

# **Cost Reduction of Water Supply Systems Through Optimization Methodologies: A Comparative Study of Optimization Approaches**

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## **Abstract**

The global water supply accounts for 7% of worldwide energy consumption. With the increasing energy tariffs, there is a growing need to reduce the energy consumption of water supply systems (WSS). Among the three primary areas for optimization in WSS, pump operation is the most critical one, representing 70% of the energy consumed in WSS. Despite several mathematical formulations and approaches for optimizing pump operation in WSS, there is a lack of comparative studies between them. Moreover, since each WSS system has unique physical characteristics, the performance and solutions generated by each formulation may vary. Therefore, this study aims to conduct a comparative analysis of three WSS optimization approaches, of which one was never implemented. These approaches are applied to two case studies with specific physical characteristics.

## **Keywords**

Cost Reduction, Mathematical Approaches, Optimization Methodologies, Water Supply Systems

## **1. Introduction**

Water supply systems (WSS) are essential infrastructures designed to transport water from distant sources to individual consumers in the required quantity and pressure levels (Manteigas et al. 2022). These constitute a crucial part of any city worldwide, with the primary objective of delivering treated drinking water to consumers (Manteigas et al. 2022).

The global water supply is responsible for a substantial portion of the world's energy consumption (Manteigas et al. 2022). As a result, reducing the energy consumption of water supply systems is extremely important to water

management entities (Manuel 2017). The high energy consumption makes energy efficiency a critical factor in the sustainability of water management entities, leading to the adoption of smart solutions capable of reducing costs. Moreover, the costs associated with energy and water management are rising faster than inflation, making it increasingly challenging for companies to rely solely on the experience of their most skilled employees. Therefore, even minor improvements in operational efficiency can result in significant cost savings for the water utility industry (Selek et al. 2012). Water supply systems are guided by operational rules that seek to meet multiple and often conflicting objectives, such as minimizing operational cost, energy consumption, flood risk, and maximizing water quality (Awe et al. 2019). Optimization can be applied to several areas of WSS, but pump operation is particularly critical (Mala-Jetmarova et al. 2017).

Supplying water with sufficient pressure to consumers necessitates pumping water to higher elevations, which incurs substantial direct costs for water supply systems (Sousa 2005). The escalating cost of pumping energy and the growing water demand have compelled water management companies to seek methodologies to maximize cost savings while maintaining system performance criteria (Nitivattananon et al. 2000).

Possible solutions to reduce the direct water pumping costs in water supply systems include adjusting pumping times to take advantage of cheaper tariffs and using variable-speed pumps (VSPs). However, effectively combining these solutions is a challenging task that requires careful consideration of various constraints, including meeting customer water requirements, maintaining appropriate pressure levels, and ensuring water quality (Coelho and Andrade-Campos 2014). Several researchers have addressed the optimal control problem of reducing energy consumption in water supply systems through a variety of methods, including linear and nonlinear programming, dynamic programming, and heuristic optimization (Pasha and Lansey 2009; Selek et al. 2012). However, improving the mathematical formulation of the operational control problem can directly enhance its efficiency. In practice, real-world optimization models tend to be more complex, featuring a more significant number of decision variables and constraints and more challenging objectives (Maier et al. 2014).

Although there are several mathematical formulations for optimizing the operation of water supply systems, there is a lack of quantitative comparison between them. As a result, it remains an open question as to which formulation is the most efficient and robust for each specific problem. Since each WSS has unique physical characteristics, the performance of optimization formulations can vary significantly. While the binary formulation, which relates to the on/off state of pumps, is the most commonly used, there are more efficient approaches for some cases. Therefore, it is essential to conduct a comparative study of the most implemented WSS optimization formulation to determine their strengths and weaknesses.

Over time, the process of formulating the problem of optimal pump scheduling has evolved, with two main approaches: implicit control problems and explicit control problems (Vega and Alem 2014). The explicit formulations directly control pump operation through decision variables that specify when the pumps operate. On the other hand, the implicit formulations dictate pump operation by defining specific system characteristics that work as external triggers to activate the pumps and dictate their operation. In this case, the implicit decision variables are related to pump flows (Bene et al. 2013), pressures (Skworcow et al. 2010), or water tank trigger levels (Quintiliani and Creaco 2019).

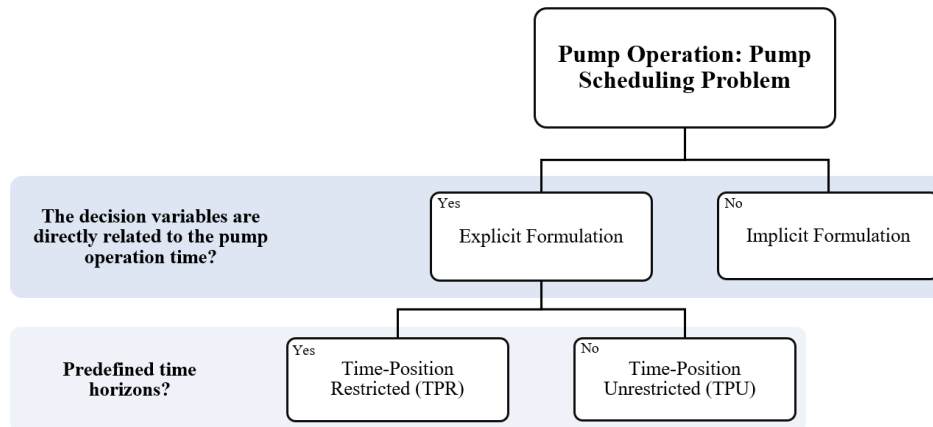


Figure 1. Categorization of pump scheduling optimization problem: explicit control problem vs. implicit control problem.

