Time-dependent traffic route selection for unsignalized junctions in tandem queueing network

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Abstract

The study proposed a methodology based on queueing theory for travel time assessment over multiple routes with unsignalized traffic junction in tandem network. The work carried out in the study gives an approach for best route selection between two points based on minimum traveling time. The queuing theories used in the study took the M/M/1 model over the unsignalized junction and evaluated the traffic performance on different mean arrival rates of the vehicles. The proposed methodology is analyzed in a pilot study with observed data of mean service rates for each of the junctions for all route choices. The study can be used as a decision-making tool for formulation of route selecting strategies based on minimum traveling time.

Keywords
Queueing theory, traffic route selection, travel time minimization, tandem queueing network, unsignalized road junctions

1. Introduction

Operations research involves a cumulative study of problem-solving techniques, to improve decision-making and efficiency. One of the core concepts of operational research is queueing theory, which is defined as the mathematical study of waiting lines or queues. Queues develop when the demand for service exceeds the limit of service that can be provided (Shortle et al., 2018).
With rising population and constantly growing network of road transport systems, there has been a consequent increase in the number of vehicles plying on roads. In recent time, the growth in the number of vehicles has surpassed the capacity of the road transport networks, leading to frequent traffic congestion and traffic snarls, especially in the major cities of the world. Excessive traffic congestion leads to increased travel time, reduced speed of travel, elevated pollution levels due to vehicular emissions and greater fuel consumption, with the subsequent rise in the cost of travel (Lindsney and Erik, 2001). According to a report published by The Times of India for the year 2017, traffic congestion in the four cities of Delhi, Kolkata, Bengaluru and Mumbai alone cost the economy Rs.1.47 lakh crores (approximately

USD 16.75 billion) annually (Dash, 2018). More time of vehicles on the road comes with various issues some of which include delay in delivery timing, wastage of fuel and resources, rise in the vehicular pollution emissions, and effects to human health due to traffic pollution and psychological effects of traffic chaos. The increase in the time of travel becomes a major factor environmentally, as excess travel time leads to excess vehicular emissions of greenhouse gases such as carbon dioxide and nitrous oxides. The transportation sector is responsible for a huge amount of air pollution, thus, minimizing the degree of vehicular exhausts becomes a primary concern for global environmentalists. (Benson, 1989; Sjodin et al., 1998). Given the severe effects of traffic congestion and increased travel time have on the economy, quality of delivery timing, environment, and social impacts, it becomes paramount to minimize these factors in road transport.

Using queueing theory, the study aims to develop a model for analysis of various route options between two fixed points, by computing the time of travel through various unsignalized junctions, which act as servers in a queueing system. As road networks generally consist of a series of junctions or intersections, the problem statement turns into a tandem queueing network model. Tandem networks consist of a series of servers, providing service in the same direction (Le Gall, 1997). The article uses concepts of queueing on these tandem networks on all possible routes between the two points, to obtain a comparative study of the travel time on all routes and determine the order of preference for these routes to minimize the time of travel. The data is analyzed at different traffic conditions to generate varying results.

2. Literature Review

Queueing theory finds applications in various real-life situations such as, analysis of a waiting line at a bank (Wenyu et al., 2005), or investigating the bottleneck for in-patient flow at a cardiac hospital (De Bruin et al., 2007) or even studying equilibrium conditions in servicing industries (Davidson, 1988). In traffic and transportation sector, queueing theory is applied at both signalized and unsignalized junctions, for determination of waiting time or delay time faced by a vehicle. (Newell, 2013). Anokye et al. (2013) studied the traffic intensities at three different routes between two points at different time periods of the day to minimize traffic congestion on signalized road intersections using a simple M/M/1 model. The proposed study also deals with multi-route road networks and aims to obtain a preferential order of all routes concerning the time of travel between two points. The study deals with queueing networks having servers in tandem, similar to the work of Haghighi and Mishev (2007), who studied tandem queues to analyze task splitting, feedback, and blocking. The proposed study also takes into account the work of Burke (1956), which states that for a tandem queueing network, if the first server functions as an M/M/1 model, then all servers on the network will also function as M/M/1 models, independently. This aids the proposed study to carry out formulations for each junction-server independently.

Solving traffic congestion problems in major cities has always been of prime importance to researchers as well. Not only does traffic congestion have economic and environmental impacts (Lindsney and Erik, 2001), but it also has psychological impacts on the people involved in the traffic (Hennessy and Wiesenthal, 1999). Minimizing traffic congestion is viewed as a vital task in road network management. As increase in traffic congestion also leads to an increase in travel time (Abdel-Aty et al., 1995; Arnott and Small, 1994), minimizing traffic or choosing the route with least traffic flow, would consequently decrease the time taken for a vehicle to travel, thus decreasing various factors like fuel consumption, engine wear, vehicular emissions and human stress (Barth and Boriboonsomsin, 2008; McKinnon, 1999). Number of studies are available for traffic route selection strategies using different methodologies. (Hellinga and Fu, 1999; Huang, n.d.; Kim et al., 2005; Rapoport et al., 2009; Wolfe et al., 2007; Xu et al., 2012). A number of works are available on traffic route optimization in the signalized junction (Cai et al. 2016; Murat and Baskan, 2006; Rouphail et al., 1992) and unsignalized junctions (Li et al., 2009; Marciano et al., 2010; Prasetijo 2007). But in rigorous literature study, insufficient research has been identified for traffic route selection in the unsignalized junction in tandem network. This study provides an approach using queuing theory to provide a methodology for selecting route choice on minimum travel time in routes of the unsignalized junction in tandem network.

