On The Capacitated Step-Fixed Charge Transportation And Facility Location Problem: A Local Search Heuristic

Gbeminiyi John Oyewole
Department of Industrial and Systems Engineering,
University of Pretoria,
Pretoria 0002, South Africa
johnigbem55@yahoo.com

Olufemi Adetunji
Department of Industrial and Systems Engineering,
University of Pretoria,
Pretoria 0002, South Africa
olufemi.adetunji@up.ac.za

Abstract

Facility location, route selection and load consolidation are integrated distribution planning problems. These three decisions are of long, medium and short terms respectively. In all these three decision areas, there could be economy or diseconomy of scale involved. Of particular interests are models with single and multiple price breaks. This paper studies a heuristic based on Linear Programming (LP) relaxation for solving this integrated distribution problem. The solution approach utilizes simple transportation problems which other known heuristics in literature have also employed. However, the solution approach ensures that a reduced number of transportation problems are solved. A numerical example is presented to ensure the comprehension of the workings of the solution method. Results obtained for the first set of computational problems show little or no statistical difference in means while the second set of problems had the modified u-v method final solution better than the other methods considered. Although computation studies on a small scale was done to compare the performance of the heuristic with the solution provided by standard optimization environment of Java Eclipse and CPLEX, large problem sizes still proof necessary to compare the runtime and objective value of each solution method.

Keywords*:* Facility location, step-fixed charge, linearization, transportation problem

1. Introduction

The classical transportation model and the solution tableau have been used to model and solve many real life distribution problems. In the classic transportation problem, it is assumed that there are no fixed charges and there is no economy (or diseconomy) of scale. The limitation of this assumption is well understood and many models relaxing these two assumptions have been published by diverse authors. This has resulted in other sub-classes of the transportation model such as the Fixed Charge Transportation Problem (FCTP), in which only the "no fixed charge" assumption is relaxed, and the Step-Fixed Charge Transportation Problem (SFCTP) where both assumptions are relaxed.

The FCTP and SFCTP, are integrated distribution problems of the simple transportation tableau problem with fixed charges and has attracted a lot of research interests. Models and solutions presented by Gray (1971) , Sandrock (1988), Adlakha and Kowalski (2003), Kowalski and Lev (2008), Adlakha et al. (2007) show that the fixed charges could occur either at the source or along the routes. Adlakha and Kowalski (2003), described the FCTP as one in which there is a variable cost and a fixed cost incurred for opening a transportation route with a shipment greater than zero. The SFCTP however, has more than one fixed charge being incurred along the route as described by Sanei et al. (2017). Research in the field of the SFCTP is growing as indicated by Altassan et al.

(2013) and El-Sherbiny and Alhamali (2013) with new problem type models and solution techniques being continually developed.

Facility location models on the other hand, as described by Ray (1966) and Sá (1969) have evolved from the simple plant un-capacitated models to the complex capacitated models with various integrated transportation and inventory models such as in Huang et al. (2015) and Carlo et al. (2017) . The field of Facility Location Problems (FLP) over the years have centred around choosing a set of locations (Plants, warehouses, depots etc.) from other potential locations where the fixed location cost is usually used as part of the model parameters in selecting the best locations to satisfy the model objective and constraints.

The integrated Step-Fixed Charge Transportation Problem and Facility Location Problem which we have called SFCTLP was indirectly modeled by Correia et al. (2010) as a capacitated transportation model problem with modular distribution costs where the step-fixed charges were shown as a number of modules in a link from source to destination. Furthermore, they presented a reformulated model with new valid inequalities to strengthen their linear programming relaxation solution. Christensen (2013) also discussed the SFCTLP and referred to it as Capacitated Facility Location Problem with Piecewise Linear Transportation Cost (CFLP with PLTC).

As seen in literature and also noted by Christensen (2013), good number of solutions have been put forward in solving separately the two main aspects of SFCLTP which are the FLP and SFCTP respectively. In the case of SFCTP, Kowalski and Lev (2008) and Altassan et al. (2013) adapted the linear programming relaxation method of Balinski (1961) to find an initial solution and Kowalski and Lev (2008) subsequently used a structured perturbation logic to arrive at an improved solution. El-Sherbiny and Alhamali (2013) presented a metaheuristic solution method to solve the SFCTP. Kim and Pardalos (2000) presented the dynamic slope scaling heuristic based on LP relaxation to solve the Fixed Charge Network Flow problem (FCNFP) with similarity in modelling to FCTP, and a possible extension to solve SFCTP. Furthermore, Christensen and Labbé (2015) provided a branch and cut solution method based on LP relaxations and compared the efficiency of their solution methods with a standard Mixed Integer Linear Programming (MILP) solver method such as CPLEX. Some solutions to the SFCTP have revolved around introducing strong inequalities for LP relaxation. These have been noted by Croxton et al. (2007) in their SFCTP called multi-commodity network design problem as strong and binding constraints. They concluded that the LP relaxations approximate the objective function when the total flow is on each arc or route.