According to Ormsbee et al. (2009), the explicit approach is further subdivided into two categories. This article refers to these categories as time-position restricted (TPR) and time-position unrestricted (TPU). The TPR explicit formulation restricts the decision variables to a predefined time interval, i.e. time horizons, and requires pumps to start operating at the beginning of the interval. On the other hand, the TPU explicit formulation allows more flexibility by allowing pumps to operate at any time within the total time horizon, and the decision variables are related to the start and end times of pump operations. An illustrative example of both approaches is presented in Figure 2.

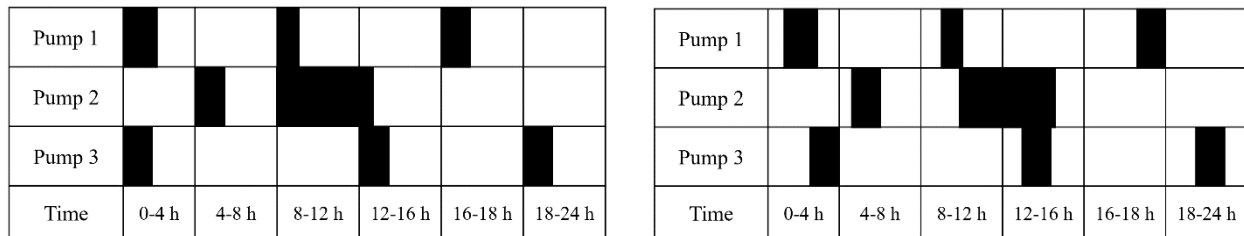


Figure 2 . Illustrative example of the difference between TPR (left) and TPU (right) explicit approaches. Adapted from Ormsbee et al. (2009).

In all formulations, the primary objective is to minimize the energy cost associated with pump operation during the total time horizon. Apart from explicit bound constraints related to the decision variables domain, there are also implicit system constraints related to the conservation of mass and energy in terms of flow rate ( $q$ ) and the satisfaction of the systems' minimum pressure ( $p$ ). These constraints are critical to ensure the feasibility of the optimized solution and that the water supply system performs optimally. Overall, the optimal pump scheduling problem is a complex bound-constraints optimization problem that requires considering the trade-off between energy consumption, system constraints, and customer satisfaction. The choice of formulation and the specific constraints and objective function used depends on each WSS's specific characteristics and goals.

Due to each WSS's unique physical characteristics and behavior, different mathematical optimization approaches will lead to different solutions when applied to these systems. Hence, it is crucial to determine the most efficient and robust formulation for each specific practical case. Currently, the selection of an optimization model is often based on the expertise of experienced professionals or the adoption of the simplest one, which may not necessarily result in the most economical pump operation. This study aims to quantitatively compare three optimization approaches for optimizing pump scheduling in two different WSSs. The approaches have different constrained optimization problems (two TPR and one TPU), and it should be noted that this TPU optimization problem has not yet been implemented in practice.

This paper is divided into five sections. It begins with this introductory section, followed by the presentation of the methodology (section 2), in which is detailed the optimization approaches developed, and the case studies implemented to compare them (section 3). It ends with the exposition of the case studies results and its discussion (section 4) and the presentation of some future work and conclusions (section 5).

## 2. Methodology

Considering the complexity of the system constraints, these are typically extracted out of the formal optimization formation and handled externally through a predictor/simulation software (Ormsbee et al.,2009) (see Figure 3). Using this problem disaggregation structure, a vector of decision variables  $\mathbf{X}$  is selected, which satisfies the explicit bound constraints (decision variables' domain). This vector  $\mathbf{X}$  is translated into controls  $s(t)$ , and it is verified if these satisfy the systems' implicit constraints ( $AH=q$ ) through the WSS Predictor, obtaining the respective vectors of flows ( $q$ ) and pressures/heads from it ( $p$ ). Through the mathematical model, the feasibility of implicit bound constraints is verified (i.e. water tank levels, number of pump switches, etc), and the value of the objective function associated with the vector  $\mathbf{X}$  of decision variables is obtained.

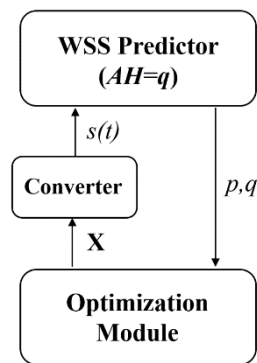


Figure 3. Optimization approaches' structure.