3. Queueing model used for unsignalized junctions in a tandem network

Queueing theory involves the mathematical study of queues or waiting lines formed at servers, or service stations, to compute queue waiting time and queue length (Sundarapandian, 2009). In road traffic networks, queueing occurs at road signals, or unsignalized intersections and junctions. At unsignalized junctions or intersections, the service time is defined as the time taken for a vehicle to cross the intersection (Heidemann, D. and Wegmann, H., 1997). For any queueing system, there are certain variable and constant parameters, such as mean arrival rate and mean service rate,

which has to be pre-defined to carry out further calculations. The relation between these parameters depends on the type of distribution model followed by the queue as well as the service discipline of the queue.
There are various models that can be implemented in queueing, however, the study assumes the model to be an M/M/1 model having a Poisson process of arrival with the service time exponentially distributed. According to Willig (1999), the M/M/1 model is the most commonly used model to analyse queueing problems. Also, a road traffic network satisfies all the conditions for the assumptions made by an M/M/1 model; the number of vehicles in the system are large, each vehicle occupies a small percentage of the road network and decisions made by the vehicle operator are independent of the decisions made by the other vehicles (Anokye et al., 2013).
Service discipline of a queue refers to the order of arrival and departure of customers in the queue. Service discipline of a queue can be First In First Out (FIFO), Last In First Out (LIFO), Random Service or Priority Service to name a few (Willig, 1999). The proposed study considers the traffic queueing to be FIFO, as the vehicle which enters the queue first, is also served first, followed by the one immediately after in and so forth. The study considers that each junction can have multiple service rates and hence multiple servers, depending on the route chosen by the vehicle. For example, if there are two possible route choices for a vehicle from a junction, then each will have a separate service time, independent of the other. In such a case, a two-lane road system is assumed, where one lane is dedicated to one server, which ensures independent queueing at the junction. Figure 1 shows a schematic of the layout of an unsignalized junction following an M/M/1 model, and Figure 2 shows a tandem queueing network with multiple servers at a junction.

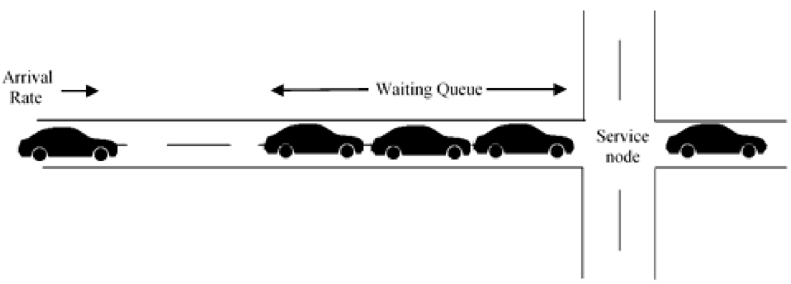

Figure 1. Layout of a single server unsignalized intersection

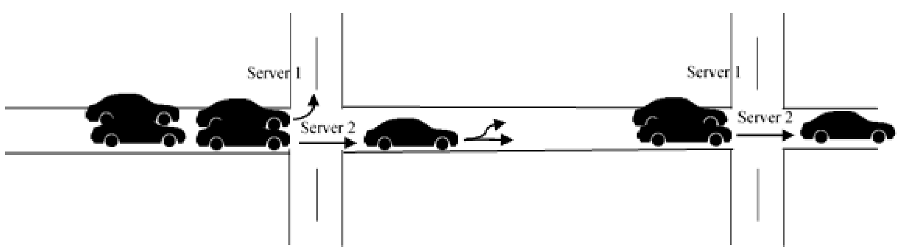

Figure 2. Tandem queueing network with multiple server junction with independent queues

When there are multiple junctions in a road network, present one after the other, all in the same direction of movement, the type of network is called a tandem queueing network. Each junction has a pre-defined service rate which is independent of the arrival rate of vehicles, and if the first junction in the network follows an M/M/1 distribution, then all the junctions on the route will also follow an M/M/1 distribution model (Shortle et al., 2018). Another important condition followed by an M/M/1 tandem queueing network is that if the arrival rate follows a Poisson process of arrival, then the arrival rate at every server of the network follows the same Poisson process of arrival, having equal mean arrival rate (Burke, 1956).
The standard parameters of an M/M/1 model are defined as follows:

- Mean arrival rate (λ): Provides the average number of vehicles that are reaching the junction points in a stipulated amount of time. For a tandem queueing on road network, the mean arrival rate λ is same for all junction points.
- Mean service rate (μ): Represents the average number of vehicles passing through the junction in a specific direction, in unit time.

