

# **A Novel Integer Programming Model for the Fixture Schedule of Turkish Professional Soccer League**

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## **Abstract**

We study a sports scheduling problem where the entire fixture of a soccer season in Turkey must be scheduled by taking into account a variety of constraints. We model the problem as an integer program; because of the computational intractability, we develop a heuristic algorithm for obtaining a suboptimal solution. Differing from the earlier models, our model minimizes the assignment of matches where a team plays against strong teams consecutively. We compare the performance of our heuristic against the existing solution and demonstrate numerically that our heuristic performs significantly better than the current solution.

## **Keywords**

OR in sports, sports scheduling, integer programming, scheduling, operations research

## **1. Introduction**

The problem of scheduling a fixture is highly complex problem since it requires the assignment of matches for each round, which leads to computational intractability. Researchers therefore resort to heuristic algorithms by exploiting certain features of these problems. Certainly, depending on the nature of the respective problem, a variety of heuristic procedures can be developed.

In any given fixture, it is almost impossible to avoid the following situations: a team having to play two consecutive matches against relatively strong teams. This situation certainly should be avoided, as it causes a serious disadvantage to the respective team. Motivated by this fact, we develop a mathematical model where the number of two consecutive matches against strong teams are minimized for every team. To be more specific, we classify teams as strong, medium, and weak based on the past data of their rankings and input them into our model. In this sense, to the best of our knowledge, our model is the first one that takes into account such undesirable situations in determining the league fixture.

Our mathematical model is still intractable, since it is developed for obtaining the entire schedule. We therefore implement an easy-to-use heuristic procedure for acquiring a suboptimal solution. Our heuristic solves the problem in a few steps, first focusing on a number of rounds, then certain rounds of the remaining ones, etc. In a sense, it partitions the entire fixture into a few sections and then optimally solves each of them, thereby providing a suboptimal solution at the end. We compare our heuristic against the existing schedule and show that it significantly outperforms the existing one. **For example, percentage improvement in terms of cost is around 44%.**

The flow of our paper is as follows. We first discuss the related literature in Section 2. Section 3 provides necessary info about the underlying scheduling problem as well as our mathematical model. In Section 4, we describe the experimental design and report numerical results. Section 5 consists of concluding remarks.

## **2. Literature Review**

Sports scheduling is among operations research fields that have been studied extensively. One can review the following survey papers in order to have basic info about the field: Rasmussen and Trick (2008), Ribeiro (2012), and Kendal et al. (2010). One classification of the related papers can be based on the objective function used. Examples of objective functions used in the respective studies are as follows. Minimizing rest mismatches (Atan and Cavdaroglu

2018), Cavdaroglu and Atan 2020), minimizing travel distances (Kendal and Westphal 2013), and minimizing breaks (Rasmussen and Trick 2007, Duran et al. 2017, Recalde et al. 2013, and Bulck 2020).

The main methodology used to tackle sports scheduling problems is integer programming (IP). Examples of related work that used IP include but not limited to Briskorn and Drexl (2009), Duran et al. (2019), Kim (2019), Elf and Junger (2003). Further, some of those papers approached the problem from the perspective of graph theory (see for instance van't Hof et al. 2010 and Yi et al. 2020).

Although there are numerous papers about sports scheduling, our focus is on those published in recent years. Examples of relatively new papers are briefly reviewed below.

Davari et al. (2020) study scheduling of multiple leagues simultaneously, considering interdependencies resulting from teams in different leagues. Their model has the property that teams belonging to the same club share the same venue, and minimizes total capacity violation. Duran et al. (2019) proposes integer programming for scheduling the South American Qualifiers to FIFA World Cup organized in 2018. The respective competition is based the tournament of a double round robin. Duran (2021) extensively surveys the literature on sports scheduling, with a particular focus on applications in Latin America. The author examines different problems addressed, techniques developed for solving them, and the implementation results of the respective methodologies.