In this study, the hydraulic simulator widely implemented EPANET was used as the WSS predictor. Through the predictor, the behavior of two different case studies (section 3) based on a real Portuguese WSS is simulated. In addition, three different optimization approaches to be compared (section 2.1) were implemented in python:

B-GA Approach: Binary Formulation (TPR Explicit Formulation) + Binary Genetic Algorithm

RC-SLSQP Approach: Real-Continuous Formulation (TPR Explicit Formulation) + Sequential Least Squares Programming (SLSQP)

DC-SLSQP Approach: Duty-Cycles Formulation (TPU Explicit Formulation) + SLSQP

When implementing the approaches with SLSQP, the scipy.optimize python library was applied. As for the genetic algorithm, a binary variant of the algorithm available in (Heris 2020) was implemented in python.

### 2.1. Optimization Approaches

The objective of the optimization models applied to these systems is to minimize the cost associated with the pump's operation. To fully understand the mathematical models used to optimize the pump operation, it is necessary to define several parameters. These include the total time horizon ( $T$ ), the number of pumps ( $P$ ), the  $N$  number of time horizons within  $T$ , the duration of each  $n$  time horizon ( $\Delta t_n^{\text{horizon}}$ ), the energy tariff applied to each time horizon ( $\$_n$ ), the hydraulic power of the pump  $p$  ( $W_p$ ), the number of water tanks ( $WT$ ), the minimum and maximum water levels of the each tank ( $N_{wt}^{\min}$  and  $N_{wt}^{\max}$ ), and the water level of each tank at each time horizon ( $g_{n,wt}$ ).

#### B-GA Approach: Binary Formulation (TPR Explicit Formulation) + Binary Genetic Algorithm

The binary formulation is the most common mathematical model used for optimizing pump operation. It involves using a set of binary decision variables, represented by  $\mathbf{X}^{\text{bin}}$  to determine whether the pump  $p$ , with  $p=1, \dots, P$ , is on or

off during a specific time horizon  $n$ , with  $n=1, \dots, N$ . If  $X_{n,p}^{\text{bin}}$  is set to zero, the pump  $p$  is off during the time horizon  $n$ . If  $X_{n,p}^{\text{bin}}$  is set to one, the pump  $p$  will be turned on. The decision variables are given by

$$\mathbf{X}^{\text{bin}} = \begin{bmatrix} X_{1,1}^{\text{bin}} & \dots & X_{N,1}^{\text{bin}} \\ \vdots & \ddots & \vdots \\ X_{1,P}^{\text{bin}} & \dots & X_{N,P}^{\text{bin}} \end{bmatrix}, \text{ (size } N \times P\text{)}. \quad (1)$$

In this study, in order to become as closest as possible to the continuum behavior, forty-eight time horizons were used ( $N=48$ ), each with a duration of 30 minutes. The mathematical model can be stated as follows:

$$\text{minimize } C(\mathbf{X}^{\text{bin}}) = \sum_{p=1}^P \sum_{n=1}^N \$n \times \frac{W_p(X_{n,p}^{\text{bin}})}{\eta_p} \times X_{n,p}^{\text{bin}} \times \Delta t_n^{\text{horizon}} \quad (2)$$

$$\text{subject to: } N_{wt}^{\text{min}} \leq g_{n,wt}(\mathbf{X}^{\text{bin}}) \leq N_{wt}^{\text{max}}, \text{ with } wt = 1, \dots, WT \text{ and } n = 1, \dots, N \quad (3)$$

$$X_{n,p}^{\text{bin}} = \{0,1\}, \text{ with } n = 1, \dots, N \text{ and } p = 1, \dots, P \quad (4)$$

$$AH = q \quad (5)$$

Equation 2 dictates the objective function of this formulation, where it is intended to minimize the cost related to the energy consumed by the pumps during the total time horizon. This cost is calculated by the sum of the energy cost of each time horizon for each pump, which is the multiplication of the energy tariff by the pump hydraulic power, and by the pump operating time. Equation 3 represents the constraint for the implicit water tank bounds, where the level of the water tank  $wt$  must be within a predefined range of values ( $N_{wt}^{\text{min}}$  and  $N_{wt}^{\text{max}}$ ). Equation 4 dictates the domain of the decision variables. Equation 5 is related to the implicit system constraints that are guaranteed by de WSS Predictor. An example of the use of this mathematical formulation can be found in (López-Ibáñez et al., 2005). Considering the problem' non-linearity and the type of decision variables, a binary genetic algorithm was used to solve this optimization problem.

#### RC-SLSQP Approach: Real-Continuous Formulation (TPR Explicit Formulation) + SLSQP

As in the previous formulations, the total time horizon is divided into time horizons, and the set of decision variables  $\mathbf{X}^{\text{rc}}$  dictate the normalized operation time of each pump during each time horizon. The pump  $p$  operating time at time horizon  $n$  is given by  $t_{n,p}^{\text{op}}$ . The decision variables are given by

$$\mathbf{X}^{\text{rc}} = \begin{bmatrix} X_{1,1}^{\text{rc}} & \dots & X_{N,1}^{\text{rc}} \\ \vdots & \ddots & \vdots \\ X_{1,P}^{\text{rc}} & \dots & X_{N,P}^{\text{rc}} \end{bmatrix}, \text{ with } X_{n,p}^{\text{rc}} = \frac{t_{n,p}^{\text{op}}}{\Delta t_n^{\text{horizon}}}, n = 1, \dots, N \text{ and } p = 1, \dots, P \text{ (size } N \times P\text{)}. \quad (6)$$

