Linear Programming Applied to the Vehicle Routing Problem with Simultaneous Delivery and Pickup: A Comparative Study

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Abstract

This work presents two Mixed-Integer Linear Programming models to solve a pickup and delivery problem from a Portuguese company. The objective is to determine the minimum travel time for the pickup and delivery of manufacturing tools of the Manufacturing Tool Repair support service, of this company. The contribution of this work involves the presentation of two three-index mathematical models. Those models reflect the integration of the Simultaneous Delivery and Pickup Problem, with the Capacitated Vehicle Routing Problem with Time Windows. The two mixed-integer linear model formulations are presented and used to solve the problem using several real-world test instances. It is also compared their performance using some benchmark instances, proving its feasibility and comparing its behavior.

Keywords
Vehicle Routing Problem with Time windows, Pickup and Delivery Problem, Mixed Linear Programming

1. Introduction

Organizations are continuously looking for new ways to improve their efficiency. However, the services that support the value chain are normally overlooked and sometimes a great opportunity for improvement is lost.
The present work aims to improve the efficiency of a Portuguese automotive factory, by increasing the efficiency of the production support services, in particularly, the Manufacturing Tool Repair (MTR) support service. The workers of the MTR service are responsible for the repair and calibration of the tools used in the machines that produce the company's products. However, before repairing and calibrating, the workers need to pickup the used tools from the production lines, and after repair, deliver them to the production lines. Since no production line must stop due to lack of manufacturing tools, and currently there is no real-time information of the tools' availability in the lines, the MTR workers need to constantly go to the shopfloor to check the tools' stock levels. This results in 30% of the total working time being allocated to transport activities, which are considered non-value adding for this service.

Due to the applicability of the VRP in real-world situations, and the fact that it is classified as an NP-hard problem, extensive research has been made to optimize it (Kumar & Panneerselvam 2012). When considering time windows constraints and the simultaneous Pickup and Delivery (P&D) a variant of the VRP, the VRPSDPTW is obtained. In this problem, every single customer may be, simultaneously, a pickup and delivery customer. Moreover, the delivering and picking service must be done within pre-defined time windows.

Zhang et al. (2020) applied two different approaches to solve the HVRPSPDTW with multiple depots: an exact and a metaheuristic approach. For the first, they solved a mixed-integer non-linear programming model using the software CPLEX, and for the latter they used a combination between the Parallel Differential Evolutionary algorithm and the

Differential Evolutionary algorithm (Par-DE). In the work of Madankumar and Rajendran (2019), an exact approach using a Mixed Integer Linear Programming (MILP) model is used. By testing it using 24 modified instances of the Vehicle Routing Problem with Time Windows (VRPTW) given by Solomon (1987), the authors concluded that this model obtains the optimal solutions within better computational times than the model of Wang and Chen (2012).
Although there are many publications in the area of vehicle route planning, there are very few that show the application of these developed methods to real problems. Furthermore, when applied to solve practical problems, sensitivity analysis or an evaluation of the effectiveness of various approaches is not always performed.

The work presented here was initiated in Romeira and Moura (2022) with a Vehicle Routing Problem with Simultaneous Pickup and Delivery and Time Windows (VRPSPDTW), and then extended in Romeira and Moura (2023) as a VRPSDPTW and a Two-Dimensional Bin-Packing Problem (2D-BPP).

The current work presents a comparative study between the three-index vehicle-flow model for the VRPSDPTW used in Romeira and Moura (2022) and a new model, a three-index commodity-flow model for the VRPSDPTW.
Subramanian et al. (2010) work presents an indirect and a direct formulation of the latter model, which uses variables that indicate the pickup, the delivery, and the simultaneous pickup and delivery flows. The authors use a branch-and-cut approach based on a previous work of Lysgaard (2003). Later on, the same authors in (Subramanian et al. 2011) proposed a branch-and-cut method over a symmetric formulation with only edge variables.

The problem addressed in this work is presented in section 2 and the mathematical models are presented in section 3. In order to prove the efficiency of the models some tests were performed using both real-world instances and benchmark instances, and the related results are presented in section 4. The conclusions and relevant remarks are presented in section 5.

2. Problem Description

A manufacturing tool has a specific lifespan defined by the number of components it can manufactured. In a perfect world this number is always the same, and thus the repair and calibration frequency will also be the same. However, due to defects or misuse this frequency can vary.