For tandem queues following an M/M/1, each junction server follows the same distribution and functions independent of the other junctions (Shortle et al., 2018). Hence the parameters defined hereafter are same for all junctions:

1. Traffic intensity (ρ): The ratio of the mean arrival rate to the mean service rate

$$\rho = \lambda/\mu \tag{1}$$

2. Average number of vehicles is the road network system for (N):

$$N = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda} \tag{2}$$

3. Average waiting time in the queue per vehicle (W):

$$W = \frac{\rho}{\mu(1-\rho)} \tag{3}$$

4. Average total time per vehicle to cross the junction (W_t):

$$W_t = W + \text{Service time} + \frac{1}{\mu}$$
$$= \frac{\rho}{\mu(1-\rho)} + \frac{1}{\mu}$$
$$W_t = \frac{1}{\mu-\lambda} \tag{4}$$

An important condition of an M/M/1 queue is that the mean arrival rate (λ) at a junction should always be less than the mean service rate (μ) of the junction to prevent the unbound growth of the queue (Berry, 2006), resulting in a traffic jam.

4. Problem description and methodology

The study aims to calculate travel time on all possible routes α between two points X and Y, using tandem queueing networks. The total travel time on any route is defined as the sum of the waiting time in the queues at every junction on the route α, and the free travel time, defined as the time of travel by a vehicle when not in a queue. The cumulative sum of the time taken to travel from one junction to another, at all junctions on the route α, gives the total time taken to travel on the route.

The proposed study defines a road network between X and Y having the following characteristics. If there are 'n' number of paths originating from X and ending at Y, they are denoted as P, where P ϵ {P_1, P_2…P_n}. Further, if a path P has 'j' number of junctions out of which 'm' number are interconnected with the other paths through by-roads, then the junctions are denoted as P_i, where i = 1, 2, 3…j. As each by-road has to lead to another path, the number of interconnecting junctions 'm' will be equal on all paths P. The remaining (i-m) junctions lead to paths which do not meet X or Y or the other paths, hence, are not considered as interconnecting junctions.

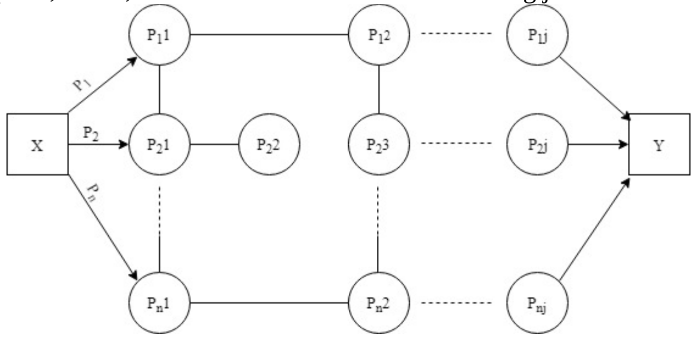

Figure 3. Road network system with interconnected by-roads

To travel from X to Y using various routes α, a vehicle can follow one of the 'n' direct paths (P_1 to P_n), or travel through the interconnecting junctions using the by-roads from one path to another. The study makes an important assumption that while traveling on any path P, a vehicle never travels in a direction opposite to that of X to Y, hence ensuring that the tandem network remains established. Figure 3 shows a road network system with interconnected by-roads. Using mathematical permutations, it is established that the maximum number of possible routes κ that a vehicle can travel by is given as:

$$\kappa = n \,.\, n^{m} = n^{(m+1)} \tag{5}$$

The routes are labeled as α_1, α_2… α_κ. Mean arrival rate for the route α, is denoted by λ_α and is the same at all the junctions along route α. If the junction is an interconnecting junction, then it will have multiple mean service rates or act as multiple servers. As is evident from the network, a junction can act as one, two or maximum three servers,

depending on the number of route choices available. Thus, the mean service rate of a junction needs to be defined by three parameters: originating junction (*p*), queue junction (*q*) and destination junction (*r*) and is thus denoted by μ_{pqr}, where *p*, *q* and *r*, are junctions having their predefined notations. The originating junction *p* is where the vehicle arrives from, the queue junction *q* is where the waiting line actually develops, and the destination junction *r* is the junction towards which the vehicle moves immediately after being served by junction *q*.
Another important assumption that the study follows is that the vehicle operator does not rely on stochastic data to make decisions about route choices. It implies that a vehicle that starts from X, predetermines the route it has to choose and then follows the route by driving in a specific lane. This ensures that there are independent single lane queues being formed at a junction, having vehicles that will travel in a particular direction after crossing the junction under different mean service rates. The assumption made, thus provides avenues for a comparative study between different routes. The traffic intensity at a junction P_i, is given by:

$$\rho_{pqr} = \frac{\lambda_\alpha}{\mu_{pqr}} \tag{6}$$

Throughout the route, the stability condition has to be satisfied at every junction, thus,

$$\max \lambda_\alpha < \mu_{min} \tag{7}$$

where μ_{min} is defined as the limiting mean service rate. Total time of waiting and service and number of vehicles in queue and getting served is defined as:

$$\mathrm{Wt}_{pqr} = \left(\frac{1}{\mu_{pqr}-\lambda_\alpha}\right) \tag{8}$$

$$\mathrm{N}_{pqr} = \left(\frac{\rho_{pqr}}{1-\rho_{pqr}}\right) = \left(\frac{\lambda_\alpha}{\mu_{pqr}-\lambda_\mathrm{P}}\right) \tag{9}$$

