mentioned above. From the analysis of the layouts and results of each heuristic, the solutions improved according to its development order, i.e., the FFDH heuristic presented the worst solution, and the IILGFR heuristic the best one. When the rotation of pieces was not considered, the exact method generated the best solution. When the rotation of pieces was considered, as in the cases of the FFDHR and RFFDHR heuristics, the solution improved, with the RFFDHR heuristic presenting the best result. This is because, beyond rotation, it also allows stacking pieces on top of each other at the same layer, using the space more efficiently. When the problem was considered non-guillotine, as in the case of the LGFR heuristic, an equal roll utilization rate was obtained, although the analysis in section 4 showed that, on average, the RFFDHR heuristic presented better results.

Regarding the regular problem, the specialized software generated the best solution. However, considering the normal irregularity of the pieces, the IILGFR heuristic generates an even better solution, surpassed only by the company's software. In summary, of all the heuristics, the one that presented the closest result to the current solution of the company was the IILGFR. However, from a computational point of view, this is the most demanding heuristic, as showed by the time required to generate a solution, mentioned in section 4. To address this limitation, a mix between the 4th and 5th heuristics can be used, where the 4th is used for larger pieces, and the 5th is used for smaller pieces, so that they fill the spaces left by the pieces introduced by the LGFR heuristic. This combination of heuristics presents a solution with great efficiency, greatly reducing the computational effort required.

6. Conclusion

All five heuristics were programmed in Python, taking also into account an exact method solution, coded in CPLEX, and a specialized software, to prove the modified heuristics performance.
Along the successive heuristics, it is notable the increasable level of complexity involved in the problem resolution. However, the most complex heuristic may not always correspond to the best solution, since the LGFR heuristic only presented a better solution than RFFDHR heuristic on 3/50 cases, presenting a worst solution on 27/50.
Furthermore, it is also notable that in all cases where the optimal solution of number of rolls was guaranteed to be reached, the optimal solution value was low (<10), which can possibly be explained by the accumulated errors.
By applying the previously presented heuristics to the company's problem, satisfactory results were obtained, highlighting the best solution, given by the IILGFR heuristic, that needs 4,29 m for the roll height, which is very close to the actual company solution (4,04 m) that is generated through a highly specialized software.
For future research, the authors suggest the use of a two-phase heuristic approach, such as in the first phase one of the presented heuristics is used to generate an initial solution and in a second phase using an improvement heuristic, that make modifications to the initial solution by switching pieces positioning, to obtain progressively better solutions.

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