Solutions presented in literature for solving both simple FLPs and integrated FLPs have been through the use of the exact methods such as branch and bound , cutting plane algorithms to the use of LP based heuristics, Lagrangian relaxation heuristics , local search heuristics and metaheuristics as noted by Wu et al. (2017). In solving the CFLP with PLTC, Christensen (2013) described the problem as NP-hard. He suggested valid in-equalities for strengthening the linear programming relaxation and also presented a lagrangian relaxation heuristic solution and compared the efficiency of the solution obtained with a standard MILP solver such as CPLEX.

In this paper, we study the SFCLTP which has been described to be NP-hard and consequently develop an LP based relaxation heuristic to solve the problem. We have considered two fixed charges in the route without loss of generality. Furthermore we introduce more valid inequality and equality approximations to the model of Christensen (2013). The LP relaxation heuristic we developed adapts the principle of linearized cost by Kim and Pardalos (2000). Our solution begins with some violation of the supply capacities but through the minimum total demand flow cost from a source to all destinations for feasible locations, we develop a simple transportation problem that ensures the demand and supply constraints are met. We also present the results of some problem instances solved based on the LP heuristic in comparison to those obtained from using the CPLEX concert technology with the code written in Java.

2 Base Model Formulation for SFCLTP

The one stage or two echelon SFCTP and SFCLTP are described as m suppliers and n demand point distribution problems, where m connotes the number of sources (factories or distribution centers) and the n refers to the number of customers, demand or sink points. There are capacities and demand for each sources or locations and demand point respectively over a time period, usually annual. The m suppliers incur a unit transportation cost per unit distance and there are more than one fixed charge in the route, link or arc depending on the number of break points in the transportation routes. The SFCLTP, further consists of fixed location costs attached to several potential locations to which the cheapest locations that satisfies the problem constraints are

to be selected. Christensen (2013) presented a mathematical model for the Multi Choice Model (MCM) of the CFLP with PLC which forms a base for our SFCLTP and represented below:
Min Z =

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{l=1}^{q}(c_{ijl}x_{ijl} + g_{ijl}v_{ijl}) + \sum_{i=1}^{n} F_i\, y_i \qquad (1)$$

Subject to (constraints):

$$\sum_{i=1}^{n}\sum_{l=1}^{q} x_{ijl} \; = \; D_j \qquad \forall\ j \qquad (2)$$

$$\sum_{l=1}^{q} v_{ijl} \; \leq \; 1 \qquad \forall\ (i,j\,) \qquad (3)$$

$$\sum_{j=1}^{m}\sum_{l=1}^{q} x_{ijr} \; \leq \; S_i y_i \qquad \forall\ i \qquad (4)$$

$$x_{ijl} \; \leq L_{ijl}\ v_{ijl} \qquad \forall\ (i,j,l\,) \qquad (5)$$

$$x_{ijl} \; \geq L_{ijl}\ v_{ijl} \qquad \forall\ (i,j,l\,) \qquad (6)$$

$$x_{ijl} \; \geq \; 0 \qquad (7)$$

$$y_i \; \in \; \{\,0,1\} \qquad \forall\ i \qquad (8)$$

$$v_{ijl} \; \in \; \{\,0,1\} \qquad \forall\ (i,j,l\,) \qquad (9)$$

Equation (1) is the total cost minimization objective function representing location and transportation cost. Equation (2) represents the compulsory demand capacity. Equation (3) enforces one transportation mode between each pair of source and destination. Equation (4) represents supply capacity not being exceeded. Equation (5) ensures an upper bound on the load distribution, while Equation (6) enforces a lower bound on distribution. Equation (7) refers to the non-negativity constraint while equation (8 and 9) refer to the binary constraints.