If the vehicles are considered to be waiting in the queue bumper to bumper, the length of the queue can be determined as:

$$\mathrm{Q}_{pqr} = \mathrm{N}_{pqr} \,.\, \mathrm{L} \tag{10}$$

Where L is the average length of a vehicle. It is assumed that when the vehicles are not in the queue, they travel at a nominal speed N_s. Hence, the time taken for a vehicle to travel from junction *p* and join the queue at junction *q*, is given as

$$\mathrm{t}_{pqr} = \frac{\mathrm{d}pq - \mathrm{Q}pqr}{\mathrm{N_S}} \tag{11}$$

Where d_{pq} is the distance between the two junctions, *p* and *q*. The total time taken for a vehicle to travel from junction *p*, join the queue at junction *q* and get serviced towards junction *r* is given by:

$$\mathrm{T}_{pqr} = \mathrm{t}_{pqr} + \mathrm{W}_{pqr} \tag{12}$$

Since the end point Y does not act as a server, it does not form a queue. Hence the time taken to travel from the last junction on the route α (α_{last}) to Y is directly the time travelled at nominal speed N_s for a distance $d_{\alpha(last)Y}$, given by:

$$\mathrm{T}_{\alpha(\mathrm{last})\mathrm{Y}} = \mathrm{d}_{\alpha(\mathrm{last})\mathrm{Y}}/\mathrm{N_s} \tag{13}$$

The total time of travel on route α, T_α is thus calculated by summation of the total time of travel (T) between all junctions on route α, and the time of travel $T_{\alpha(last)Y}$. As the formulation suggests, the total travel time on a route will depend on a number of factors such as the mean service rates of junctions on that route, the distance of travel on the route, the nominal speed of travel and the mean arrival rate of vehicles. Using the methodology described, an experimental study carried out, analyze real-time data to obtain results for all possible routes between two points X and Y in the pilot study.

5. Data collection for experimental study

The study considers the data for a road network starting from Manduadih Railway Station (X) to Varanasi Junction Railway Station (Y), both present in the city of Varanasi, in the Uttar Pradesh state of India. To travel from Manduadih to Varanasi Junction, there are two direct paths available. The first path is defined as P1 = A, and the second path is defined as P2 = B. Path A has 4 junctions designated A1, A2, A3 and A4 of which A1, A2 and A3 are interconnecting junctions. Path B has 5 junctions, B1, B2, B3, B4 and B5 of which, B1, B2 and B4 are interconnecting junctions. A1 is the junction that connects to B1, A2 connects to B2, and A3 connects to B4. Figure 4 shows the Google Map view of the layout of the junctions and paths A and B. Figure 5 is a detailed layout of the junctions and the road network with their designations and distances.

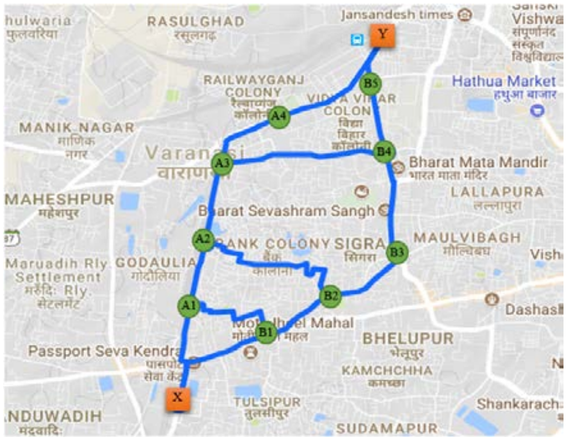

Figure 4. Google Map view of the road network

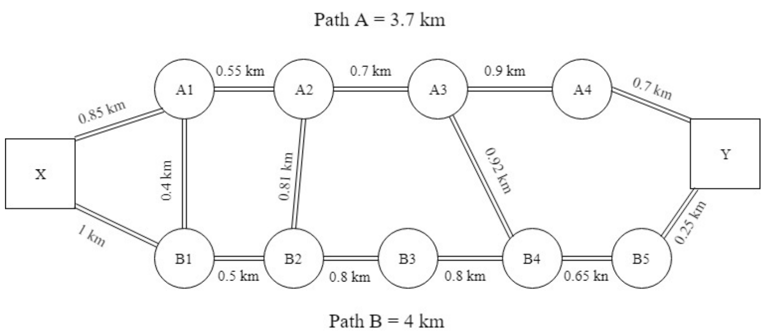

Figure 5. Schematic layout of road network

The mean service rate for each junction was calculated by observation, manually. The observer collected the data using measurement at a point method, by standing at a point of interest and noting the time taken by a vehicle to cross the junction (Mathew, 2010). The data was collected over a period of 12 days from 28th of May, 2018 to 8th of June, 2018. For each service rate at each junction, 10 readings each were taken between 9 am to 11 am, 2 pm to 4 pm and 7 pm to 9 pm on the same day. The 30 observations collected were averaged and noted as the mean service time of the junction. For multi-server junctions, the observational process was carried out separately for each server. The mean service rates of the junctions were calculated by taking the reciprocal of the mean service time and rounded up to the nearest whole numbers since they represent a number quantity. Table 1 provides the mean service times and mean service rates of all junctions. The mean service rates are provided in vehicles per hour, which gives the average number of vehicles passing through the junction in one hour.

