Reliability analysis of a standby system with two stage Erlangian repair

Venkata S.S. Yadavalli
Department of Industrial & Systems Engineering, University of Pretoria, 0002, Pretoria, South Africa
Sarma.yadavalli@up.ac.za

Shagufta Abbas
Department of Industrial & Systems Engineering, University of Pretoria, 0002, Pretoria, South Africa
Abbas.mujahid@gmail.com

V.S. Vaidyanathan
Department of Statistics, Pondicherry University,
Puducherry – 605 014, India
Vaidya.stats@gmail.com

P. Chandrasekhar
Department of Statistics, Loyola College,
Chennai – 600 034, India.
drchandrasekharin@yahoo.co.in

Abstract

By making a detailed study of a two unit cold standby system with constant failure rate and two stage Erlangian repair time distribution, measures of system performance and its statistical inferential aspects are discussed through classical and Bayesian approaches.

Keywords

Bayes estimator, confidence limits, Slustky theorem, standby system, steady state availability.

1. Introduction

Reliability theory is concerned with statistical description of a system and has been studied in detail using the failure time and repair time density functions. The failures and repairs in any system are influenced by several factors such as system configuration, the environmental conditions under which the system operates and the varying failures (minor and major) and so on, which cannot be controlled or assessed well in advance. For a detailed study of systems operating in random environments, see Chandrasekhar and Natarjan (2001) and Chandrasekhar et al. (2005). In real life situations with problems involving system configurations, it is essential to carry out an analysis of measures of system performance. These problems often require the applications of statistical tools such as point estimation, interval estimation, hypotheses testing and Bayesian inference. Most of the times, it is possible that some statistical information pertaining to the parameters of both lifetime and repair time distributions is available. In such a scenario, Bayesian approach provides statistical methodology to incorporate the prior information with the data at hand. Analysis of systems using the above mentioned statistical tools is scarce in literature.

In recent times, there has been great interest in analyzing the system from a Bayesian perspective. However, all the Bayesian research work till date has been on constant failure and service rates. In this paper, we study in detail a two unit cold standby system with constant failure rate λ and constant repair rate μ (both unknown) and two repair stages. Several authors have studied extensively two unit standby redundant systems in the past. Osaki and Nakagawa (1976) give a bibliography of the work on two unit systems. Our interest in this paper is on statistical inference procedures of a standby system with two stage Erlangian repair. The choice of the Erlangian distribution is motivated by the fact that an Erlangian variate with shape parameter k is the sum of k independent and identically distributed (iid) exponential variates. Hence, an Erlangian repair model can be thought of as a model with repair in k exponential phases, where repair at each phase is exponential with rate μ.

In our model, we perform a simple experiment by observing m lifetimes and n repair times. Given this experiment, the likelihood is of the form

$$L(\text{parameters}|\text{data})=\lambda^m e^{-\lambda u}\left(\frac{\mu^{2n}}{\Gamma(2)^n}e^{-\mu v}\prod_{j=1}^{n} y_j\right)$$
$$=\mu^{2n}e^{-(\lambda u+\mu v)}\lambda^m\prod_{j=1}^{n} y_j \qquad (1.1)$$

where u and v are the sums of m observed lifetimes and n repair times respectively. For the system under consideration, in the subsequent sections, we have described maximum likelihood and Bayesian procedures. Flexible priors for lifetime and repair time parameters are introduced under the assumption that priors for life time and repair time parameters are independent. By using these conjugate prior distributions, we evaluate the posterior distributions along with Bayes estimators. The model and the assumptions, expressions for system reliability, MTBF, availability and associated statistical inference together with numerical illustration are discussed in detail in the following sections.

2. Model (Two unit cold standby system with a single repair facility)

2.1 Assumptions

The assumptions of the model are as follows:

(i) The system has two statistically independent and identical units each with constant failure rate say λ and one perfect repair facility.

(ii) A standby unit will not fail.

(iii) The repair time distribution is a two stage Erlangian with probability density function (pdf) given by

$$g(t)=\mu^2 e^{-\mu t}t,\ 0<t<\infty;\mu>0 \qquad (2.1)$$

(iv) Once a unit is repaired, it is as good as new.