3 Problem Structure And Formulation for SFCLTP

For the linearization and relaxation of our solution for the SFCLTP, a valid inequality constraint for the break point as used in the work of Sanei et al. (2017) was introduced to the model of Christensen (2013) to further strengthen the LP lower bound relaxation . Secondly, the Balinski (1961) method of binary variable relaxation which Kowalski and Lev (2008) and Altassan et al. (2013) employed in relaxing their binary fixed charge variables was also employed. This method uses some constraints in the problem to relax the binary constraints in the cost objective term. Which is a general approach commonly employed for LP relaxations.

3.1 Model Assumption

We make the following assumptions in our model:

1. Deterministic input
2. One stage or Two echelon problem
3. Two step-fixed charge cost without loss of generality
4. Single period and single item distribution problem.

3.2 Model Parameters

i : Index for set of Sources (Plants, Warehouses etc.)
j : Index for set of Destinations (Customers, Warehouses, depots etc.)
m : Number of sources (or plants)
n : Number of destinations (or demand point)
c_{ij} : Unit cost of shipment on route (i, j)
S_i : Supply Capacity for Source i

D_j : Demand Capacity for Destination j
H_{ij} : First level fixed cost
I_{ij} : Second level fixed cost
A_{ij} : Break point for selecting the route fixed charges

Decision variables
x_{ij} : Allocations (or load distributions) along route (i, j)
y_i : Location variable for plant or source (0 or 1)
g_{ij} : Binary fixed charge variable before the break point
z_{ij} : Binary fixed charge variable after the break point

Objective function of the Original Problem:
Minimize Z_O =

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij} + \sum_{i=1}^{m} F_i\, y_i + \sum_{i=1}^{m}\sum_{j=1}^{n} H_{ij}\ g_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} I_{ij}\ z_{ij} \qquad (10)$$

Subject to (constraints):

$$\sum_{j=1}^{n} x_{ij} \le S_i y_i \qquad \forall\ i = 1 \dots m \qquad (11)$$

$$\sum_{i=1}^{m} x_{ij} = D_j \qquad \forall\ j = 1 \dots n \qquad (12)$$

$$\sum_{i=1}^{m} S_i y_i \ge \sum_{j=1}^{n} D_j \qquad \forall\ i = 1 \dots m, \quad \forall\ j = 1 \dots n \qquad (13)$$

$$x_{ij} \le M_{ij}\, g_{ij} \qquad \forall\ i = 1 \dots m,\ \forall\ j = 1 \dots n \qquad (14)$$

$$x_{ij} - A_{ij} \le M_{ij} z_{ij}\ \ \forall\ i = 1 \dots m,\ \forall\ j = 1 \dots n \qquad (15)$$

$$x_{ij} \ge 0 \qquad (16a)$$

$$y_i = 0\ or\ 1 \quad z_{ij} = 0\ or\ 1 \quad g_{ij} = 0\ or\ 1 \qquad (16b)$$

$$g_{ij} = \begin{cases} 1 & x_{ij} > 0 \\ 0 & Otherwise \end{cases}, \quad z_{ij} = \begin{cases} 1 & x_{ij} > A_{ij} \\ 0 & Otherwise \end{cases} \qquad (16c)$$

$$M_{ij} = \min(S_i, D_j)\ \ \forall\ i = 1 \dots m,\ \forall\ j = 1 \dots n$$

Equation (10) is the objective function. The first term is a variable cost, the second term is the facility location cost and third term is the route step-fixed charge cost. Equation (11) is the supply capacity constraint of each location or sources which should not be exceeded. Equation (12) is the demand constraint of each customer which must be met. Equation (13) is the aggregate constraint for supply and demand balance. Equation (14) is a constraint that ensures that an upper bound on the load distribution before the break point is established. Equation (15) is a constraint that ensures that a lower bound on the load distribution after the break point is established. Equation (16a) refers to the non-negativity constraint while (16b) refer to the binary integer constraints. Equation (16c) ensures the objective function selects the fixed charges based on the break points.