The repair and calibration of the manufacturing tools is done by the MTR support service. Besides this, the MTR service is responsible for picking the tools that reached their lifespan from the production lines, delivering the repaired and calibrated tools to the production lines (henceforth defined as customers), and for solving the issues related to the tools in the customers. However, only the repair and calibration are defined as value-adding activities for this service. All the remain are non-value adding and thus, should be eliminated or minimized.

The MTR service is organized in 3 shifts. Both the morning and evening shifts are comprised by 3 workers and the night and weekend shift by 2 workers each. To perform the P&D routes the workers follow 3 different standardized routes (A,B, and C) presented in Figure 1. The P&D process takes, on average, 18, 15 and 7 minutes for standardized routes A, B and C, respectively.

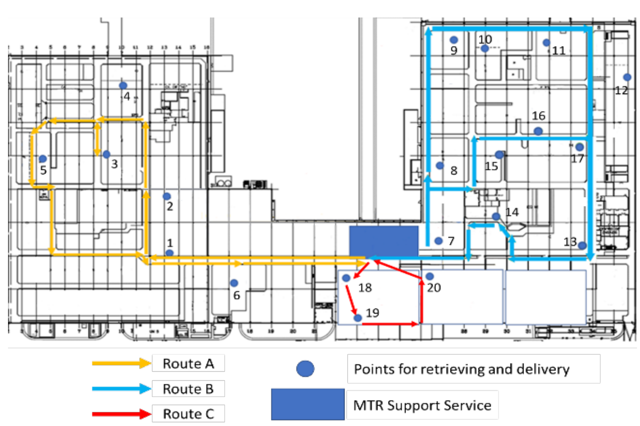

Figure 1. Standardized routes for the P&D of manufacturing tools.

It is important to note that, this service needs to guarantee that no customer will ever stop producing due to lack of manufacturing tools. This rule together with the fact that currently there is no real-time information regarding the available tools to collect in each customer, forces the MTR workers to continuously go on site to check the existing stocks in each customer.

To deal with above, the MTR workers perform one P&D route in the beginning of the shift (to guarantee that all customers have the necessary tools for the shift) and another at the middle. Then, depending on the customer, the workers may need to go on site to check the stocks available, which leads to an average of 6 additional routes per shift. So, we can say that on average 30% of the workers time is wasted doing the P&D activity, which hinders the available time to repair and calibrate tools (value-added activity).

The MTR services has 2 different vehicles to perform the P&D activity: a manual vehicle and an automatic vehicle, as can be seen in Figure 2. The manual vehicle can transport up to 90 tools and is the one adapted to carry tools with a special support that can only fit in this vehicle (henceforth defined as extraordinary tools). For this vehicle, the allocation of tools is straightforward, because a tool will correspond to a specific space in the vehicle.

Figure 2. MTR service vehicles: A – Manual vehicle; B – Automatic Vehicle.

Contrarily, the automatic vehicle carries the manufacturing tools inside baskets, which are then placed in a trailer attached to the vehicle. The trailer is 120cm height and 120cm width, has 2 sides equally divided into 3 levels and has a capacity limit of 22 baskets. The baskets have 4 different standardized dimensions and depending on this can carry a maximum number of tools. Figure 3 presents the different baskets, their dimensions (width and height, respectively) and the maximum number of tools they can carry.

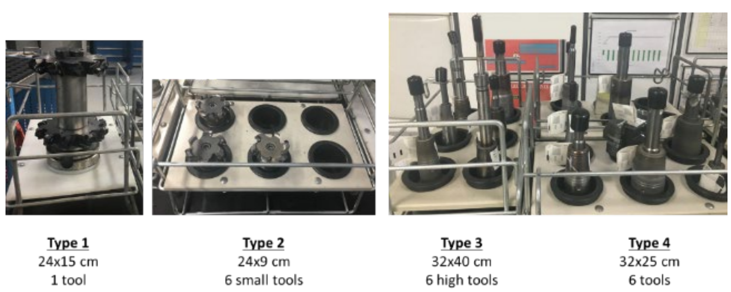

Figure 3. Baskets used to transport tools in the automatic vehicle.