(v) There is a perfect switch with negligible switchover time.

2.2 Analysis of the system

The state of the system is described by discrete valued stochastic process $\{X(t), t\in[0,\infty)\}$, where $X(t)$ denotes the number of units failed at time t. It may be noted that the stochastic process $\{X(t)\}$ is a Markov Process (since a two stage Erlangian variate is the sum of two iid exponential variates and exponential distribution satisfies lack of memory property) on $\{0,1,2\}$. We note that at any given instant of time t, the system is found in any of the following mutually exclusive and exhaustive states S_i, $i=0,1,\ldots,4$. Here S_0 corresponds to the situation, wherein both the units are operable but only one unit is operating online and the other unit is kept in cold standby. S_1 (S_2) represents the state of the system in which one unit is operating online and the other unit is in the first (second) stage of repair. It is clear that the states S_0, S_1 and S_2 are the system upstates. Similarly, S_3 (S_4) denotes the situation that one unit is in the first (second) stage of repair and the other unit is waiting for repair and are the system downstates. Clearly the Markov process $\{X(t)\}$ has the infinitesimal generator given by

$$Q = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{pmatrix} -\lambda & \lambda & 0 & 0 & 0 \\ 0 & -(\lambda+\mu) & \mu & \lambda & 0 \\ \mu & 0 & -(\lambda+\mu) & 0 & \lambda \\ 0 & 0 & 0 & -\mu & \mu \\ 0 & \mu & 0 & 0 & -\mu \end{pmatrix} \end{array} \quad (2.2)$$

Let $p_i(t)$ be the probability that the system is in state S_i , $i=0,1,\ldots,4$ at time t with the initial condition $p_0(0)=1$. Initially, we assume that both the units are operable and obtain the system performance measures.

2.2.1 System reliability

The system reliability $R(t)$ is the probability that the system does not fail upto time t. To derive an expression for system reliability, it is necessary to study the transitions of the Markov process $\{X(t)\}$ into the states S_0, S_1 and S_2 without passing through S_3 and S_4. The differential – difference equations corresponding to these upstates are given below which are obtained using the infinitesimal generator of the process given in (2.2).

$$\frac{dp_0(t)}{dt} = -\lambda p_0(t)+\mu p_2(t) \quad (2.3)$$

$$\frac{dp_1(t)}{dt} = \lambda p_0(t)-(\lambda+\mu)p_1(t) \quad (2.4)$$

$$\frac{dp_2(t)}{dt} = \mu p_1(t)-(\lambda+\mu)p_2(t) \quad (2.5)$$

The equations (2.3) – (2.5) are solved by using Laplace transformation. Suppose $L_i(s)$ represents the Laplace transform of $p_i(t), i=0,1,2$. Taking Laplace transform, solving and inverting, we get the solution for $p_i(t), i=0,1,2$ as follows:

$$p_0(t) = \sum_{i=1}^{3} \frac{(\alpha_i+\lambda+\mu)^2}{\prod_{j=1,j\neq i}^{3}(\alpha_i-\alpha_j)} e^{\alpha_i t} \quad (2.6)$$

$$p_1(t) = \lambda \sum_{i=1}^{3} \frac{(\alpha_i+\lambda+\mu)}{\prod_{j=1,j\neq i}^{3}(\alpha_i-\alpha_j)} e^{\alpha_i t} \quad (2.7)$$

$$p_2(t) = \lambda\mu \sum_{i=1}^{3} \frac{1}{\prod_{j=1,j\neq i}^{3}(\alpha_i-\alpha_j)} e^{\alpha_i t} \quad (2.8)$$

Adding (2.6), (2.7) and (2.8), we obtain the system reliability as

$$R(t) = \sum_{i=1}^{3} \frac{[(\alpha_i+\lambda+\mu)^2 + \lambda(\alpha_i+\lambda+2\mu)]}{\prod_{j=1,j\neq i}^{3}(\alpha_i-\alpha_j)} e^{\alpha_i t} \quad (2.9)$$

where $\alpha_i, i=1,2,3$ are the roots of the equation
$s^3+(3\lambda+2\mu)s^2+\left(3\lambda^2+4\lambda\mu+\mu^2\right)s+\lambda^2(\lambda+2\mu)=0$.