4 Solution Method

As indicated in earlier sections, the solution method presented in this work is based on linearizing the cost objective function through the relaxation of some of the constraints. The relaxed constrains are used in objective function to relax the location y_i and the route fixed charge g_{ij} and z_{ij} variables. Figure 1 represents the linearization cost structure of the objective function Z_O before and after linearization. The linearized model is solved to optimality as the first transportation problem and a lower bound to the SFCLTP is obtained from the result of the first transportation problem solution. The linearized cost solution obtained however may produce some supply capacity infeasibilities at the sources, which have to be resolved before the desired solutions are obtained. Our LP heuristic resolves the supply infeasibilities by identifying the combination of potential sources that meet the demand. Using the linearized cost version of equation (10) and assuming total flow of all the demand load from a source (i) and all destinations(j), we compute the total flow for each of the sources $(i = 1 \dots m)$. Croxton et al. (2007) suggested that the LP relaxation of the cost objective function would give an

approximation of the original problem with the "lower convex envelope", when the total flow on each route is used in computation.
Finally, Combination(s) of the sources (i) that meet all the demand $\sum_{j=1}^{n} D_j \quad \forall\ j = 1 \dots n$ with the least cost combination using the linearized version of cost equation (10) is selected for the final transportation problem. To solve the final transportation problem, we use the relaxed cost combination of $c_{ij,}H_{ij}\ and\ I_{ij}$ to determine the final load distribution (x_{ij}) to be used in equation (1).

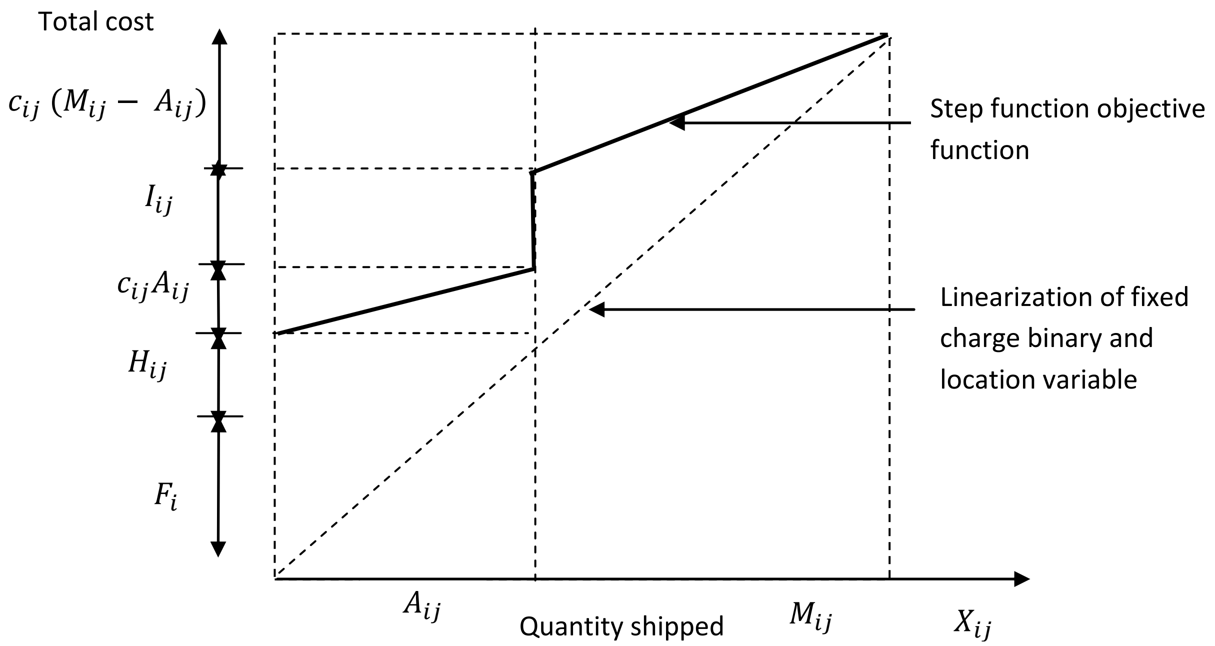

Figure 1. Adapted Linearization And Relaxation Structure For SFCLTP (Kowalski and Lev, 2008)

4.1 Linearized Cost Solution

The principle of linearized cost as used by Kim and Pardalos (2000) is adapted for the linearization to solve the first transportation problem. We select the constraints (11), (14) and (15) respectively, place an equality upper bound on them and use them in relaxing the fixed location cost and fixed charge variables y_i , g_{ij} and z_{ij} respectively in the objective function equation (1).
The constraints given by (11), (14) and (15) are transformed respectively into

$$\sum_{j=1}^{n} x_{ij} = \quad S_i y_i \qquad \forall\ i = 1 \dots m \qquad (17)$$

$$x_{ij}/M_{ij} = g_{ij} \qquad (18)$$

$$(x_{ij} - A_{ij})/\ M_{ij} = z_{ij} \qquad (19)$$

We substitute this equation (17) into the aggregate constraint of equation (13) above to obtain

$$\sum_{i=1}^{m} S_i (\frac{\sum_{j=1}^{n} x_{ij}}{S_i}) \geq \sum_{j=1}^{n} D_j \qquad \forall\ i = 1 \dots m, \qquad \forall\ j = 1 \dots n \qquad (20)$$