Table 1. Mean service time and mean service rates of all junctions in the road network

Originating Junction (*p*)	**Queue Junction (*q*)**	**Destination Junction (*r*)**	**Observed Mean Service Time (sec)**	**Mean Service Rate μ_{pqr} (veh/s)**	**Mean Service Rate μ_{pqr} (veh/hr)**	**Mean Service Rate Rounded Up μ_{pqr} (veh/hr)**
X	A1	A2	2.975	0.336134	1210.084	1210
B1	A1	A2	2.473	0.404367	1455.722	1456
X	A1	B1	2.799	0.357270	1286.174	1286
A1	A2	A3	3.255	0.307220	1105.991	1106
A1	A2	B2	2.946	0.339443	1221.996	1222
B2	A2	A3	2.679	0.373274	1343.785	1344
A2	A3	A4	3.232	0.309406	1113.861	1114
B4	A3	A4	2.486	0.402253	1448.109	1448
A2	A3	B4	2.889	0.346141	1246.106	1246
A3	A4	Y	3.191	0.313381	1128.173	1128
X	B1	B2	2.429	0.411692	1482.091	1482

A1	B1	B2	1.976	0.506073	1821.862	1822
X	B1	A1	2.058	0.485909	1749.271	1749
B1	B2	B3	2.179	0.458926	1652.134	1652
A2	B2	B3	1.998	0.500501	1801.802	1802
B1	B2	A2	2.013	0.496771	1788.376	1788
B2	B3	B4	2.200	0.454545	1636.364	1636
B3	B4	B5	2.254	0.443656	1597.161	1597
A3	B4	B5	1.974	0.506586	1823.708	1824
B3	B4	A3	2.016	0.496032	1785.714	1786
B4	B5	Y	2.335	0.428266	1541.756	1542

For the road network under study, the number of direct paths is 2, hence n = 2, while the number of interconnecting junctions is three, so m = 3. Thus, the maximum number of possible routes κ that a vehicle can travel by is $\kappa = 16$. Hence, the possible routes are labeled from α_1 to α_{16}. Table 2 enlists all the possible route options along with their route names and limiting mean service rates, for the given road network.

Table 2. Possible routes for given road network with limiting mean service rates

Route Name	Junction order on route	Limiting Mean Service Rate (veh/hr)
α_1	X – A1 – A2 – A3 – A4 – Y	1106
α_2	X – A1 – A2 – A3 – B4 – B5 – Y	1106
α_3	X – A1 – A2 – B2 – B3 – B4 – B5 – Y	1210
α_4	X – A1 – A2 – B2 – B3 – B4 – A3 – A4 – Y	1128
α_5	X – A1 – B1 – B2 – B3 – B4 – B5 – Y	1286
α_6	X – A1 – B1 – B2 – A2 – A3 – A4 – Y	1114
α_7	X – A1 – B1 – B2 – B3 – B4 – A3 – A4 – Y	1128
α_8	X – A1 – B1 – B2 – A2 – A3 – B4 – B5 – Y	1246
α_9	X – B1 – B2 – B3 – B4 – B5 – Y	1482
α_{10}	X – B1 – B2 – B3 – B4 – A3 – A4 – Y	1128
α_{11}	X – B1 – B2 – A2 – A3 – A4 – Y	1114
α_{12}	X – B1 – B2 – A2 – A3 – B4 – B5 – Y	1246
α_{13}	X – B1 – A1 – A2 – A3 – A4 – Y	1106
α_{14}	X – B1 – A1 – A2 – A3 – B4 – B5 – Y	1106
α_{15}	X – B1 – A1 – A2 – B2 – B3 – B4 – B5 – Y	1222
α_{16}	X – B1 – A1 – A2 – B2 – B3 – B4 – A3 – A4 - Y	1128

To estimate queue length using average vehicle length, an assumption that all vehicles are of a similar model has to be made. A report titled Fuel Efficiency Standards of Heavy Duty Vehicles in India (2014), by Shakti Sustainable Energy Foundation, Central Road Research Institute, India, has stated that registrations of LDVs in India have increased from 62% in 1951 to 91% in 2011. The report also stated that by 2013, LDVs would account for approximately 94% of total vehicle kilometers traveled in intra-city transportation. Hence, to make a fair assumption, all vehicles our considered to be LDVs. For increasing the specificity of the model, LDV sales statistics for the fiscal year 2017-18 are used. According to a report by Modi (2018), the highest selling LDV in India, for the fiscal year 2017-18 was the Tata Ace, having sold 208,443 units. Thus, the pilot study considers all the vehicles to be Tata Ace models. By this assumption, the average length of the vehicle is taken as the length of the Tata Ace model which is 3.8 m. (Tata Ace, n.d). The nominal speed of the vehicle is assumed to be 40 km/hr. Using the above data and assumptions, the time of travel on all routes α of the road network are calculated, and a comparative study is made in the results and analysis section which follows.

5. Results and analysis

The proposed queueing model is applied to the observed data to obtain the following datasets. For each route α, the time of travel T_{pqr} between two junctions, p, and q, is calculated using the described methodology, at N_s = 40 km/hr, L = 3.8m and for a varying mean arrival rate from 100 veh/hr to λ_{max}, where $\lambda_{max} < \mu_{min}$. μ_{min} is the limiting value of the mean service rate for route α. The sum of the individual values of travel time for all junctions on route α along with the travel time from the last junction on route α till Y, gives the value of the total travel time for the entire route T_α, as a function of mean arrival rate.