2.2.2 Mean time before failure

The system MTBF is the expected or average time to failure and is given by
MTBF= $L_0(0)+L_1(0)+L_2(0)$

$$= \frac{(2\lambda^2+4\lambda\mu+\mu^2)}{\lambda^2(\lambda+2\mu)} \quad (2.10)$$

2.2.3 System availability

The system availability $A(t)$ is the probability that the system is in operable condition at any arbitrary point of time t. To obtain the availability function, we have to study the transitions of the Markov process $\{X(t)\}$ into the states S_i , $i=0,1,\ldots,4$. Using the infinitesimal generator given in (2.2), we get the following system of differential – difference equations.

$$\frac{dp_0(t)}{dt} = -\lambda p_0(t) + \mu p_2(t) \tag{2.11}$$

$$\frac{dp_1(t)}{dt} = \lambda p_0(t) - (\lambda+\mu) p_1(t) + \mu p_4(t) \tag{2.12}$$

$$\frac{dp_2(t)}{dt} = \mu p_1(t) - (\lambda+\mu) p_2(t) \tag{2.13}$$

$$\frac{dp_3(t)}{dt} = \lambda p_1(t) - \mu p_3(t) \tag{2.14}$$

$$\frac{dp_4(t)}{dt} = \lambda p_2(t) + \mu p_3(t) - \mu p_4(t) \tag{2.15}$$

Solving (2.11) – (2.15) with the condition $\sum_{i=0}^{4} p_i(t) = 1$, we obtain the solution as follows.

$$p_0(t) = \frac{\mu^4}{\prod_{i=1}^{4}\alpha_i} + \lambda\mu^2 \sum_{i=1}^{4} \frac{(\alpha_i+\mu)^2}{\alpha_i(\alpha_i+\lambda)\prod_{j=1,j\neq i}^{4}(\alpha_i-\alpha_j)} e^{\alpha_i t} \tag{2.16}$$

$$p_1(t) = \frac{\lambda\mu^2(\lambda+\mu)}{\prod_{i=1}^{4}\alpha_i} + \lambda \sum_{i=1}^{4} \frac{(\alpha_i+\mu)^2(\alpha_i+\lambda+\mu)}{\alpha_i\prod_{j=1,j\neq i}^{4}(\alpha_i-\alpha_j)} e^{\alpha_i t} \tag{2.17}$$

$$p_2(t) = \frac{\lambda\mu^3}{\prod_{i=1}^{4}\alpha_i} + \lambda\mu \sum_{i=1}^{4} \frac{(\alpha_i+\mu)^2}{\alpha_i\prod_{j=1,j\neq i}^{4}(\alpha_i-\alpha_j)} e^{\alpha_i t} \tag{2.18}$$

$$p_3(t) = \frac{\lambda^2\mu(\lambda+\mu)}{\prod_{i=1}^{4}\alpha_i} + \lambda^2 \sum_{i=1}^{4} \frac{(\alpha_i+\mu)(\alpha_i+\lambda+\mu)}{\alpha_i\prod_{j=1,j\neq i}^{4}(\alpha_i-\alpha_j)} e^{\alpha_i t} \tag{2.19}$$

$$p_4(t) = \frac{\lambda^2\mu(\lambda+2\mu)}{\prod_{i=1}^{4}\alpha_i} + \lambda^2\mu \sum_{i=1}^{4} \frac{[\lambda + 2(\alpha_i+\mu)]}{\prod_{j=1,j\neq i}^{4}(\alpha_i-\alpha_j)} e^{\alpha_i t} \tag{2.20}$$