Substituting equation (17) (18) and (19) into the objective function (equation (1)) and using the derived equation (20) would give a linear programming problem model of the Original problem, given as :
Min Z_{LPO} =

$$\sum_{i=1}^{m} (F_i \frac{\sum_{j=1}^{n} x_{ij}}{S_i}) + \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} H_{ij}\ \frac{x_{ij}}{M_{ij}} + \sum_{i=1}^{m}\sum_{j=1}^{n} I_{ij}\ \frac{x_{ij} - A_{ij}}{M_{ij}} \qquad (21)$$

Subject to

$$\sum_{i=1}^{m} x_{ij} = D_j \qquad \forall\ j = 1 \dots n \tag{22}$$

$$\sum_{i=1}^{m} S_i\left(\frac{\sum_{j=1}^{n} x_{ij}}{S_i}\right) \geq \sum_{j=1}^{n} D_j \qquad \forall\ i = 1 \dots m, \qquad \forall\ j = 1 \dots n \tag{23}$$

$$x_{ij} \geq 0 \tag{24}$$

Equation (21) could also be restated as

$$Z_{LP0} = \sum_{i=1}^{m}\left(F_i\ \frac{\sum_{j=1}^{n} x_{ij}}{S_i}\right) + \sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij} + \frac{H_{ij}}{M_{ij}} + \frac{I_{ij}}{M_{ij}}\right) x_{ij} - \sum_{i=1}^{m}\sum_{j=1}^{n} I_{ij}\ \frac{A_{ij}}{M_{ij}} \tag{25}$$

Equations(21) to (24) gives a linear programming problem which can be solved to optimality by any known optimal linear programming solver.

4.2 Resolution of Linearization Infeasibility and Final Transportation Problem

We note that in the solution of the new relaxed model represented by equation (21) to (24), there could be supply infeasibilities due to constraints (22) and (23) not strictly enforcing the supply requirements. We however resolve these supply infeasibilities through our heuristic presented below:

Step 1: Select individuals or groups of potential sources ($i \in m$) such that $\sum_{i=1}^{m} S_i\, y_i \geq \sum_{j=1}^{n} D_j$
Step 2: For the individuals or groups selected, Use equation (25) only to calculate $Z_{LP0}^{i}\ \forall\ i = 1 \dots m,$ where the load distribution is given as $x_{ij} : x_{ij} = D_j\ \forall\ j = 1 \dots n$.
(thus Z_{LP0}^{1} is calculated using $x_{11}, x_{12}, x_{13}, \dots \dots x_{1n},$)
Step 3: From the individuals or groups of $i \in m$ selected in step 1, choose the minimum Z_{LP0}^{i} or combinations of Z_{LP0}^{i} ($\sum_{i}^{m} Z_{LP0}^{i}$) if groups are selected. We note that Z_{LP0}^{i} values could be positive or negative depending on the problem parameters. This is evident by the third term in equation (25) which has the possibility of making the equation negative. If the combinations of Z_{LP0}^{i} are positive we select the minimum value. However if the combinations of Z_{LP0}^{i} are negative values we select the minimum absolute value.
Step 4: Solve the transportation problem created by using the sources selected in Step 4 and the relaxed cost combination of ($c_{ij} + \frac{H_{ij}}{M_{ij}} + \frac{I_{ij}}{M_{ij}}$) subject to simple demand and supply constraints to obtain the load distribution x_{ij} for equation (1). The transportation problem to be solved is given as

$$Min\ Z_F = \sum_{i=1}^{m}\sum_{j=1}^{n}\left(c_{ij} + \frac{H_{ij}}{M_{ij}} + \frac{I_{ij}}{M_{ij}}\right) x_{ij} \tag{26}$$

$$\sum_{j=1}^{n} x_{ij} \leq S_i y_i \qquad \forall\ i = 1 \dots m \tag{27}$$

$$\sum_{i=1}^{m} x_{ij} = D_j \qquad \forall\ j = 1 \dots n \tag{28}$$

$$x_{ij} \geq 0 \tag{29}$$

Step 5: Using the value of Z_O, compare the load distribution x_{ij} obtained by solving the transportation model given by equation (19) to (22) by any feasible approach such as the North-West Corner (NWC) method and optimally using Simplex or the Modified u-v distribution method known as (Modi). Select the minimum Z_O obtained by the methods.