Januario et al. (2016) exploit concepts from graph theory for solving a variety of sports scheduling problems. Specifically, they contribute to the literature by showing how edge coloring can be utilized for such problems. In another study, Januario et al. (2016b) analyze the weakness of the canonical method, characterizing the conditions in which that weakness of the canonical method, which is entrapment in local search procedures, occurs. Through this characterization, the authors shed light on a relation between "the connectivity of the analyzed neighborhood and the riffle shuffle, a method of shuffling playing cards." Kim (2019) proposes an integer programming-based model for scheduling of multiple round robin tournaments, focusing on Korea baseball league. The criteria included in this study include fairness of match, fairness of attendance, and total travel distance for teams. Their results reveal that the current schedule can be improved significantly through the implementation of the proposed model.

Kim et al. (2020) analyse the fairness of The World Baseball Confederation tournament. The authors propose a more fair schedule and provide objective critiques on the current schedule, providing views on how it can be more successful. Liang et al. (2021) develop a Variable Neighborhood Search algorithm for Major League Baseball. Their model aims to minimize total traveling distance of teams. The authors demonstrate that the resulting schedule significantly outperforms the existing schedule in terms of total traveling distance and the respective standard deviation.

Peng et al. (2021) use reinforcement learning (RL) for the problem of scheduling multi-team tournaments. The authors first model the problem as a Markov decision process, and then propose an RL environment where a variant of temporal difference method is employed. Sagir et al. (2019) developed a mathematical model for the sports scheduling problem faced by the Malaysian football federation. Their model is combined with the Analytic Hierarchy Process, which is used to obtain importance levels defined for each potential fixture.

## **2.1. Contribution**

We re-visit the fixture scheduling problem for Turkish Professional Soccer league in light of strength criteria of teams. Particularly, we develop a novel mathematical model that takes into account strength of each team while determining the league fixture. To deal with computational intractability, we propose a heuristic algorithm that quickly leads to a suboptimal solution. Finally, we numerically demonstrate that our heuristic procedure can significantly improve the current schedule of fixture.

## **3. Problem Description and Mathematical Formulation**

This section first contains necessary information about Turkish Professional Soccer League.

The Turkish Super League (TSL) regularly contains 18 teams, which implies that there are 34 rounds. At each round, 9 matches must be played. There is usually break between the first and the second halves of the fixture. The fixture scheduling has to deal with the decision of who will play with whom at each round. Matchday schedule, however, is

concerned with the days and times of each match for a given round. The soccer federation In Turkey first determines the fixture based on a certain mechanism; thereafter, the matchday schedules are determined.

As stated earlier, we classify teams on the basis of their strength. To avoid a subjective classification, we considered the total points of the teams in the past 5 seasons. Further, as the format of the league slightly changed in terms of number of teams at the start of the Covid-19, we specified those 5 seasons as 2014-15, 2015-16 ... 2018-19. The total points of the teams are weighted over years using the following weights: 0.2, 0.3, 0.5, 0.6, and 0.7. The reasoning here is that the total points gained in the recent past have more weights than those gained in the far past.

The weighted total points of teams and the resulting classification are given in Table 1. The reason why one of the teams has 0 total points is because it was just elevated to the Super league from a lower league, and has no points gained in the past.

Table 1. Teams, their weighted total points, and their strength category. T.I and T.P. refer to team index and total points, respectively.

T.I.	T.P.	Strength	T.I.	T.P.	Strength
1	164.1	Strong	10	74.8	Medium
2	156.1	Strong	11	74.6	Medium
3	156	Strong	12	65	Weak
4	144.4	Strong	13	64.3	Weak
5	126	Strong	14	58.7	Weak
6	102.9	Medium	15	56	Weak
7	98.8	Medium	16	31.8	Weak
8	96.8	Medium	17	28	Weak
9	84.3	Medium	18	0	Weak

### 3.1. Mathematical Formulation

The problem we are concerned with can be formulated mathematically as follows.

The variables of our formulation are given below.

$assign_{ij} = 1$ , if team  $i$  has a game against team  $j$  at round  $r$   
 $= 0$ , o.w.