where $\alpha_i, i=1,2,3,4$ are the roots of the equation
$s^4+(3\lambda+4\mu)s^3+(3\lambda^2+10\lambda\mu+6\mu^2)s^2+(\lambda^3+8\lambda^2\mu+9\lambda\mu^2+4\mu^3)s+\mu(2\lambda^3+4\lambda^2\mu+2\lambda\mu^2+\mu^3)=0.$
Hence, the system availability is obtained by adding (2.16), (2.17) and (2.18) and is given by

$$A(t) = \frac{\mu^2(\lambda+\mu)^2}{\prod_{i=1}^{4}\alpha_i} + \lambda\mu^2 \sum_{i=1}^{4} \frac{(\alpha_i+\mu)^2}{\alpha_i(\alpha_i+\lambda)\prod_{j=1,j\neq i}^{4}(\alpha_i-\alpha_j)} e^{\alpha_i t} + \lambda \sum_{i=1}^{4} \frac{(\alpha_i+\mu)^2(\alpha_i+\lambda+\mu)}{\alpha_i\prod_{j=1,j\neq i}^{4}(\alpha_i-\alpha_j)} e^{\alpha_i t}$$

$$+ \lambda\mu \sum_{i=1}^{4} \frac{(\alpha_i+\mu)^2}{\alpha_i\prod_{j=1,j\neq i}^{4}(\alpha_i-\alpha_j)} e^{\alpha_i t} \tag{2.21}$$

Allowing $t\to\infty$ on both the sides of (2.21), the system steady state availability is obtained as

$$A_\infty = \frac{\mu(\lambda+\mu)^2}{(2\lambda^3+4\lambda^2\mu+2\lambda\mu^2+\mu^3)}, \tag{2.22}$$

which is in agreement with Chandrasekhar and Natarajan (1994).

2.3 ML estimator of system reliability

Let $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_n$ be random observations on exponential failure times and Erlangian repair times with the pdf given by (2.1). It is well known that $\bar{X}$ and $\frac{\bar{Y}}{2}$ are the ML estimators of $\frac{1}{\lambda}$ and $\frac{1}{\mu}$ respectively, where $\bar{X}$ and $\bar{Y}$ are the corresponding sample means. Thus, the ML estimator of $R(t)$ is given by

$$\hat{R}(t) = \sum_{i=1}^{3} \frac{\lfloor (2\bar{X}+\bar{Y}+\hat{\alpha}_i\bar{X}\bar{Y})^2 + \bar{Y}(4\bar{X}+\bar{Y}+\hat{\alpha}_i\bar{X}\bar{Y}) \rfloor}{(\bar{X}\bar{Y})^2\prod_{j=1,j\neq i}^{3}(\alpha_i-\alpha_j)} e^{\hat{\alpha}_i t} \tag{2.23}$$

where $\hat{\alpha}_i$, i=1,2,3 are the roots of the cubic equation

$$\bar{X}^3\bar{Y}^2s^3+\bar{X}^2\bar{Y}(4\bar{X}+3\bar{Y})s^2+\bar{X}(2\bar{X}+\bar{Y})(2\bar{X}+3\bar{Y})s+\bar{Y}(4\bar{X}+\bar{Y})=0 \quad (2.24)$$

2.4 Confidence limits for A_∞

In section 2.3, we have seen that $\bar{X}$ and $\frac{\bar{Y}}{2}$ are the ML estimators of $\frac{1}{\lambda}$ and $\frac{1}{\mu}$ respectively. Let $\theta_1=\frac{1}{\lambda}$ and $\theta_2=\frac{1}{\mu}$. Clearly, A_∞ given in (2.22) is simplified to

$$A_\infty=\frac{\theta_1(\theta_1+\theta_2)^2}{(\theta_1^3+2\theta_1^2\theta_2+4\theta_1\theta_2^2+2\theta_2^3)} \quad (2.25)$$

and hence ML estimator of A_∞ is given by

$$\hat{A}_\infty=\frac{\bar{X}(2\bar{X}+\bar{Y})^2}{(4\bar{X}^3+4\bar{X}^2\bar{Y}+4\bar{X}\bar{Y}^2+\bar{Y}^3)} \quad (2.26)$$