2.1 Optimizing the MTR service

In the work of Romeira et al. (2021) an e-Kanban system for production planning and control was developed. This system collects real-time information about production and stocks levels in all production stages. To this system, we added a Manufacturing Tool stock menu, which shows us the tools that need to be collected in each customer and the tools available to be deliver in the MTR service. This data is continuously processed to trigger a P&D route, as presented in the framework of Figure 4.

A P&D route can be triggered when, the minimal stock is reached in a production line or when there are enough tools to fill the selected vehicle. The analysis that leads to this is the output of the Data Pre-processing process, which will be presented ahead in next section.

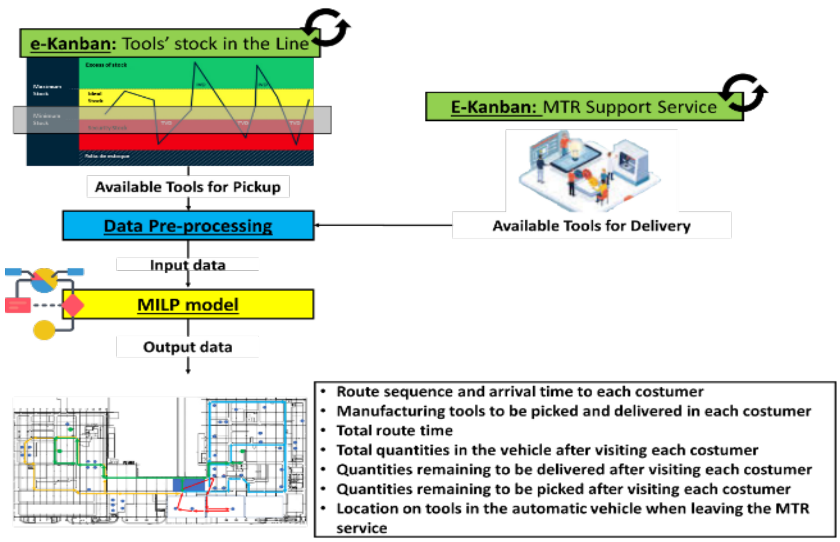

Figure 4. Decision support system framework for the Pickup and Delivery activity of the MTR service.

Once a route is triggered, the customers to be visited, the tools to pickup and delivery in each one and the priority level of each customer are inputted into the MILP model and the P&D route that minimizes the total travel time is obtained and shared with the MTR worker. In the MILP model several important constraints of the problem are considered:

- Each customer is visited by only one vehicle and exactly once per route;
- The MTR service time windows;
- Customers time windows (which indicate the visiting priority level);
- Vehicles' capacities.

A. Data pre-processing process

The data pre-processing process, presented in Figure 5, is composed of a set of heuristics that, using the MILP model proposed, determine the manufacturing tools to P&D in each customer and the route to follow. This math-heuristic uses the input data from the e-Kanban system and triggers a route when the minimal stock is reached, or the selected vehicle's capacity is matched.

When a route is triggered due to the minimal stock, the customer that gave this alert is now considered a priority customer. After this, the system verifies if the priority tools are ready to be delivered, if not an urgent alert is sent to the MTR workers. Having the priority tools available, the vehicle is selected (Vehicle Selection procedure) and then, to take full advantage of the vehicles capacity, the system verifies if there are non-priority tools to be picked-up or delivered from or to these priority customers (Availability of tools to P&D procedure). Once the P&D tools are defined, the MILP model for the VRPSDPTW creates and delivers the P&D route.

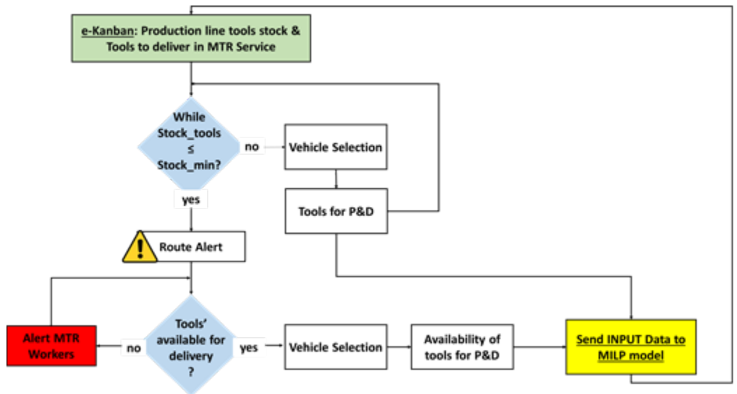

Figure 5. Decision support system framework for the Pickup and Delivery activity of the MTR service.