Table 3 shows the calculations and results for one junction system μ_{XA1A2}, for the route α_1. The mean arrival rate varies from 100 vehicles per hour to 1100 vehicles per hour (as λ_{max} has to be less than μ_{min} = 1106 veh/hr). For the junction system X-A1-A2, the distance d_{XA1} is 0.85 km. The estimated values are the traffic intensity ρ_{XA1A2}, the average waiting plus service time per vehicle Wt_{XA1A2}, the average number of vehicles in the system N_{XA1A2}, whose value is rounded up to a whole number as it represents the number of vehicles, the distance of the queue Q_{XA1A2}, the free travel time t_{XA1A2} at speed N_s from X to beginning of the queue and the total time of travel from X to A1 towards A2, T_{XA1A2}. The time is also tabulated in minutes for better analysis.

Table 3. Calculated data for travel time T_{XA1A2} from junction X to junction A1 in the direction of junction A2 at varying mean arrival rates

Mean Arrival Rate $\lambda_{\alpha 1}$ **(veh/hrs)**	**Mean Service Rate** μ_{XA1A2} **(veh/hrs)**	**Traffic Intensity** ρ_{XA1A2}	**Average waiting and service time per vehicle** Wt_{XA1A2} **(hrs)**	**Average vehicles in the system** Rounded Up N_{XA1A2}	**Distance of queue** Q_{XA1A2} **(km)**	**Free travel time** t_{XA1A2} **(hrs)**	**Total Time from X to A1 towards A2** T_{XA1A2} **(hrs)**	**Total Time from X to A1 towards A2** T_{XA1A2} **(min)**
100	1210	0.082645	0.000901	1	0.0038	0.021155	0.022056	1.323354
200	1210	0.165289	0.000990	1	0.0038	0.021155	0.022145	1.328706
300	1210	0.247934	0.001099	1	0.0038	0.021155	0.022254	1.335234
400	1210	0.330579	0.001235	1	0.0038	0.021155	0.02239	1.343374
500	1210	0.413223	0.001408	1	0.0038	0.021155	0.022563	1.353807
600	1210	0.495868	0.001639	1	0.0038	0.021155	0.022794	1.367661
700	1210	0.578512	0.001961	2	0.0076	0.02106	0.023021	1.381247
800	1210	0.661157	0.002439	2	0.0076	0.02106	0.023499	1.409941
900	1210	0.743802	0.003226	3	0.0114	0.020965	0.024191	1.451448
1000	1210	0.826446	0.004762	5	0.0190	0.020775	0.025537	1.532214
1100	1210	0.909091	0.009091	10	0.0380	0.020300	0.029391	1.763455

Using the total time of travel between all junctions on route α, the time of travel on route α itself is obtained. Table 4 summarizes the total time of travel between all junctions on route α_1 and the total time of travel on route α_1 for varying mean arrival rates. The mean arrival rate for all junctions varies from 100 veh/hr to 1100 veh/hr, even though there exist junctions with much higher mean service rates on the route α_1, because the limiting value of the mean service rate μ_{min} is 1106 veh/hr. The table estimates the values of the total time of travel from X to A1 towards A2, T_{XA1A2}, total time of travel from A1 to A2 towards A3, T_{A1A2A3}, total time of travel from A2 to A3 towards A4, T_{A2A3A4}, total time of travel from A3 to A4 towards Y, T_{A3A4Y}, time of travel from last server A4 to Y at nominal speed N_s, T_{A4Y} and the total time of travel on route α_1, $T_{\alpha 1}$ given by the summation of all the previously calculated travel times. The value of T_{A4Y} remains constant throughout as there is no queue formation at Y, so the time of travel is only dependent on the distance between A4 and Y.

Table 4 Total time of travel between all junctions on route α_1

Mean Arrival Rate $\lambda_{\alpha 1}$ **(veh/hr)**	T_{XA1A2} **(min)**	T_{A1A2A3} **(min)**	T_{A2A3A4} **(min)**	T_{A3A4Y} **(min)**	T_{A4Y} **(min)**	$T_{\alpha 1}$ **(min)**
100	1.323354	0.878942	1.103472	1.402666	1.05	5.758434
200	1.328706	0.885525	1.109946	1.408955	1.05	5.783132
300	1.335234	0.893742	1.118010	1.416764	1.05	5.813750
400	1.343374	0.904286	1.128334	1.426718	1.05	5.852711
500	1.353807	0.918310	1.142020	1.439841	1.05	5.903978
600	1.367661	0.932177	1.155332	1.452236	1.05	5.957406
700	1.381247	0.961383	1.183528	1.478787	1.05	6.054945
800	1.409941	1.003978	1.223983	1.515827	1.05	6.203730
900	1.451448	1.087762	1.301874	1.590358	1.05	6.481442
1000	1.532214	1.334038	1.525016	1.773150	1.05	7.214418
1100	1.763455	9.776200	4.885414	3.264857	1.05	20.73993

After travel time for all 16 routes have been calculated, the results are compared using bar graphs. Table 5 is the summary of the comparison of the travel time of all routes at discrete values of mean arrival rates. The total time of travel T_α over route α is presented for values at mean arrival rates λ_α of 500 veh/hr, 900 veh/hr, 1000 veh/hr, 1100 veh/hr, 1200 veh/hr and 1300 veh/hr. Figure 6 is the depiction of Table 4 in the form of column graphs for a cogent understanding of the trends at various mean arrival rates. Table 6 gives the preference of routes in ascending order of their time of travel for values at mean arrival rates λ_α of 500 veh/hr, 900 veh/hr, 1000 veh/hr, 1100 veh/hr, 1200 veh/hr and 1300 veh/hr.