When  $X_{n,p}^{\text{rc}}$  is set to zero, the pump  $p$  is off during the time horizon  $n$ . Dissimilar from the previous formulation, when  $X_{n,p}^{\text{rc}}$  is set between zero and one, the pump  $p$  is turned on during a period of time of  $X_{n,p}^{\text{rc}} \times \Delta t_n^{\text{horizon}}$ . In each time horizon, the pumps can only start operating at the beginning of that time horizon. For this study, in this formulation, it was defined six time horizons ( $N=6$ ), one for each energy tariff. The mathematical model is translated in:

$$\text{minimize } C(\mathbf{X}^{\text{rc}}) = \sum_{p=1}^P \sum_{n=1}^N \$n \times \frac{W_p(X_{n,p}^{\text{rc}})}{\eta_p} \times X_{n,p}^{\text{rc}} \quad (7)$$

$$\text{subject to: } N_{wt}^{\text{min}} \leq g_{n,wt}(\mathbf{X}^{\text{rc}}) \leq N_{wt}^{\text{max}}, \text{ with } wt = 1, \dots, WT \text{ and } n = 1, \dots, N \quad (8)$$

$$X_{n,p}^{\text{rc}} = [0,1], \text{ with } n = 1, \dots, N \text{ and } p = 1, \dots, P \quad (9)$$

$$AH = q \quad (10)$$

Regarding the model constraints, they are similar to those of the previous model. An example of the use of this mathematical formulation can be found in (Bernardete Coelho and Andrade-campos, 2013). To solve this model, the SLSQP algorithm was used.

DC-SLSQP Approach: Duty-Cycles Formulation (TPU Explicit Formulation) + SLSQP

In this formulation, pumps are allowed to start operating at any point in the given total time horizon. The concept of duty cycle  $d$  is introduced, which represents a pumping operation. For example, if a pump is only allowed to operate once per day, there is only one duty cycle per day. The set of decision variables  $\mathbf{X}^{\text{dc}}$  determines the starting and duration times of each duty cycle. In this case, there are two decision variables associated with each duty cycle, having values between 0 and  $T$ . The decision variables are given by

$$\mathbf{X}^{\text{dc}} = \begin{bmatrix} X_{1,1}^{\text{dc}} & \dots & X_{2D,1}^{\text{dc}} \\ \vdots & \ddots & \vdots \\ X_{1,P}^{\text{dc}} & \dots & X_{2D,P}^{\text{dc}} \end{bmatrix}, \text{ (size } 2D \times P\text{)}. \quad (11)$$

During the total time horizon, pumps may operate multiple times. However, to minimize pump starting's and correspondent maintenance costs, a maximum number of duty cycles ( $D$ ) per pump is set. For each pump, the first  $D$  variables decision dictates the starting time, and the lasts  $D$  dictates the duration of each duty-cycle. In these case studies, it was defined a maximum of six duty-cycles ( $D=6$ ), i.e., pumps can be switched on six times, in order to be similar to the previous formulation and thus the results can be comparable. The optimization model is formulated as:

$$\text{minimize } C(\mathbf{X}^{\text{dc}}) = \sum_{p=1}^P \sum_{d=1}^D \$_d(X_{d,p}^{\text{dc}}, X_{d+D,p}^{\text{dc}}) \times \frac{W_p(X_{d,p}^{\text{dc}}, X_{d+D,p}^{\text{dc}})}{\eta_p} \times X_{d+D,p}^{\text{dc}} \quad (12)$$

$$\text{subject to: } N_{wt}^{\text{min}} \leq g_{d,wt}(\mathbf{X}^{\text{dc}}) \leq N_{wt}^{\text{max}}, \text{ with } wt = 1, \dots, WT \text{ and } d = 1, \dots, 2D \quad (13)$$

$$X_{d+1,p}^{\text{dc}} - (X_{d,p}^{\text{dc}} + X_{d+D,p}^{\text{dc}}) \geq 0 \text{ with } d = 1, \dots, D \text{ and } p = 1, \dots, P \quad (14)$$

$$X_{d,p}^{\text{dc}} = [0, T], \text{ with } d = 1, \dots, 2D \text{ and } p = 1, \dots, P \quad (15)$$

$$AH = q \quad (16)$$

In addition to the typical constraints applied to this problem related to the variable's domain and minimum and maximum water tank levels, it is necessary to add another one related to the pump operations (Equation 14). This hard constraint ensures that the starting time for a particular duty cycle is greater than the stopping time of the previous duty cycle. As in the previous approach, SLSQP was used to solve this model.

### 3. Case Studies

#### 3.1. Case Study 1: Water Supply System with water inlet from below

To compare the three optimization approaches, it was used a water supply subsystem of a real Portuguese WSS. As shown in Figure 4, it is composed of a water tank  $F$ , a fixed speed pump  $P$ , and two consumption points  $Vc$  and  $R$  (nodes 3 and 5, respectively).