Here,  $i=1,\dots,I$ ,  $r=1,\dots,R$ ,  $j=1,\dots,I$  s.t.  $i \neq j$ .  $I$  equals 18 and  $R$  equals 17, as there are 18 teams.

$y_{SSS_{irs_1s_2}} = 1$ , if a strong team  $i$  has a game against strong teams  
 $s_1$  and team  $s_2$  at rounds  $r$  and  $r + 1$ ,  
 $= 0$ , o.w.

Here,  $i=1,\dots,S$ ,  $r=1,\dots,R$ ,  $s_1=1,\dots,S$ ,  $s_2=1,\dots,S$ .  $S$  equals 5 as there are 5 strong teams.

$y_{MSS_{irs_1s_2}} = 1$ , if a medium team  $i$  has a game against strong teams  $s_1$  and team  $s_2$  at rounds  $r$  and  $r + 1$ ,  
 $= 0$ , o.w.

Here,  $i = S+1, \dots, S+M, r=1, \dots, R, s1=1, \dots, S, s2=1, \dots, S$ .  $S$  equals 5 and  $M$  equals 6 as there are 6 medium teams.

$y\_WSS_{irs1s2} = 1$ , if a weak team  $i$  has a game against strong teams  $s1$  and team  $s2$  at rounds  $r$  and  $r + 1$   
 $= 0$ , o.w.

Here,  $i = S+M+1, \dots, I, r=1, \dots, R, s1=1, \dots, S, s2=1, \dots, S$ .  $S$  equals 5 and  $M$  equals 6.

The objective function is expressed as:

$$\begin{aligned} \text{Min. } & c\_SSS \times \sum_{i \text{ in } 1 \dots S, r \text{ in } 1 \dots R, s1 \text{ in } 1 \dots S, s2 \text{ in } 1 \dots S, s1 \neq s2} y\_SSS\_irs1s2 \\ & + \\ & c\_MSS \times \sum_{i \text{ in } S+1, \dots, S+M, r \text{ in } 1 \dots R, s1 \text{ in } 1 \dots S, s2 \text{ in } 1 \dots S, s1 \neq s2} y\_MSS\_irs1s2 \\ & + \\ & c\_WSS \times \sum_{i \text{ in } S+M+1, \dots, I, r \text{ in } 1 \dots R, s1 \text{ in } 1 \dots S, s2 \text{ in } 1 \dots S, s1 \neq s2} y\_WSS\_irs1s2. \end{aligned}$$

Here,  $c\_SSS$ ,  $c\_MSS$ , and  $c\_WSS$  are cost of playing against two strong teams for strong, medium, and weak teams, respectively.

The constraints of the mathematical model are in the following form.

$$y\_SSS_{irs1s2} \geq \text{assign}_{irs1} + \text{assign}_{i(r+1)s2} - 1 \quad (1)$$

$$y\_SSS_{irs1s2} \leq \text{assign}_{irs1} \quad (2)$$

$$y\_SSS_{irs1s2} \leq \text{assign}_{i(r+1)s2} \quad (3)$$

Here,  $i=1, \dots, 5, r=1, \dots, R-1, s1=1, \dots, 5, s2=1, \dots, 5, i \neq s1, i \neq s2, s1 \neq s2$ .

$$y\_MSS_{irs1s2} \geq \text{assign}_{irs1} + \text{assign}_{i(r+1)s2} - 1 \quad (4)$$

$$y\_MSS_{irs1s2} \leq \text{assign}_{irs1} \quad (5)$$

$$y\_MSS_{irs1s2} \leq \text{assign}_{i(r+1)s2} \quad (6)$$

Here,  $i=S+1, \dots, S+M, r=1, \dots, R-1, s1=1, \dots, 5, s2=1, \dots, 5, s1 \neq s2$ .

$$y\_WSS_{irs1s2} \geq \text{assign}_{irs1} + \text{assign}_{i(r+1)s2} - 1 \quad (7)$$

$$y\_WSS_{irs1s2} \leq \text{assign}_{irs1} \quad (8)$$

$$y\_WSS_{irs1s2} \leq \text{assign}_{i(r+1)s2} \quad (9)$$

Here,  $i=S+M, \dots, I, r=1, \dots, R-1, s1=1, \dots, 5, s2=1, \dots, 5, s1 \neq s2$ .