Table 5. Total time of travel T_α on route α at different values of mean arrival rate λ_α

Route	$T_{\alpha\,(min)}$ at $\lambda_\alpha = 500$ (veh/hr)	$T_{\alpha\,(min)}$ at $\lambda_\alpha = 900$ (veh/hr)	$T_{\alpha\,(min)}$ at $\lambda_\alpha = 1000$ (veh/hr)	$T_{\alpha\,(min)}$ at $\lambda_\alpha = 1100$ (veh/hr)	$T_{\alpha\,(min)}$ at $\lambda_\alpha = 1200$ (veh/hr)	$T_{\alpha\,(min)}$ at $\lambda_\alpha = 1300$ (veh/hr)
α_1	5.903978	6.481442	7.214418	20.73993	Unbound queue	Unbound queue
α_2	6.218345	6.616814	7.022371	15.87513	Unbound queue	Unbound queue
α_3	7.409586	7.698366	7.898741	8.379439	15.28452728	Unbound queue
α_4	9.927098	10.36639	10.74525	12.72770	Unbound queue	Unbound queue
α_5	6.679698	6.867966	6.975551	7.149490	7.595067581	Unbound queue
α_6	7.688457	8.154151	8.658809	13.69357	Unbound queue	Unbound queue
α_7	9.197211	9.535992	9.822062	11.49775	Unbound queue	Unbound queue
α_8	8.002823	8.289522	8.466763	8.828773	10.16108519	Unbound queue
α_9	6.249776	6.386943	6.459349	6.554375	6.701396097	6.97475039
α_{10}	8.767289	9.054969	9.305860	10.90264	Unbound queue	Unbound queue
α_{11}	7.258535	7.673127	8.142608	13.09845	Unbound queue	Unbound queue
α_{12}	7.572901	7.808499	7.950561	8.233657	9.267413709	Unbound queue
α_{13}	6.749571	7.285779	7.965389	21.30326	Unbound queue	Unbound queue
α_{14}	7.063938	7.421151	7.773343	16.43847	Unbound queue	Unbound queue
α_{15}	8.255178	8.502702	8.649712	8.942774	11.0915919	Unbound queue
α_{16}	10.77269	11.17073	11.49622	13.29104	Unbound queue	Unbound queue

Tables 3 and 4 affirm the general trend of a queueing model, by showing that with an increase in the mean arrival rate of vehicles λ_α at a junction q, there is a consequent increase in the traffic intensity ρ_{pqr}, and hence an increase in the waiting time Wt_{pqr} at the junction. The increase in waiting time is marginal at first, however, as the value of the mean arrival rate λ_α approaches the mean service rate of the junction $\mu_{pqr,}$ there is a drastic rise in the number of vehicles in the queue. It should be noted that when the mean arrival rate is equal to the mean service rate, the number of vehicles in the queue will be infinite, hence, the situation must never be allowed to occur. Table 5 and Figure 6 depict the trend displayed by each route α over increasing mean arrival rate λ_α. As the value of λ_α crosses the limiting mean service rate value μ_{min} for the route α, the queue becomes unstable and grows unbound. Hence, the route α is discarded as an option. As it is evident from Table 5, at lower mean arrival rate λ_α values, up till 800 veh/hr, the order of routes remains the same. This implies that until heavy traffic follow does not occur, the time of travel remains fairly independent of the queue length or the waiting time and is dominated by the distance of travel. The shorter the distance of the route, the lesser is the total time of travel T_α, thus, higher is the preference of the route. At higher values of λ_α, as traffic congestion increases, the waiting time increases manifold, which verifies the findings of Abdel-Aty et al. (1995) and Arnott and Small (1994). The change in traffic flow over each route causes a change in the order of preference of routes. Some routes cross their stability condition, resulting in a traffic jam at the junction with service rate μ_{min}. These routes are represented by blanks in Figure 6 and are prominent at high mean arrival rate values such as 1200 veh/hr and 1300 veh/hr. Table 7 gives the order of preference of the routes with respect to the total time of travel, at different mean arrival rates, λ_α. As stated earlier, at lower mean arrival rates the shorter route α_1 is given more preference as the traffic queueing is minimal at all junctions. As the mean arrival rate increases, route α_9 becomes the most preferred route, even though it is longer than α_1, and remains as the 1st preference for all mean arrival rates. It must be noted that for values above 1300 veh/hr, route α_9 remains as the sole route choice as other routes cross their respective stability conditions. This continues till α_9 reaches its own limiting mean service rate μ_{min} of 1482 veh/hr. The mean arrival rates give a general idea of traffic flow. Traffic data taken from the local traffic department can help establish the relation between the mean arrival rate and the time of the day. This would imply a change in the preferred route at different time periods throughout the day.