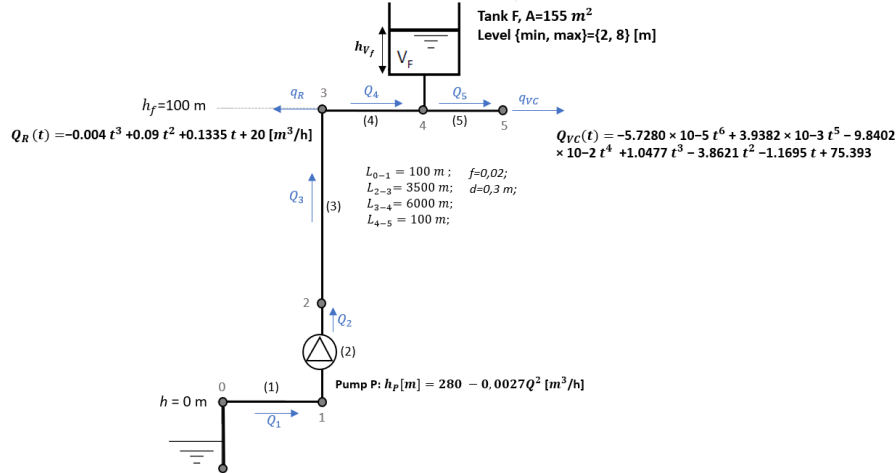


Figure 4. Water supply subsystem used as case study 1.

Water tank  $F$  has a quota of 100 mm, 155 m<sup>2</sup> of area, and the water inlet is positioned at the bottom of the tank. For safety reasons, the tank's water level can only fluctuate between 2 and 8 m. This tank supplies consumers in the  $Vc$  region and also supplies consumers in the  $R$  region when the pump  $P$  is not in operation. The consumptions of these two regions are defined by two polynomials whose behaviors are present in Figure 5.

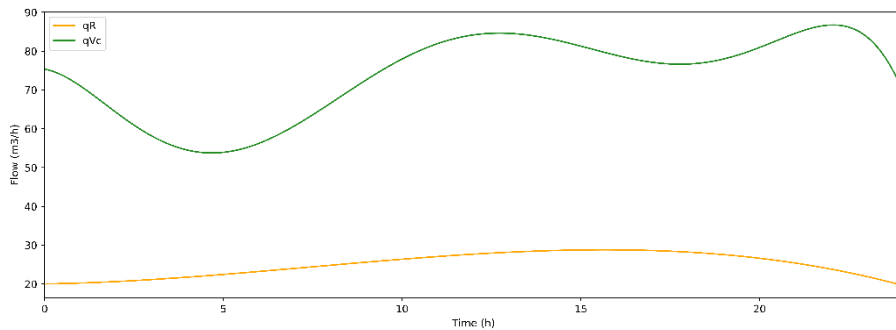


Figure 5. Daily consumption in m<sup>3</sup>/h for regions  $Vc$  and  $R$ .

The pump  $P$  has a hydraulic curve  $h_p$  ( $h_p = 280 - 0.0027Q^2$ ) and is at a 0 m quota. This pump is responsible for sending water to tank  $F$  and to consumer  $R$ . The cost of energy (tariff) consumed by the pump  $P$  varies throughout the day, as listed in Table 1.

Table 1. Energy tariffs during the day.

| Time Intervals [h] | Cost [€/kWh] |
|--------------------|--------------|
| [0,2[              | 0,0737       |
| [2,6[              | 0,006618     |
| [6,7[              | 0,0737       |
| [7,9[              | 0,10094      |
| [9,12[             | 0,18581      |
| [12,24[            | 0,10095      |

### 3.2. Case Study 2: Water Supply System with water inlet from above

Case study 2 closely resembles the first one, with the distinctions being the location of the water feed in the tank and the removal of the consumption point  $R$ . While the previous case study featured inlet and outlet points positioned beneath the tank, the current case study employs an inlet point above and an outlet point below, as shown in Figure 6. In this case, when activated, pump  $P$  supplies only the water tank  $F$  that provides the  $Vc$  regions with water.

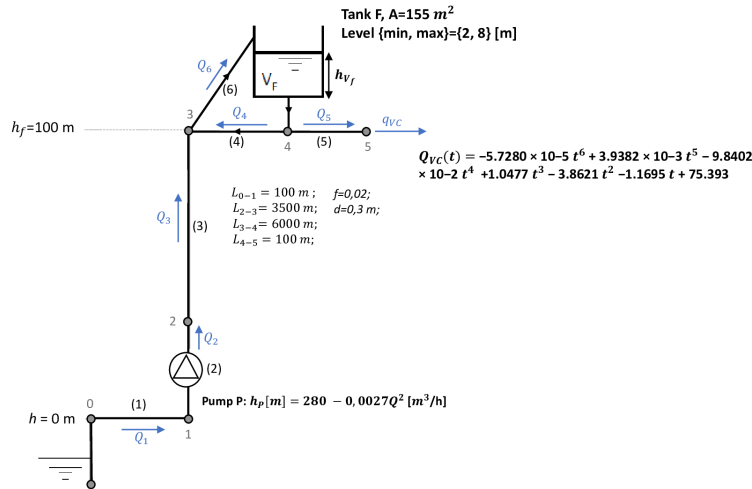


Figure 6. Water supply subsystem used as case study 2.