By the asymptotic property of ML estimators, it is clear that

$$\sqrt{n}[\hat{A}_\infty\text{-}A_\infty]\xrightarrow{d}N(0,\sigma^2(\boldsymbol{\theta})) \text{ as } n\to\infty,$$

where $\boldsymbol{\theta}=(\theta_1,\theta_2)$ and

$$\sigma^2(\boldsymbol{\theta})=\theta_1^2\left(\frac{\partial A_\infty}{\partial\theta_1}\right)^2+\frac{\theta_2^2}{2}\left(\frac{\partial A_\infty}{\partial\theta_2}\right)^2 \quad (2.27)$$

The partial derivatives $\left(\frac{\partial A_\infty}{\partial\theta_i}\right)$, i=1,2 are given by

$$\left(\frac{\partial A_\infty}{\partial\theta_1}\right)=\frac{2\theta_2^2(3\theta_1^3+6\theta_1^2\theta_2+4\theta_1\theta_2^2+\theta_2^3)}{(\theta_1^3+2\theta_1^2\theta_2+4\theta_1\theta_2^2+2\theta_2^3)^2} \quad (2.28)$$

$$\left(\frac{\partial A_\infty}{\partial\theta_2}\right)=\frac{-2\theta_1\theta_2(3\theta_1^3+6\theta_1^2\theta_2+4\theta_1\theta_2^2+\theta_2^3)}{(\theta_1^3+2\theta_1^2\theta_2+4\theta_1\theta_2^2+2\theta_2^3)^2} \quad (2.29)$$

Substituting (2.28) and (2.29) in (2.27) and simplifying, we get

$$\sigma^2(\theta)=\frac{6\theta_1^2\theta_2^4(3\theta_1^3+6\theta_1^2\theta_2+4\theta_1\theta_2^2+\theta_2^3)^2}{(\theta_1^3+2\theta_1^2\theta_2+4\theta_1\theta_2^2+2\theta_2^3)^4} \quad (2.30)$$

Thus, $\hat{A}_\infty$ is a CAN estimator of A_∞. One can also use the method of moments to generate CAN estimator of A_∞, see Sinha (1986).

Using Slutsky theorem and a property of consistent estimator, it can be shown that the confidence limits for the steady state availability of the system are given by $\hat{A}_\infty \pm k_{\frac{\alpha}{2}}\frac{\hat{\sigma}}{\sqrt{n}}$,where $k_{\frac{\alpha}{2}}$ is the upper $100\left(1-\frac{\alpha}{2}\right)\%$ quantile of standard normal distribution and $\hat{\sigma}$ is obtained from (2.30) and is given by

$$\hat{\sigma}=\sqrt{\frac{3\bar{X}^2\bar{Y}^4(24\bar{X}^3+24\bar{X}^2\bar{Y}+8\bar{X}\bar{Y}^2+\bar{Y}^3)^2}{2(4\bar{X}^3+4\bar{X}^2\bar{Y}+4\bar{X}\bar{Y}^2+\bar{Y}^3)^4}} \quad (2.31)$$

In the next section, Bayes estimator of MTBF under squared error loss function is obtained.

3. Bayes estimation of MTBF in a two unit cold standby system

In this section, we derive the Bayes estimator of MTBF by considering Gamma distributions with parameters (α,β) and (δ,ω) as natural conjugate priors for the lifetimes and repair times respectively. In other words, λ and μ have the following prior distributions with the probability density functions as follows.

$$\tau_1(\lambda|\alpha,\beta)=\frac{\alpha^\beta}{\Gamma(\beta)}e^{-\alpha\lambda}\lambda^{\beta-1},\ 0<\lambda<\infty;\ \alpha,\ \beta>0 \quad (3.1)$$

$$\tau_2(\mu|\delta,\omega)=\frac{\delta^\omega}{\Gamma(\omega)}e^{-\delta\mu}\mu^{\omega-1},\ 0<\mu<\infty;\ \delta,\ \omega>0 \quad (3.2)$$