On the contrary, if there are no priority customers, a vehicle is selected (Vehicle Selection procedure) and then, the system determines if there are enough tools to P&D to fill the selected vehicle (Tools to P&D procedure). Once the tools to P&D are defined the route is obtained.
The Vehicle Selection procedure (Figure 6) is the procedure responsible for selecting the vehicle in function of the tools to be transported and the availability of the vehicles.

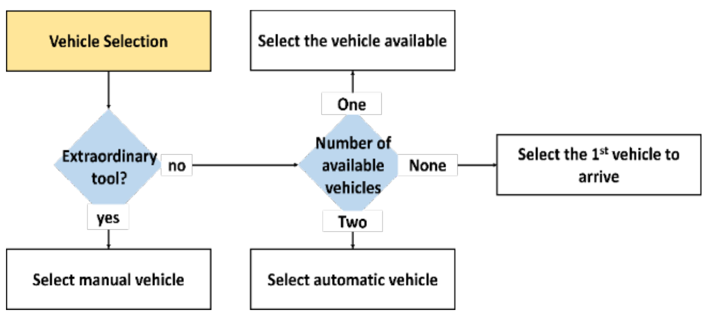

Figure 6. The Vehicle Selection procedure.

3. Mixed-Integer Linear Programming models

In this section, we present two different flow formulations for the VRPSDPTW problem. The VRPSDPTW is NP-hard in a strong sense. But since the Pickup and Delivery activity of the MTR service deals with a small number of production lines (customers) and tools (P&D demand), the idea was to solve the P&D routes optimality. The first mathematical model used is a three-index vehicle-flow formulation (subsection 3.2) and the second one a three-index commodity-flow formulation (subsection 3.3).

Both models are used to solve the same problem. So, it will be expected that the solutions obtained will be the same, since they will return the optimal solution. However, the idea is to check the performance of each of the models in terms of computation times and relative MILP gap tolerance.

3.1 General models notation and data

Considering the VRPSDPTW is defined on a direct graph $G(C, A)$, where each customer is represented by a set of nodes and with different geographical location. The set of customers and the set of edges in G are represented as $C = \{1, \dots, c\}$ and $A = \{(i, j): i, j \in C, i \neq j\}$, respectively. The length of each arc is given by t_{ij}, which consists in the time needed to travel from customer i to j. Moreover, each customer has a P&D demand, which is represented by p_i and d_i, respectively. To deliver the required demand, a set of vehicles $V = \{1, \dots, v\}$ is available, and their capacity is given by Q_k. The customer must be visited within a pre-defined time windows $[a_i, b_i]$ and the service time associated to each one corresponds to st_i. Furthermore, the depot node (MTR support service) also has a time window, $[a_0, b_0]$,which defines the total time available to execute the P&D requirements in each shift.

In order to fulfill these time windows, a decision variable must be useds_{ik}. This variable gives the arrival time of the vehicle at each customer and the depot. Another decision variable used in the following models is x_{ijk}. This is a binary variable that indicates if arc (i, j) is traversed by the vehicle k or not.

3.2 A Three index vehicle-flow model for the VRPSDPTW problem

This model has a binary variable assigned to each arc of the graph. Besides the decision variables presented in the previous subsection, there are other integer variables that must be considered.

- l_{ik}, related to the number of tools after the vehicle k visits a customer i.
- ld_{ik}, related to the number of tools that must be delivered by vehicle k to customer i and to all the other subsequent customers.
- lp_{ik}, related the number of picked-up tools after the vehicle k visits customer i.

Considering those decision variables and the general model components, the objective function of the model is defined as:

$$min \sum_k \sum_i \sum_j c_{ij} x_{ij} \tag{1}$$

s.a.