The Least Cost (LC) approach of solving the transportation problems ensures that maximum allocations are made to the minimum relaxed cost position, while NWC approach ensures that allocations are made to the least cost positions from a northwest corner of the transportation tableau. An optimal transportation cost solution using the Modified u-v distribution method or Modi would perform dual variable (u and v) analysis and ensure that $m + n - 1$ variable positions are occupied.

5. Numerical Solution Obtained

This section consists of the numerical computation done using a random problem size of (4× 4). Furthermore, computation studies were done to further gain insight into the heuristic performance when compared to the standard MILP solver such as CPLEX.

5.1 Numerical Example

In order to explain the workings of the LP based heuristic presented in section 4.2 above, we have adapted the problem created by Kowalski and Lev (2008), represented the problem in Table (1) and (2) below and also shown the solution below.

Table 1 : Supply, demand, location (set up) costs and unit cost parameters

. i	S_i	F_i	j = 1	2	3	4
			c_{ij}			
1	25	100	1	3	1	3
2	25	200	2	2	3	2
3	25	250	2	1	2	1
4	25	150	1	3	1	3
D_j			10	30	20	15

Table 2 : Two tier fixed charges on route ($\boldsymbol{i, j}$)

i	H_{ij} , I_{ij}	H_{ij} , I_{ij}	H_{ij} , I_{ij}	H_{ij} , I_{ij}
1	10 ; 20	10 ; 10	10 ; 30	10 ; 10
2	10 ; 30	10 ; 20	10 ; 20	10 ; 20
3	10 ; 20	10 ; 30	10 ; 10	10 ; 30
4	10 ; 20	10 ; 10	10 ; 30	10 ; 10
	j =1	2	3	4

The break point $A_{ij} = 5$ $(constant)$ through route i, j
From section 4.1, using the linearized cost model represented by equation (21) to (24) we obtained the value of $Z_{LPO} = 488.75$

25	10	30	20	15
25	0	0	0	0
25	0	0	0	0
25	0	0	0	0
	10	30	20	15

This clearly gives an infeasible solution as noted in section 4.

Step 1:

Using first step of the heuristic we obtain the combinations of locations to resolve the infeasible solution obtained above i.e. (1,2,3), (1,2,4), (1,3,4), (2,3,4) and (1,2,3,4) as having $\sum_{i=1}^{m} S_i\, y_i \geq \sum_{j=1}^{n} D_j$

However, it is clear that the combination of (1,2,3,4) would not be cost saving based on the extra fixed location cost.

Step 2:

We compute objective value Z_{LP0}^{i} for the four sources (no optimization done here) where :

Z_{LP0}^{1} is calculated using $x_{11,}\ \ x_{12,}\ \ x_{13,}x_{14,} = 488.75$

Z_{LP0}^{2} is calculated using $x_{21,}\ \ x_{22,}\ \ x_{23,}x_{24,} = 794.5$

Z_{LP0}^{3} is calculated using $x_{31,}\ \ x_{32,}\ \ x_{33,}x_{34,} = 867.75$

Z_{LP0}^{4} is calculated using $x_{11,}\ \ x_{12,}\ \ x_{13,}x_{14,} = 638.57$

The location (1,2,3) gives a sum of (488.75+794.5+867.75) = 2151

Similarly locations (1,2,4), (1,3,4), (2,3,4) gives 1921.82, 1995.07 and 2300.82 respectively.

Step3:

Since the combinations of Z_{LP0}^{i} are positive , we select the minimum value. The location (1,2,4) possess the minimum sum of linearized cost and thereby chosen.

Step 4:

We use the relaxed cost combination of $c_{ij} + \frac{H_{ij}}{M_{ij}} + \frac{I_{ij}}{M_{ij}}$ given below as the new unit cost and solve the second transportation problem given by equation (26) to (29) to obtain the final load distribution to be used to calculate Z_O in equation (10) above. The relaxed cost combination matrix is presented below.