$$\text{assign}_{ij} = \text{assign}_{jri} \quad (10)$$

Here,  $i=1, \dots, I, j=1, \dots, I, r=1, \dots, R, i \neq j$ .

$$\sum_{j \text{ in } 1 \dots I: j \neq i} \text{assign}_{irj} = 1 \quad (11)$$

Here,  $i=1, \dots, I, r=1, \dots, R$ .

Constraints 1 to 3 ensure that if  $i^{\text{th}}$  team plays two consecutive games against strong teams, then the respective variable (i.e.,  $y_{SSS_{i^{\text{th}}s_1s_2}}$ ) has to be 1. Constraints 4 to 9 play the same role. Constraint 10 makes sure that match assignments are made correctly: if  $i^{\text{th}}$  team is assigned to  $j^{\text{th}}$  team at round  $r$ , then the respective variable,  $assign_{jri}$  has to be 1. Finally, Constraint 11 requires that each team has a match at each round.

#### 4. Experimental Design and Numerical Results

In order to compare our heuristic procedure against the current solution, we need to specify the values of certain cost parameters. These parameters are defined for the following pairs: (strong, strong), (strong, medium), (medium, strong), (medium, medium). This implies that, for example, there is a cost if a team plays strong and medium teams consecutively. The respective cost values for those pairs are set to (16,4,4,2) for strong teams; for medium teams, we use (32,8,8,4) whereas those pairs are set to (48,12,12,6) for weak teams. The reasoning is the following. First, the cost of playing strong teams consecutively should be significantly higher than that of playing two medium teams as well as a strong team and then a medium team. Additionally, the cost of playing medium teams should be the lowest one, as it does cause the least amount of disadvantage to the respective team. What is more, the negative impact of playing strong-strong matches on medium teams is higher than that for strong teams (16 vs. 32). The same argument holds as far as medium and weak teams are concerned.

The way our heuristic algorithm works is as follows. It first finds an optimal solution for the first 3 rounds, then for rounds 4 to 6, and then rounds 7 to 9. Finally, it finds the optimal solution for the remaining rounds, thereby generating a suboptimal solution.

We used AMPL to solve the underlying integer programming problems. For a 3-round problem, it takes only a couple of minutes to reach the optimal solution via AMPL.

The number of matches corresponding to each strength pair for each team for the existing solution and the heuristic algorithm are given in Tables 2 and 3. Notice that while there is no match corresponding to the (strong, strong) pair for the heuristic procedure, there are 14 matches corresponding to that pair in the existing solution. Further, percentage improvement in terms of cost is around 44% (1054 versus 592).

Table 2. Number of matches of each strength pair for the current solution.

T.I.	S-S	S-M	M-S	M-M
1	1	2	0	2
2	0	2	1	2
3	1	1	1	2
4	1	2	1	2
5	0	1	1	1
6	1	2	1	1
7	0	2	1	1
8	1	1	1	2
9	1	1	1	0
10	1	1	1	1
11	1	1	0	2

12	1	2	1	2
13	1	2	1	2
14	1	2	1	2
15	0	2	4	0
16	1	2	1	2
17	1	2	1	2
18	1	2	1	2

Table 3. Number of matches of each strength pair for the heuristic solution

T.I.	S-S	S-M	M-S	M-M
1	0	0	2	0
2	0	0	0	3
3	0	1	0	3
4	0	0	3	1
5	0	1	2	2
6	0	0	1	2
7	0	2	1	1
8	0	1	2	1
9	0	1	1	3
10	0	1	1	1
11	0	2	1	1
12	0	3	2	1
13	0	2	2	1
14	0	2	2	1
15	0	2	1	3
16	0	2	1	2
17	0	1	2	2
18	0	2	2	3

#### **4.1. An Extended Model**

We extended our mathematical model with the inclusion of the effect of playing against strong and medium teams as well as medium and strong teams consecutively. In fact, the inclusion of medium-medium can also be handled; we

however observe that in that case it takes significant amount of time to solve a small problem with 3 rounds optimally. This implies that our heuristic cannot be implemented efficiently in that case.