The performance of a pump is defined by its characteristic curve, which in this study can be expressed as  $h_p = 280 - 0.0027Q_p^2$ . Essentially, a higher manometric head ( $h_p$ ) corresponds to a lower flow rate ( $Q_p^2$ ) and, consequently, an increased power consumption by the pump ( $W_p = \frac{\rho \times g \times h_p \times Q_p^2}{\eta_p}$ ). In case study 1, since the water inlet is positioned at the bottom of the tank, the manometric head is directly linked to the water level in the tank. Therefore, it is more cost-effective to pump water when the tank water level is at its lowest while being mindful of the energy tariff. On the contrary, in case study 2, the water inlet is located at the top of the tank, and the manometric head is solely dependent on the tank quota, which remains constant. As a result, the pumping cost is only influenced by energy tariff rates. The purpose behind altering the design of the case study was to evaluate the efficacy of three optimization approaches across two different case studies and determine whether their performance varied between the two scenarios.

## 4. Case Studies Results

Three different optimization approaches (OA) were tested in the optimization of two case studies. To compare their performance, the following Key Performance Indicators (KPI) were chosen: number of time horizons/number of decision variables, processing time, number of objective function evaluations, pump starting's, and total cost. In Table 2, the results obtained for each optimization approach are exposed. The optimization approaches were run on an 11<sup>th</sup> Gen Intel(R) CORE(TM) i7-1185G7 CPU @ 3.00 GHz with 16 GB RAM.

Table 2. Summary of results of each case study and optimization approach.

| KPI/OA   | Case Study 1 |          |          | Case Study 2 |          |          |
|--|--------------|----------|----------|--------------|----------|----------|
|  | B-GA         | RC-SLSQP | DC-SLSQP | B-GA         | RC-SLSQP | DC-SLSQP |
| Number of Time Horizons/Number of Decision Variables | 48/48        | 6/6      | 6/(6+6)  | 48/48        | 6/6      | 6/(6+6)  |
| Processing Time (s)                                  | 6249.78 s    | 1.17 s   | 26.73s   | 5759.67 s    | 1.50 s   | 2.77 s   |
| Number of Objective                                  | 100100       | 5        | 136      | 100100       | 13       | 14       |



|                      |         |         |         |         |         |         |
|----------------------|---------|---------|---------|---------|---------|---------|
| Function Evaluations |         |         |         |         |         |         |
| Pump Starting's      | 6       | 3       | 3       | 5       | 2       | 2       |
| Total Cost (€)       | 111.95€ | 109.30€ | 108.90€ | 124.55€ | 124.24€ | 124.18€ |

#### **4.1 Case Study 1: Discuss of Results**

Figures 7, 8, and 9 show each optimization approach's solution through the status of the pump (full black line), the evolution of the tank water level (full blue line), the energy tariffs prices (dotted orange line) and its division in time slots (vertical full gray line), the water tank limits (dotted dark blue line), and the pump flow rate (green line).

When comparing the mathematical formulations of each approach, it is possible to affirm that the solutions space of the binary formulation is a subset of the discrete formulation, and the solution space of this last one is a subset of the duty-cycles formulation. Therefore, it is possible to write these mathematical models starting from the duty-cycles formulation, and as a consequence, theoretically, is possible to achieve any solution from the first and second models through the third one.

Upon analyzing the water supply system in case study 1, it is noticeable that, due to the typical pump characteristic curve, the placement of the water inlet below the water tank and the consumption point  $R$  directly influences the water pumped. As a result, the lower pump energy consumption corresponds to the lower water tank level. Therefore, it would be more cost-effective to pump water when the tank level is as low as possible. Thus, the DC-SLSQP approach provides an advantage over the other two approaches since it has a time-position unrestricted explicit formulation, allowing for greater flexibility in choosing the start-up time. In contrast, the other two approaches (B-GA and RC-SLSQP) have a fixed start-up time.

Considering the mentioned above, it was expected that the DC-SLSQP approach would achieve more economical results in this case study. The results in Table 2 and Figures 7, 8, and 9 confirm that statement. It should be noted that the observed discrepancies in final costs are not substantial, which can be assigned to the relatively straightforward nature of the case study involving only one pump and one tank. As a result, more complex case studies must be tested in future work to determine whether these differences can become significant. Additionally, it is essential to consider the solution space of each approach and evaluate whether the processing time of the DC-SLSQP approach remains comparable to the RC-SLSQP approach in more complex case studies. As the system's complexity increases, the solutions space also expands, which may result in longer processing times. By examining more intricate scenarios involving multiple pumps and tanks, a more comprehensive understanding of the efficacy of the approaches can be gained, allowing for more informed decision-making.

Another observation concerns the efficiency of the DC-SLSQP approach, which was found to be heavily influenced by the quality of the initial solution supplied. Multiple tests with several initial solutions revealed that the better the quality of the initial solution, the better the solution is obtained. However, to have an impartial comparison between the optimization approaches, the same initial pumping operation was defined for each one.

Regarding the significant disparities in processing time and the number of objective function evaluations observed between the B-GA and the other two approaches, it can be attributed to the inherent characteristics of the binary genetic algorithm.