4	3.8	3	5.166667
6	3.2	4.5	3.416667
5	2.6	3	2.166667
4	3.8	3	5.166667

Step 5:

The Load distribution obtained when Transportation problem (26) to (29) is solved optimally and using least cost approach $Z_F = 259.25\ \ and\ Z_O = 750$ is presented below.

25	10	15	0	0
25	0	10	0	15
25	0	0	0	0
25	0	5	20	0
	10	30	20	15

Using the Northwest corner approach when solving transportation problem (19) to (21) gives
$Z_F = 276.95\ and\ Z_O = 720$. This is shown below.

25	10	0	0	15
25	0	25	0	0
25	0	0	0	0
25	0	5	20	0
	10	30	20	15

The CPLEX Values obtained for Z_O are given below. $Z_O = 710$

25	5	0	20	0
25	0	25	0	0
25	0	0	0	0
25	5	5	0	15
	10	30	20	15

5.2 Computation study

To further gain a little insight into the performance of this LP based heuristic in relation to the solution provided by CPLEX, we conducted a small scale computation study. This study was however based on the objective values of the problem alone, though articles in this field would also compare the runtime of the different solution methods. Our primary interest in this paper is to observe the trend of the objective values for the little pilot study. Our SFCLTP model was coded using the Java Eclipse environment with the CPLEX 12.8 concert technology as the MILP solver. The LP heuristic was partly coded in Java and the transportation optimizations were solved with Microsoft excel solver and TORA software. A total of 25 randomly generated problems were solved. These problems were classified under two sets of problem instances. The first set of problem instances had two problem sizes (4×4 and 7×7) with 5 problems solved for each size. The LP based heuristic was solved using the NWC to obtain the final load distribution according to the LP heuristic step 5.

For the second set of problems, three problem sizes (6×4, 8×5 and 9×7) were considered with 5 problem instances generated under each size. The step 5 of our LP based heuristic was however solved using the NWC, LC and optimally using the modified u-v distribution method (Modi) in TORA optimization software to obtain the final load distribution. This was done to further gain an insight on the performance of the feasible transportation methods and optimal transportation method at arriving at the final load distribution. Table 3 below shows the parameter range used for the random problem generation. The orders of magnitude of the parameters utilized for the range have been selected to reflect the reality of the proportions of the parameters compared to one another.

Table 3: Data range used for the small scale computation study

Problem size (No. of Instances)	Parameter	Range of values	Breakpoint per problem size
4×4 (5)	F_i c_{ij}	$100-550$ $1-10$	$A_{ij} = 5$
7 × 7 (5)	S_i	$10-100$	$A_{ij} = 5$
6 × 4 (5)	D_j	$5-50$	$A_{ij} = 5$
8 × 5 (5)	H_{ij}	$10-30$	$A_{ij} = 10$
9 × 7 (5)	I_{ij}	$10-30$	$A_{ij} = 25$

The mean value, and the 5 different problem instances of the CPLEX and LP heuristic (using NWC solution) generated are shown in Table 4 below. Also, the percentage mean difference have been included to show the gap obtained from the CPLEX values. The larger the mean percentage, the poorer the results of the LP heuristic in comparison to CPLEX values. Similarly to Table 4, Table 5 below presents the results obtained using the northwest corner, least cost and modified u-v distribution method using only the mean values of the different 5 problem instances generated.

Table 4: First set of results obtained

Problem Size	Instance No	CPLEX (Z_O)	LP heuristic (Z_O) (NWC)	% mean Difference LP heuristic and CPLEX
4×4	1	710	720	2.4%
	2	735	735	
	3	705	710	
	4	685	725	
	5	580	610	
Mean values	($\overline{Z_O}$)	683	700	
7×7	1	1315	1315	1.0%
	2	1425	1480	
	3	1610	1620	
	4	1910	1920	
	5	1755	1765	
Mean values	($\overline{Z_O}$)	1603	1620	

Table 5: Second set of results obtained

Problem Size	Mean CPLEX ($\overline{Z_O}$) Value	Mean LP heuristic ($\overline{Z_O}$)(NWC)	Mean LP heuristic ($\overline{Z_O}$) (LC)	Mean LP heuristic ($\overline{Z_O}$) (Modi)	% mean difference LP-NWC and CPLEX	% mean difference LP-LC and CPLEX	% mean difference LP-Modi and CPLEX
6×4	1665	1915	1810	1731	15.01%	8.70%	3.96%
8×5	2471	2964	2514	2494	19.95%	1.74%	0.93%
9×7	2961	3392	3102	3015	14.56%	4.76%	1.82%