Examples of the resulting additional decision variables for this model are defined below.

$y\_SSM_{irms} = 1$ , if a strong team  $i$  has a game against a strong team  $s$  and a medium team  $m$  at rounds  $r$  and  $r + 1$

$y\_SSM_{irms} = 0$ , o.w.

Here,  $i=1,\dots,S$ ,  $r=1,\dots,R$ ,  $s=1,\dots,S$ ,  $m=1,\dots,M$ .

$y\_MMS_{irms} = 1$ , if a medium team  $i$  has a game against a medium team  $m$  and a strong team  $s$  at rounds  $r$  and  $r + 1$   
 $= 0$ , o.w.

Here,  $i=1,\dots,M$ ,  $r=1,\dots,R$ ,  $m = 1,\dots,M$ ,  $s=1,\dots,S$ .

$y\_WSM_{irms} = 1$ , if a weak team  $i$  has a game against a strong team  $s$  and a medium team  $m$  at rounds,  $r$  and  $r + 1$   
 $= 0$ , o.w.

Here,  $i=1,\dots,W$ ,  $r=1,\dots,R$ ,  $s=1,\dots,S$ ,  $m=1,\dots,M$ .  $W$  equals 7 as there are 7 weak teams.

Our extended model certainly contains additional parameters as well as constraints owing mainly to the inclusion of these decision variables. Since those parameters and constraints are structurally the same as the existing ones, there is no need to introduce them in the paper.

Results are presented in Table 5. The percentage improvement over the current solution in terms of total cost is around 48%. This value was 44% for the first model. Slight increase in the improvement value may be attributed to the fact that the extended model handles more complexity as compared to the former one. On the other hand, std. deviation values associated with the extended model are significantly higher than those for the first model. Specifically, the std. dev. values for strong, medium, and weak teams are now 6.6, 14.9, and 25, respectively (see Table 4 for the comparison).

Table 4. Standard deviation and maximum difference values for the two solutions. S.D.(H) and M.D.(H) stand for standard deviation and maximum difference in cost for the heuristic. S.D.(C) and M.D.(C) stand for standard deviation and maximum difference in cost for the current solution.

Strength	S.D. (H)	M.D. (H)	S.D. (C)	M.D.(C)	Perc. Impr. in S.D.	Perc. Impr. in M.D.
Strong	4.1	10	9.3	22	55	54
Medium	5.3	12	11.1	32	52	62
Weak	7.5	18	9.1	24	17	25

Table 5. Number of matches of each strength pair for the solution of the revised heuristic.

T.I.	S-S	S-M	M-S
1	0	1	0

2	0	1	0
3	0	0	0
4	1	0	0
5	0	2	1
6	0	2	0
7	0	0	0
8	0	0	1
9	1	0	0
10	1	1	0
11	0	1	1
12	1	1	0
13	0	0	1
14	1	2	0
15	0	1	0
16	1	1	0
17	1	0	1
18	1	1	0

## **T5. Conclusion**

The problem of scheduling the fixture is computationally intractable in terms of optimality objective. In this research, we tackle the fixture schedule problem for Turkish professional soccer league by using the notion of strength criteria and utilizing an easy-to-use heuristic procedure. Particularly, the idea that teams can be classified based on their strength enables us to have a mathematical model for obtaining a near-optimal solution.

Our numerical results are promising, showing the strength of our heuristic algorithm as compared to the current solution. This implies that such a heuristic based on a simplified mathematical model can be used for obtaining a better solution in terms of strength criterion.

We are of the view that computational intractability of fixture schedule problems may be addressed through a variety of criteria that are similar to the strength criterion we proposed. In that case, it would be interesting to observe how distinct heuristic algorithms perform if multiple types of criteria are considered.

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## **Biography**

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