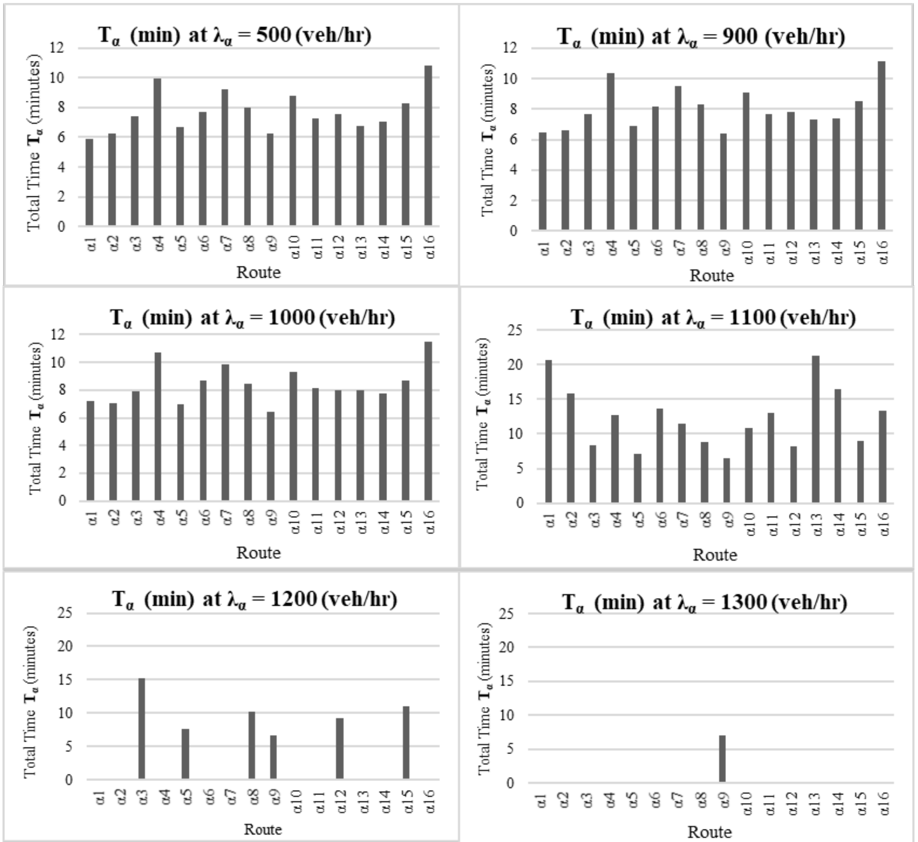

Figure 6. Column graphs depicting total time of travel T_α on route α at different values of mean arrival rate λ_α

Table 6 Order of preference of routes in ascending order of total time of travel T_α

Order pf preference of routes	**Mean Arrival Rate λ_α (veh/hr)**					
	500	**900**	**1000**	**1100**	**1200**	**1300**
1st	α_1	α_9	α_9	α_9	α_9	α_9
2nd	α_9	α_1	α_5	α_5	α_5	-
3rd	α_2	α_2	α_2	α_{12}	α_{12}	-
4th	α_5	α_5	α_1	α_3	α_8	-
5th	α_{13}	α_{13}	α_{14}	α_8	α_{15}	-
6th	α_{14}	α_{14}	α_3	α_{15}	α_3	-
7th	α_{11}	α_{11}	α_{12}	α_{10}	-	-
8th	α_3	α_3	α_{13}	α_7	-	-
9th	α_{12}	α_{12}	α_{11}	α_4	-	-
10th	α_6	α_6	α_8	α_{11}	-	-
11th	α_8	α_8	α_{15}	α_{16}	-	-
12th	α_{15}	α_{15}	α_6	α_6	-	-
13th	α_{10}	α_{10}	α_{10}	α_2	-	-
14th	α_7	α_7	α_7	α_{14}	-	-
15th	α_4	α_4	α_4	α_1	-	-
16th	α_{16}	α_{16}	α_{16}	α_{13}	-	-

The results can also be implemented in crisis situations where one or more junctions are shut down due to unforeseen circumstances. In the case of diversions, Table 6 can be used to find the next best option of travel, provided the traffic conditions at all other junctions remain unaffected. For example, if the road from junction B2 to B3 has to undergo repairs or construction, then at high mean arrival rates of 1100 veh/hr, route α_9 and route α_5 cannot be used for travel anymore. Thus, route α_{12} becomes the most favorable route of travel.

6. Conclusion:

The work proposed in the study provides a route selection methodology between two locations with number of route choices having different junction points and traffic conditions. Taking the parameters of least distance and traffic intensity at each of the junction points, various traveling time is calculated for each of the route choices for different arrival rates of the vehicles. The proposed study provides a method using queueing theory concepts to formulate least travel time for covering a distance in urban traffic situations that can help decision-makers to formulate strategies for effective traffic routing. Also, the proposed methodology provides a ranking of best route alternatives in terms of minimized time at different arrival rates of vehicles giving a comparative analysis over different route choices. The work can be considered for further study with future work related to traffic fuel analysis over different routes, route selection over unsignalized as well as a signalized junction, and multi-arrival and multi-server junction model for making the study more realistic for real-world implementations.

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