6. Discussion Of Solutions Obtained

From the results of the little computation study conducted it is easily seen that the CPLEX solution outperformed the LP based heuristic as per the individual instances and mean values considered both in Tables 4 and 5. Results obtained for the first set of problems as per Table 4, shows the mean objective value gap difference of 2.4% and 1.0% respectively. A quick analysis using the paired t-test of means for the (4×4) problem size at 0.05 level of significance gave a p-value of 0.09130, showing there is not quite statistical significance in the difference of the means obtained, while the second and larger problem size (7×7) gave a p-value of 0.15440 showing no statistical significance in the difference of means. This further shows that the LP heuristic may have the possibility of having very good solutions as the problem size increases. Furthermore we observe that the NWC used to obtain the final distribution load for the LP heuristic had a good performance of being close to 0% difference.

However, in the second set of problems as presented in Table 5, the NWC performance was consistently worse when compared to LC and Modi. This shows that for certain problem structures the NWC could quickly obtain good results. The LP based heuristic using the Modi and LC distributes the load using the minimum relaxed cost of the problem parameters, while NWC does not. The LP-Modi in problem size (8×5) obtained the best mean difference of 0.93%. Showing it has the likelihood of generating good results irrespective of problem size and parameter structure.

Finally, the numerical problem considered showed the possibility of having near solutions to the CPLEX values when a minimum value among the NWC, LC and Modi is used for the final load distribution of the LP based heuristic.

7. Conclusion And Future Directions

In this paper, we considered the Facility Location (FL) and Step-Fixed Charge Transportation Problem (SFCTP). A Linear Programming (LP) based heuristic was developed which was decomposed into two major transportation problems. The relaxed objective function of the first transportation problem in equation (25) was used in selecting the combinations of locations to be considered for the final transportation problem. In addition, solving the first transportation problem could help in generating a good lower bound for other computation methods such as Lagrange relaxations. A Numerical solution was presented to show the workings of the heuristic while a small scale pilot computation study was done to gain a little insight into the heuristic performance as regards the objective value obtained. Two sets of problem sizes were considered. The first set of problem sizes was solved using the NWC method as a solution method for the final transportation problem. The statistical results obtained showed a little or no significant differences in the mean values of the CPLEX solution and the LP based heuristic with increase in problem size. However, the NWC method lagged behind the LC and Modi methods for the second set of problem sizes. The Modi method for the final transportation problem was superior in all the problem instances of the second set of problems. The optimal distribution pattern of the Modi method (minimum cost) seemed to have a strong link to the good results obtained.
Conclusively, large scale computation study still have to be conducted to gain proper insight into the performance of the heuristic under different parameter ranges and also due to the reason that exact methods which may obtainable using the solution methods present in the CPLEX optimization studio could become inefficient as the problem size increases. Also, comparing the runtimes of the different solution methods would also need to be considered for an efficient study. Finally a comparative study between this LP based heuristic and solution methods such as the lagrangian relaxation heuristic, metaheuristics such as genetic algorithm , tabu search and hybrid solutions which uses both LP based relaxation and metaheuristics would be possible areas for exploring.

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Biographies

Gbeminiyi John Oyewole : Is currently a doctoral student in the Department of Industrial and Systems Engineering at the University of Pretoria, South Africa. He obtained his Master's degree in Industrial and Production Engineering at the University of Ibadan, Nigeria. He has worked in field of Logistics, transportation and as a supply chain officer at the Candel Agrochemical Plant in Lagos Nigeria. He is also an UNLEASH award winner in the category of sustainable production and consumption, whose team was able to design sustainable packaging materials for transportation using pallets. His research interests are in Facility location problems, applied optimization, operations research, supply chain and engineering Designs.

Olufemi Adetunji: is currently a senior lecturer at the Department of Industrial and Systems Engineering at the University of Pretoria, South Africa. He obtained his PhD at the same University. He has performed various research work with National Research Foundation (NRF) South Africa and with other leading governmental and private agencies within South Africa. He has also published a lot of articles in leading local and International Journals. His research Interests are in Supply Chain design and Engineering , Lean Manufacturing and applied optimization