It can be shown that the posterior distributions of λ and μ given the sample observations $X_1, X_2,\ldots,X_m$ and $Y_1, Y_2,\ldots,Y_n$ are respectively given by

$$q_1(\lambda|x_1, x_2, \ldots, x_m) = \frac{(\alpha+u)^{m+\beta}}{\Gamma(m+\beta)} e^{-\lambda(\alpha+u)} \lambda^{(m+\beta)-1},\ 0<\lambda<\infty;\ \alpha, u, m, \beta>0 \quad (3.3)$$

$$q_2(\mu|y_1, y_2, \ldots, y_n) = \frac{(\delta+v)^{2n+\omega}}{\Gamma(2n+\omega)} e^{-\mu(\delta+v)} \mu^{(2n+\omega)-1},\ 0<\mu<\infty;\ \delta, v, n, \omega>0 \quad (3.4)$$

In other words, λ and μ are distributed as Gamma with parameters $(\alpha+u, m+\beta)$ and $(\delta+v, 2n+\omega)$ respectively.

Bayes estimator of MTBF say MTBF*, given the sample observations is defined as

MTBF* = E[MTBF | sample observations]

$$= \int_0^\infty \int_0^\infty \frac{(2\lambda^2+4\lambda\mu+\mu^2)}{\lambda^2(\lambda+2\mu)} q_1(\lambda|x_1, x_2, \ldots, x_m) q_2(\mu|y_1, y_2, \ldots, y_n)\, d\lambda\, d\mu \quad (3.5)$$

$$= \int_0^\infty \int_0^\infty \left(\frac{1}{\mu}+\frac{2}{\lambda}+\frac{\mu}{2\lambda^2}\right) \sum_{j=0}^{\infty} \frac{(-1)^j}{2^j} \left(\frac{\lambda}{\mu}\right)^j q_1(\lambda|x_1, x_2, \ldots, x_m) q_2(\mu|y_1, y_2, \ldots, y_n)\, d\lambda\, d\mu,\ \lambda<\mu$$

$$= \frac{1}{\Gamma(m+\beta)\Gamma(2n+\omega)} \left[\sum_{j=0}^{\infty} \frac{(-1)^j}{2^j} \frac{(\delta+v)^{j+1}}{(\alpha+u)^j} \Gamma(m+\beta+j)\Gamma(2n+\omega-j-1) + 2\sum_{j=0}^{\infty} \frac{(-1)^j}{2^j} \frac{(\delta+v)^j}{(\alpha+u)^{j-1}} \Gamma(m+\beta+j-1)\Gamma(2n+\omega-j) + \frac{1}{2}\sum_{j=0}^{\infty} \frac{(-1)^j}{2^j} \frac{(\delta+v)^{j-1}}{(\alpha+u)^{j-2}} \Gamma(m+\beta+j-2)\Gamma(2n+\omega-j+1) \right] \quad (3.6)$$

4. Numerical Illustration

The performance of the Bayes estimate of MTBF i.e., MTBF* is illustrated in this section through simulated data. The estimates are obtained using (3.5). Monte Carlo integration method is used to evaluate the integrals in (3.5) in two steps. First, the inner integral is evaluated by generating random observations using the posterior density of λ treating μ as unknown. The outer integral is then evaluated using random observations generated from the posterior density of μ. The values of hyper parameters in the posterior density functions are fixed as $m=n=50$; $\alpha=2.5$; $\beta=3.0$; $\delta=0.75$; $\omega=1.5$. u and v are determined by taking the sums of iid samples of sizes m and n generated respectively from exponential distribution and Erlangian distribution with pdf given in (2.1). For generating samples, the following choices of λ and μ namely, $\lambda=3, 6, 9, 12$ and $\mu=2, 4, 6, 8$ are used. The results of the simulation based on 10,000 Monte Carlo runs are presented below.

Table 1: Bayes estimate of MTBF

μ \ λ	3.0	6.0	9.0	12.0
2.0	0.01662	0.03390	0.07491	0.11261
4.0	0.02634	0.05689	0.14142	0.15921
6.0	0.02723	0.11658	0.16159	0.26393
8.0	0.03311	0.14515	0.26254	0.39131

From the above table, it can be observed that for fixed repair rate (μ), the Bayes estimate of MTBF increases as the failure rate (λ) increases. Similarly, for fixed λ, the Bayes estimate of MTBF increases as μ increases. In other words, whenever the two unit cold standby system with single repair facility under consideration exhibits high failure and repair rates, the estimated mean time before failure is also high.