$$\sum_k \sum_{i \neq j} x_{ijk} = 1, \forall j \neq 1 \in C \tag{2}$$

$$\sum_k \sum_{j \neq i} x_{ijk} = 1, \forall i \neq 1 \in C \tag{3}$$

$$\sum_{i \neq h} x_{ihk} - \sum_{j \neq h} x_{hjk} = 0, \qquad \forall\, k \in V; \forall h \in C \tag{4}$$

$$\sum_{j \neq 1} x_{1jk} \leq 1, \forall k \in V \tag{5}$$

$$\sum_{i \neq 1} x_{i1k} \leq 1, \forall k \in V \tag{6}$$

$$s_{ik} + st_i + c_{ij} \leq s_{jk} - M^T\left(1 - x_{ijk}\right), \forall k \in V;\ \forall i, j \neq i \in C \tag{7}$$

$$a_i \leq s_{ik} \leq b_i, \forall\, i \in C, k \in V \tag{8}$$

$$ld_{ik} \geq ld_{jk} + d_i - M^1\left(1 - x_{ijk}\right), \forall k \in V;\ \forall i, j \neq 1 \in C \tag{9}$$

$$l_{jk} \geq ld_{jk} - d_j + p_j, \forall k \in V;\ \forall j \neq 1 \in C \tag{10}$$

$$l_{jk} \geq l_{ik} + d_j + p_j - M^2\left(1 - x_{ijk}\right), \forall k \in V;\ \forall i, j \neq 1 \in C \tag{11}$$

$$d_j \leq ld_{ik} \leq Q_k, \forall k \in V;\ \forall j \neq 1 \in C \tag{12}$$

$$p_i \leq l_{ik} \leq Q_k, \forall k \in V;\ \forall i \neq 1 \in C \tag{13}$$

$$lp_{jk} \geq lp_{ik} + p_j - M^3\left(1 - x_{ijk}\right), \forall k \in V;\ \forall i \neq 1, j \in C \tag{14}$$

$$p_i \leq lp_{ik} \leq Q_k + p_j, \forall k \in V;\ \forall i \in C \tag{15}$$

$$l_{ik} = lp_{ik} + ld_{ik} - d_i, \forall k \in V;\ \forall i \neq 1 \in C \tag{16}$$

$$x_{ijk} \in \{0,1\} \tag{17}$$

$$l_{ik}, ld_{ik}, lp_{ik} \geq 0, integer \tag{18}$$

This model minimizes the total route cost (1) and this objective function is related to constraints (2) and (3), which ensure that each customer is visited exactly once by a vehicle. Constraint (4) states that if a vehicle enters a node, then it must leave it and constraints (5) and (6) guarantee that if a vehicle leaves the MTR service, then it must return to it. Inequalities (7) and (8) are related to the time windows constraints. The first one specifies the vehicle arrival time to a customer and the second one guarantees that the vehicle arrives within the related time window. With constraint (9), the delivery quantity that must be loaded at the MTR is specified. Additionally (7) and (9) force a specific order for customers visiting within the routes, which ensures that no sub-tours are generated without the MTR. Inequalities (10) and (11) indicate the number of tools in the vehicles after visiting the first customer and the other customers in the routes, respectively. Constraints (12) and (13) guarantee that the vehicle's capacity is not violated. To have more information on pickup loadings, three more constraints were added to the model. With inequality (14) the pickup quantity that must be unloaded in the MTR is determined. Constraint (15) guarantees that the vehicle's capacity is not violated. Constraint (16) correlates the load variables to each other. Constraints (7), (9), (11) and (14) are disjunctive constraints that are linearized by using large multipliers (''big-M values''). To create valid inequalities, one set $M^T = s_{0k}$ and $M^1 = M^2 = M^3 = Q_k$.

3.3 A Three index commodity-flow model for the VRPSDPTW problems

This model, in addition to the variables used by the three-index vehicle-flow formulation, requires a new set of continuous variables (integer variables) associated to the arcs that represent the amount of demand that flows along them:

- α_{ijk}, is the number of tools to be delivered and carried by vehicle k along arc (i, j).
- μ_{ijk}, is the number of picked-up tools carried by the vehicle k along arc (i, j).