In terms of the number of pump starts, given the decision variable count in the DC-SLSQP and RC-SLSQP formulations, these are restricted to 6 starts. Both formulations reach a solution with the same number of pump starts, thereby having an equal effect on maintenance concerns. As the B-GA approach possesses more decision variables, it is constrained to 24 pump starts, having achieved a solution with 6 pump starts.

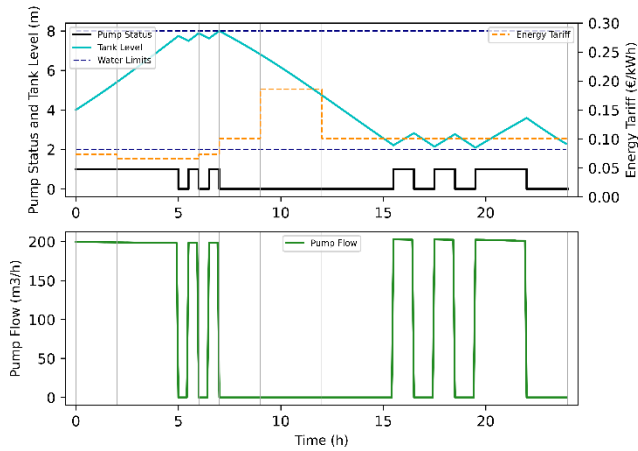


Figure 7. Results of B-GA approach in case study 1.

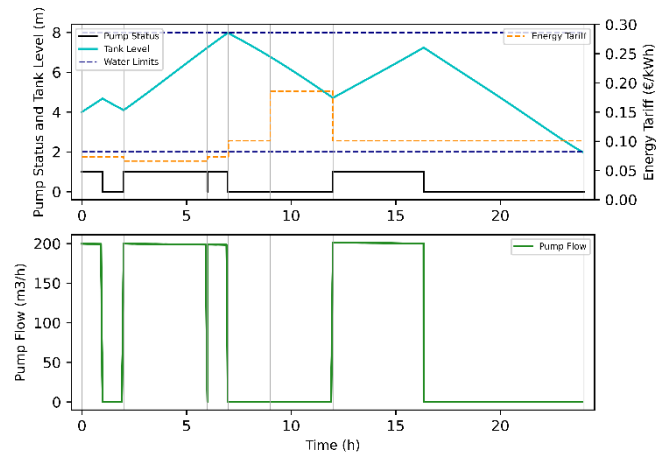


Figure 8. Results of RC-SLSQP approach in case study 1.

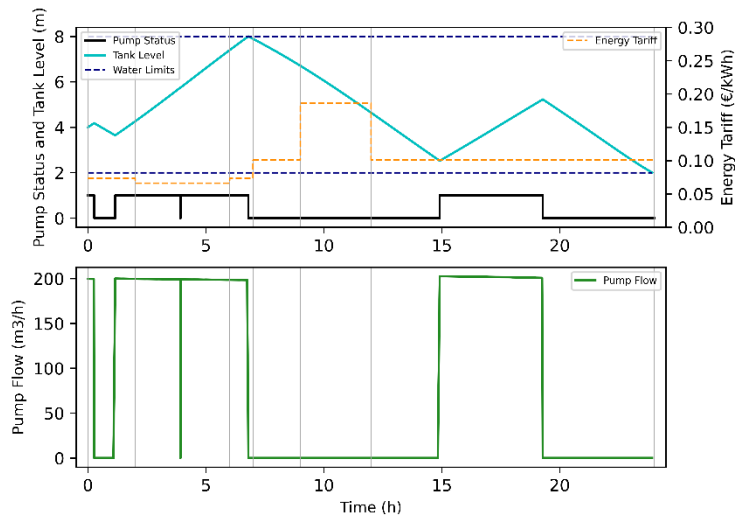


Figure 9. Results of DC-SLSQP approach in case study 1.

#### 4.2 Case Study 2: Discuss of Results

In case study 2, the water inlet to the tank was altered, and the consumption at the pump outlet was eliminated. As a consequence, the water level and the consumption point no longer impact the quantity of water pumped. As a result, the pump will always pump the same amount of water whenever it is activated, and the most cost-effective solution will depend solely on pumping at the most economical energy tariffs.

The solutions obtained are shown in Table 2 and Figures 10, 11, and 12. Upon analyzing the figures and the results presented in Table 2, it is evident that all three approaches arrived at a similar final solution. This outcome was anticipated as the economical solution solely depends on the cost of energy tariffs. Thus, the advantage of the DC-SLSQP approach in terms of the freedom to choose the initial point of operation does not produce any significant differences in the final cost. Therefore, the RC-SLSQP approach is more advantageous for water supply systems with these types of physical properties/design as it reaches similar final solution with the lowest processing time.

It is also important to mention that the B-GA approach's solution is slightly higher because it is a binary approach with time-horizons of 30 minutes. This means that when the pump is activated, it must operate at least 30 minutes. As a result, the total amount of water pumped is slightly higher compared to the other approaches, which do not have this time constraint. Regarding the number of pump starts, the conclusions are the same as in case study 1.