5. Conclusion

An attempt is made in this paper to study in detail a two unit cold standby system with a single repair facility. Mathematical expressions for $R(t)$, MTBF, $A(t)$ and A_∞ are obtained. Further, asymptotic confidence limits for A_∞, ML estimator of $R(t)$ and Bayes estimator of MTBF are obtained. Also the performance of the Bayes estimator of MTBF is illustrated through simulation study.

6. References

Chandrasekhar, P. and Natarajan, R. Confidence limits for steady state availability of a two unit standby system, *Microelectronics and Reliability*, Vol. 34, No. 7, pp. 1249 – 1251,1994

Chandrasekhar, P. and Natarajan, R. A system operating in a random environment, *Statistical Methods*, Vol.3, No. 2, pp. 98 – 107, 2001

Chandrasekhar, P., Natarajan, R., Yadavalli, V.S.S. and Malada, A. An n unit system operating in a random environment, *OPSEARCH*, Vol. 42, No.3, pp. 280 – 287, 2005

Osaki, S. and Nakagawa, T. Bibliography for reliability and availability of stochastic systems, *IEEE Trans. on Reliability.*, Vol.R-25, pp. 284 – 287, 1976

Sinha, S.K. , Reliability and Life testing, *Wiley Eastern Pvt. Ltd*., New Delhi, 1976.

Biography

Dr VSS Yadavalli is a Professor and Head of Department of Industrial & Systems Engineering, University of Pretoria. Professor Yadavalli has published over 150 research paper mainly in the areas of Reliability and inventory modeling in various international journals like, International Journal of Production Economics, International Journal of Production Research, International Journal of Systems Science, IEEE Transactions on Reliability, Stochastic Analysis & Applications, Applied Mathematics & Computation, Annals of Operations Research, Computers & Industrial Engineering etc. Prof Yadavalli is an NRF (South Africa) rated scientist and attracted several research projects. Professor Yadavalli was the past President of the Operations Research Society of South Africa. He is a `Fellow' of the South African Statistical Association. He received a `Distinguished Educator Award 'from Industrial Engineering and Operations Management Society in 2015.

Dr Shagufta Abbas is currently working as an assistant professor in department of Mathematics in Govt Degree College for Women, Lahore. She completed her PhD in 2016 from Department of Industrial and Systems Engineering , University of Pretoria, South Africa. Her areas of interest include distribution theory, reliability theory and mathematical analysis.

V.S.Vaidyanathan obtained M.Sc. degree in Statistics in the year 1995 and M.Phil. degree in Statistics in the year 1999 from University of Madras, India. He was awarded Ph.D. degree in Statistics in the year 2011 from University of Madras, India for his thesis "Contributions to Clustering and Classification algorithms". He joined the Department of Statistics, Loyola College, Chennai, India in the year 2000 as Lecturer in Statistics and later got appointed in Pondicherry University, India in the year 2010. Currently, he serves as Assistant Professor in the Department of Statistics, Pondicherry University, India. His research interests include Statistical Inference for queueing and stochastic models, mixture models and distribution theory

Dr. P. Chandrasekhar has obtained his M. Sc (Statistics) and M. Phil (Statistics) degrees from the University of Madras in 1974 and 1984 respectively. Further, he has received his Ph. D degree in 1996 from the same University. His doctoral thesis is entitled "Stochastic models of redundant systems operating in random environments". His biographical profile was included in Marquis who's who in the world in 16th edition. He was the faculty member in the department of Statistics, Loyola College, Chennai, India from 1978 to 2010. He was the University Grants Commission - Emeritus Fellow from 2011 to 2013. He has nearly 50 research publications - both National and International. His areas of research are stochastic modelling in Reliability Theory and Statistical Inference for Queueing Models.