This model has the same objective function as the previous one (1). The previous constraints from (2) to (8) and (17) are also considered in this model. Moreover, another set of constraints is considered:

$$\sum_{i \neq j} \alpha_{ihk} - \sum_{l \neq j} \alpha_{jlk} = d_j, \forall\ k \in V; \forall j \neq 1 \in C \tag{19}$$

$$\sum_{l \neq j} \mu_{jlk} - \sum_{i \neq j} \mu_{ijk} = p_j, \forall\, k \in V; \forall j \neq 1 \in C \qquad (20)$$

$$\alpha_{ijk} + \mu_{ijk} \leq M^4 x_{ijk}, \forall\, k \in V; \forall i, j \neq i \in C \qquad (21)$$

$$\alpha_{ijk} \leq Q_k, \forall k \in V;\ \forall i, j \neq i \in C \qquad (22)$$

$$\mu_{ijk} \leq Q_k, \forall k \in V;\ \forall i, j \neq i \in C \qquad (23)$$

$$\sum_{i} \alpha_{ijk} \geq d_j, \forall k \in V;\ \forall j \neq 1 \in C \qquad (24)$$

$$\sum_{j} \mu_{ijk} \geq p_i, \forall k \in V;\ \forall i \neq 1 \in C \qquad (25)$$

$$\alpha_{ijk}, \mu_{ijk} \geq 0, integer \qquad (26)$$

Constraints (19) and (20) are the balance equations to satisfy the delivery and the pickup demands of a customer, respectively. The consideration of those equations also eliminates sub-tours occurrences. Constraints (21), (22) and (23) ensure that the capacity of the vehicle is not exceeded and the "big-M value" of the disjunctive constraint (21) is set to $M^4 = Q_k$. Constraints (24) and (25) are bounds of the total quantities of tools entering and leaving a customer's location. By considering these two constraints, we improve the solution solving process. The model ends with constraint (26), that indicates the variables domain.

4. Computational results

The main objective is to optimize the MTR support service activities, related to the pickup and delivery of tools to the production lines. Once the tools to delivery and collect are sorted a P&D route that minimizes the total travel time must be computed.

The number of production lines is relatively small however the number of tools to P&D can be higher than the vehicle's capacity. For that reason, two types of instances were created. In the first 5 instances, the total number of tools to P&D is always less or equal to the vehicle's capacity, which for the manual vehicle corresponds to 90 tools and for the automatic vehicle to 22 baskets. All the remaining instances the total number of tools to be delivered and collected are higher than the vehicles' capacity. In total, 11 problem instances with real data for each type of vehicle were developed and tested.

Besides the difference in the quantity of tools to P&D, the difference between the two types of instances is related to the customer's time windows, which were obtained randomly.
In order to teste the effectiveness of the two models, bigger and more challenging problems instances, from the literature, specifically the benchmark tests (PDPTW) from Li and Lim (2001) were used. However, since the primary goal of both the real problems and those benchmark problems is to minimize the number of vehicles or routes, and due to the computational complexity of the VRPSDPTW, only the first 20 customers of each problem instances (IC2, IR2 and IRC2) were considered.

All the problem instances were solved using the CPLEX software, and the experiments were run on an Intel(R) Core (TM) i7-10750H CPU @ 2.60GHz 2.59 GHz with 16.0Gb of memory.

4.1 Real data test results

The next two tables (Table 1 and Table 2) present the results achieved for the automatic and manual vehicles in terms of number of vehicles needed to deliver and collect all the required tools, objective function value, that gives us the total time of a route, computational time (CPU) in seconds and the relative MILP gap tolerance in percentage.

According to the results presented in Table 1 and 2, the number of vehicles and objective function values are the same, as to be expected. Although the computational (CPU) time of the two models is similar and very small, the Commodity-Flow (C-F) has slightly higher values. For this model, in most of the instances the optimal solution is achieved. The vehicle-flow (V-F) model in half of the instances has a considerable GAP. The difference between the average GAP values of the two models is 80%, which can be consider very high.

Related to the size of the problems in terms of number of variables and constraints, the average values were, for the V-F model 442 and 620 respectively and for the C-F model 764 and 2422 respectively. This difference (more or less 58% for the variables and 26% for the constraints) can be the reason for the difference, not only, for the computational times, but also for the GAP's values.

Table 1. Vvehicle-flow and Commodity-flow model results for the real problem instances with the manual vehicle.