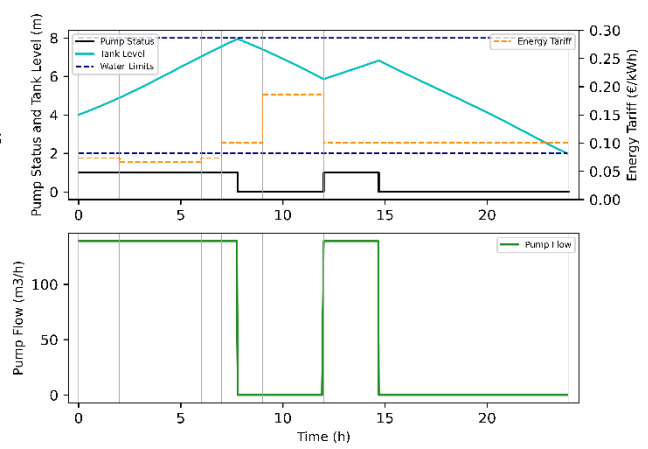
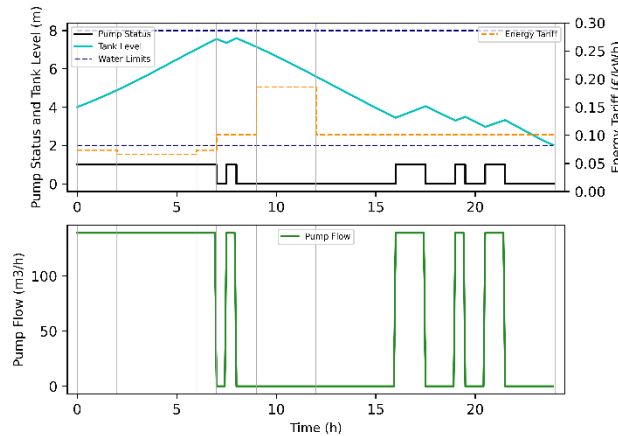


Figure 10. Results of B-GA approach in case study 2. Figure 11. Results of RC-SLSQP approach in case study 2.

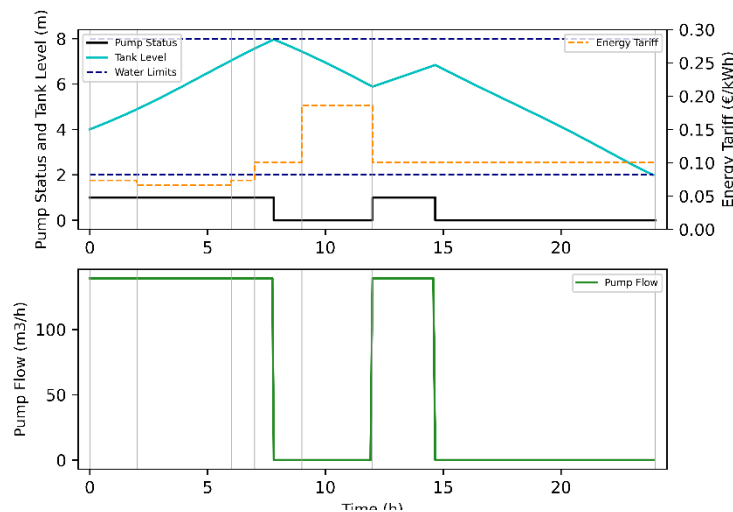


Figure 12. Results of DC-SLSQP approach in case study 2.

## 5. Conclusions and Future Work

Optimization applications in water supply systems are several, but it is critical to emphasize the optimization of pump operations, as it is an area where energy consumption is significant. With increasing water consumption, energy consumption is also increasing. Therefore, optimizing pumping station energy consumption is essential for economic and environmental sustainability. Reducing energy consumption in pump operations makes it possible to lower operational costs, reduce greenhouse gas emissions, and increase the overall efficiency of WSS. As such, optimization of pump operation is crucial to achieving a sustainable and efficient water supply system.

This study compared three optimization approaches - binary formulation with a genetic algorithm, continuous formulation with SLSQP, and duty-cycles formulation with SLSQP. The results indicate that for water supply systems where the pump operation point (and, by consequence, the pumped flow) is affected by its physical characteristics, i.e. water tank level and consumption points, the duty-cycles formulation with SLSQP (DC-SLSQP) is more effective at obtaining economical solutions due to its increased operational freedom. However, it is essential to pay careful attention to the initial solution provided to the optimization model as the final solution can be quite sensitive to it. On

the other hand, in water supply systems where the pumped flow is not dependent on any physical aspect of the system, such as in case study 2, the advantage of using the DC-SLSQP approach mentioned above no longer applies. In this case, the RC-SLSQP approach may be more advantageous since it can yield similar results to the DC-SLSQP approach but with lower processing times.

As for future work, it is necessary to compare the performance of these optimization approaches in more complex case studies (with variable speed pumps) and compare them to approaches with other formulations, not only explicit but also implicit ones. Moreover, considering the results of duty-cycles formulation, testing a resolution methodology involving several initial solutions is advisable. This will ensure the best operation of the formulation and provide a more comprehensive analysis of the problem. By testing multiple initial solutions, it is possible to identify the most appropriate solution that provides better results while considering the sensitivity of the duty-cycle formulation.

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