	Vehicle Flow				**Commodity Flow**			
Instance	**Nº Vehicles**	**FO**	**CPU**	**GAP**	**Nº Vehicles**	**FO**	**CPU**	**GAP**
1	1	5.02	1.09	11.16	1	5.02	1.11	0.00
2	1	6.55	0.05	0.00	1	6.55	0.03	0.00
3	1	3.46	0.05	0.00	1	3.46	0.03	0.00
4	2	5.03	0.02	0.00	2	5.03	0.87	0.00
5	1	4.49	0.01	0.00	1	4.49	0.02	0.00
6	2	5.80	33.47	7.29	2	5.80	53.47	0.00
7	2	6.32	0.14	49.37	2	6.32	3.19	17.26
8	2	6.47	0.14	31.60	2	6.47	3.19	0.00
9	2	6.73	0.13	29.72	2	6.73	2.03	0.00
10	3	7.85	0.45	51.08	3	7.85	5.00	19.04
11	3	8.66	352.98	4.78	3	8.66	403.17	0.00
Average			**35.32**	**16.82**			**42.92**	**3.30**

Table 2. Vehicle-flow and Commodity-flow model results for the real problem instances with the automatic vehicle.

	Vehicle Flow				**Commodity Flow**			
Instance	**Nº Vehicles**	**FO**	**CPU**	**GAP**	**Nº Vehicles**	**FO**	**CPU**	**GAP**
1	1	8.77	0.06	0.00	1	8.77	0.08	0.00
2	1	9.24	0.05	0.00	1	9.24	0.14	0.00
3	1	12.35	0.08	0.00	1	12.35	0.16	0.00
4	1	9.23	0.05	0.00	1	9.23	0.05	0.00
5	2	9.27	0.06	0.00	2	9.27	0.17	0.00
6	1	8.77	0.11	18.56	1	8.77	0.39	0.00
7	2	10.76	0.31	11.37	2	10.76	0.87	0.00
8	2	12.68	0.13	0.00	2	12.68	0.27	0.00
9	3	12.83	0.61	30.55	3	12.83	1.21	10.37
10	3	14.85	0.30	18.18	3	14.85	2.37	0.00
11	3	14.85	0.45	23.91	3	14.85	2.48	7.81
Average			**0.20**	**9.32**			**0.74**	**1.65**

4.2 Benchmark test results

As mentioned in the beginning of this section, to test the effectiveness and efficiency of the models, bigger instances were also solved. Using the PDPTW, of Li and Lim (2001), some of the benchmarks were selected and adapted. Only the first 20 customers were considered, and the results are presented in Table 3.

However, due to the size of those instances, if the optimal solution is not reached within the pre-defined time limit (2 hours), the column of the objective function (costs) presents the best integer solution, and the CPU displays a horizontal bar.

The instances are symmetric and are grouped in clustered customers (IC type), uniformly distributed customers (IR type) and a mix of IC and IR types. As can be seen in Table 3 and like in the real problems (presented in Table 1 and Table 2), the results in terms of solution GAP obtained with V-F model are slightly worse than the C-F model. The procedure based on V-F model provides the fastest running times since the CPU time is rather lower (55% on average) than the CPU time of the C-F model. For the IRC problems (more complex instances), the optimal solution was not achieved for the last two instances, within the time limit pre-established. And only one instance was solved optimality with the two models, the IC201.

Table 3. Vehicle-flow and Commodity-flow model results for the adapted benchmark problem instances.

Instances	**Vehicle-Flow Model**			**Commodity-Flow Model**		
	Costs	**CPU (sec)**	**GAP (%)**	**Costs**	**CPU (sec)**	**GAP (%)**
IC201	252	0.01	0.00	252	0.01	0.00
IC202	202	1.69	11.88	202	3.07	5.70
IC203	194	8.69	2.36	194	8.60	0.00
IC204	186	42.01	0.96	186	9.69	0.00
IC205	251	2.75	1.40	251	1.17	0.00
IC206	230	1.64	3.28	230	3.29	1.42
IC207	218	1.89	1.87	218	2.06	1.04
IC208	205	3.74	3.78	205	2.68	0.00
IR202	399	146.84	0.87	399	166.36	0.00
IR203	354	391.19	0.71	354	734.03	0.00
IR204	289	42.68	0.81	289	66.10	0.00
IR205	350	1.86	1.19	350	2.42	0.00
IR206	318	11.40	2.83	318	31.03	0.00
IR207	318	61.82	0.00	318	175.81	0.00
IR208	260	2.53	1.08	260	2.01	1.04
IR209	312	3.87	2.26	312	3.23	0.00
IR210	353	79.06	0.69	353	103.96	0.00
IR211	293	253.83	0.00	293	99.98	0.00
IRC202	341	156.23	3.16	341	1310.44	0.00
IRC203	309	-	20.95	307	-	10.69
IRC204	268	-	17.91	283	-	9.76
Average		**63.88**	**3.71**		**143.47**	**1.41**

5. Conclusions

The main idea started with the need to improve the MTR support service's efficiency of a Portuguese company. This service is one of the most time-consuming activities because it consists of delivering and collecting the manufacturing tools, from the MTR workshop to the production lines and vice-versa.

To focus on improving this distribution process, it was proposed to develop and implement a support system that informs workers when to do a P&D route, according to the tools needed for production and the tools available for collection. The data related to what tools were available for collection in the production lines and what tools were available for delivery in the MTR service is known. Every time that a P&D route must be done, we are facing a VRPSDPTW. Since the number of production lines, vehicles and tools is not very high, it was decided to solve this problem using a linear programming model.

However, the need arose to test more than one approach, to validate its efficiency in terms of computational time and solutions. Hence the proposal presented in this paper to make a comparative study of the performance of two models: Vehicle-Flow model and the Commodity-flow model.

With the two models, most of the real-world instances tested were solved optimality in a reduced computational time and good upper bounds could be generated for the remaining instances.
The models were also tested, and its performance compared with the well-known benchmark tests for Li and Lim (2001). Even with these more complex and larger instances, the conclusions drawn for the real instances were the same. In conclusion, after the tests performed, the commodity-flow model has a better performance regarding computational times and GAP values, than the vehicle- flow model, being proven its efficiency.

References

Li, H., Lim, A., A metaheuristic for the pickup and delivery problem with time windows, 13th IEEE international conference on tools with artificial intelligence (ICTAI) pp. 160-170, 2001.

Lysgaard, J., *CVRPSEP: a package of separation routines for the capacitated vehicle routing problem,2003*.

Kumar, S. N., & Panneerselvam, R., A Survey on the Vehicle Routing Problem and Its Variants, 2012(03), 66–74, 2012. https://doi.org/10.4236/IIM.2012.43010

Madankumar, S., & Rajendran, C. (2019). A mixed integer linear programming model for the vehicle routing problem with simultaneous delivery and pickup by heterogeneous vehicles, and constrained by time windows. Sadhana - Academy Proceedings in Engineering Sciences, 44(2), 1–14,2019. https://doi.org/10.1007/s12046-018-1048-y

Romeira, B., Cunha, F., & Moura, A. (2021). Development and Application of an e-Kanban System in the Automotive Industry. In *IEOM Monterrey 2021 Conference*. Monterrey.

Romeira, B., & Moura, A. (2022). Optimizing Route Planning for Minimising the Non-added-Value Tasks Times: A Simultaneous Pickup-and-Delivery Problem. In *11th International Conference on Operations Research and Enterprise Systems* (pp. 153–160). https://doi.org/10.5220/0010821000003117

Romeira, B., & Moura, A. (2023). Minimizing the Non-Value Task Times: A Pickup and Delivery Problem with Two-Dimensional Bin-Packing. In *Communications in Computer and Information Science*. Springer.

Solomon, M. M., Algorithms for the Vehicle Routing and Scheduling Problems with Time Window Constraints. Https://Doi.Org/10.1287/Opre.35.2.254, 35(2), 254–265,1987. https://doi.org/10.1287/OPRE.35.2.254

Subramanian, A., Uchoa, E., & Ochi, L. S., New lower bounds for the vehicle routing problem with simultaneous pickup and delivery. *Lecture Notes in Computer Science (Including Subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, *6049 LNCS*, 276–2872010. https://doi.org/10.1007/978-3-642-13193-6_24/COVER

Subramanian, A., Uchoa, E., Pessoa, A. A., & Ochi, L. S. (2011). Branch-and-cut with lazy separation for the vehicle routing problem with simultaneous pickup and delivery. *Operations Research Letters*, *39*(5), 338–341. https://doi.org/10.1016/J.ORL.2011.06.012

Wang, H. F., & Chen, Y. Y. (2012). A genetic algorithm for the simultaneous delivery and pickup problems with time window. Computers and Industrial Engineering, 62(1), 84–95,2012. https://doi.org/10.1016/j.cie.2011.08.018

Zhang Q, Liu S, Zhang R, Qin S. Column generation algorithms for mother plate design in steel plants. OR Spectrum,;43:127–53,2020. doi:10.1007/S00291-020-00610-Z.

Biography

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