On The Facility Location and Fixed Charge Solid Transportation Problem: A Lagrangian Relaxation Heuristic

Gbeminiyi John Oyewole
Department of Industrial and Systems Engineering,
University of Pretoria,
Pretoria 0002, South Africa
johnigbem55@yahoo.com

Olufemi Adetunji
Department of Industrial and Systems Engineering,
University of Pretoria,
Pretoria 0002, South Africa
olufemi.adetunji@up.ac.za

Abstract

A Lagrangian Relaxation Heuristic (LRH) to solve the integrated distribution planning problem of selecting transportation mediums (conveyances), route selection, load consolidation and location of facilities has been studied in this paper. The problem is known as Facility Location and Fixed charge Solid Transportation with real life applications of integrating several planning horizons of small, medium and long terms. A numerical example was proposed and solved using two different sets of Lagrange multipliers as starting values for the iterations performed and termed the first and second trial runs respectively. The CPLEX standard optimization software was used as a base for comparing the LRH obtained. A simple infeasibility resolution of the better LRH (second trial) lower bound solution was applied to obtain the CPLEX values. Extensive computations of various problem sizes and instances will however be required to validate the good performance of LRH (second trial) run obtained compared to using standard optimization solvers such as CPLEX.

Keywords: Facility location, Solid Transportation problem, Lagrange Relaxation Heuristic, Fixed Charge Problem.

1. Introduction

Distribution problems which involve merging various individual optimization problems have continued to be explored by researchers. Supply chain and Integrated distribution issues are clear examples of these. Usually, these individual optimization problems have different planning horizons from short, medium to long term. The suboptimal solutions obtained when individual solutions are desired amidst the integrated distribution problems have resulted in the search for a global solution.

Facility location problems (FLP) which involve searching for the best locations out of other competing locations for distribution of products or services based on the objective criterion under consideration are still being explored in literature. The FLP has emerged from the simple plant location problem to various complex variants over the years. Amongst those that developed early models on the FLP were Ray (1966) , Sá (1969) and Akinc and Khumawala (1977) . They considered the simple plant location models. Holmberg and Ling (1996), Revelle and Laporte (1996), Agar and Salhi (1998), Hindi and Pieńkosz (1999) extended the simple plant location model with capacitated plant location models of the multi objective, multiproduct type and large scale single source capacitated models. The FLP has been formulated as mixed integer and mostly pure integer programming problems as seen in the works mentioned earlier. References could also be made to Krarup and Pruzan (1983), Klose and Görtz (2007), Arabani and Farahani (2012) and Boujelben et al. (2016) on the various formulation of FLP over the years. Recently, facility location models are being integrated into other planning and distribution models such as in the location inventory problem of Puga and Tancrez (2017), FLP with plant size selection of Wu et al. (2017), location inventory and routing models of Hiassat et al. (2017) and transportation location problem of Carlo et al. (2017). These variants of Facility location models have been solved using several optimization techniques developed over the years. These techniques range from the exact methods to heuristics. Ray (1966), Holmberg and Ling (1996) and Yang et al. (2012) have employed Exact methods such as Branch and Bound(BB), cutting plane algorithm to solve small scale instances of the FLP variants . However, due to the inefficiency of exact methods in solving large problem sizes in a reasonable time, other Heuristics such as the Linear Programming Relaxation(LPR), Lagrange relaxation(LR), Lagrange Relaxation Heuristics(LRH), local search heuristics and metaheuristics are being developed to tackle both small to large problems sizes. A lot of researches have been done in the application of these heuristic methods in the FLP variants such as in the works of Ghiani et al. (2002), Nezhad et al. (2013), Wang and Lee (2015) , Wu et al. (2017) and Ou-Yang and Ansari (2017).

On the other hand, the classical Transportation Problem (TP) which is being solved by the transportation tableau has also been transformed like the FLP into various variants. Among the variants which are currently being explored by researchers are the Fixed Charge Transportation Problem (FCTP), Fixed Charge Solid Transportation problem (FCSTP) and the Step Fixed Charge Solid transportation problem(SFCSTP) as noted in the works of Roberti et al., (2014), Calvete et al. (2016), Zhang et al. (2016) and Sanei et al. (2017) . Both the FCSTP and SFCSTP are distribution problems which involve m-sources, n- destinations and k-conveyances. As indicated by Sanei et al. (2017) the problem both FCSTP and SFCSTP seek to solve is to determine the quantities that the chosen capacitated k-conveyances will be able to ship from the m-sources and n- destinations under a mix of route fixed charges at the best minimum cost. They also showed that the FCSTP and the SFCSTP are NP-hard problems in which exact methods, may be deficient in tackling certain problem sizes and structure in a reasonable time. As a result, they presented the LRH in solving the SFCSTP.

The LRH utilizes Lagrangian Relaxation (Use of Lagrangian multipliers) in dualizing difficult constraints in the Original Problem (OP) which will be added to the objective function. The OP is thus split into simpler sub problems (SP) based on the decision variable types. The summation of each SP result into a Lower Bound(LB). The LB is thus known as the relaxed version of the OP. The type of constraints selected to participate in the Lagrangian relaxation also affects the strength of the LB derived (Cornuéjols et al., 1991). The LRH reformulation, creates a lower bound upon which an upper bound(UB) would be determined. An iterative procedure then follows in which the Lagrange multipliers, LB, UB and other optimization methods are utilized in arriving at a good solution. Among the best known optimization methods utilized during the application of the Lagrangian heuristics for both the FLP and FCSTP has been the sub gradient optimization method. It has been argued by Fisher (1981) , and supported by Holmberg and Ling (1996) and Ghiani et al. (2002) that using the sub gradient method can give a strong lower bound that is less than or equal to the optimal solution of the Original Problem.

In this paper we consider an integrated distribution problem between the Facility Location Problem (FLP) and the Fixed Charge Solid Transportation Problem (FCSTP). As noted earlier most FLPs are formulated as pure integer problems and most TPs as mixed integer problems. We however, formulate this Facility Location Problem and Fixed Charge Solid Transportation Problem as a mixed integer problem to show the desired product quantities been distributed by the selected conveyances between the m sources and n destinations. By doing this more decisions can be taken by the operational personnel involved in decision making. We have termed this problem as Fixed Charge Solid Location and Transportation Problem (FCSLTP). The objective for the FCSLTP is to find the optimal locations amongst several competing locations (e.g. Warehouses, depots) to distribute products at a unit cost to meet customers' orders by selecting from competing capacitated conveyances or transport mediums under route fixed charges. Furthermore, we have adapted the LHR method presented by Sanei et al. (2017) to arrive at a solution for the FCSLTP and also compared it with the solution obtainable by a standard general purpose solver such as the IBM CPLEX. Figure 1 below shows a schematic representation of the FCSLTP.

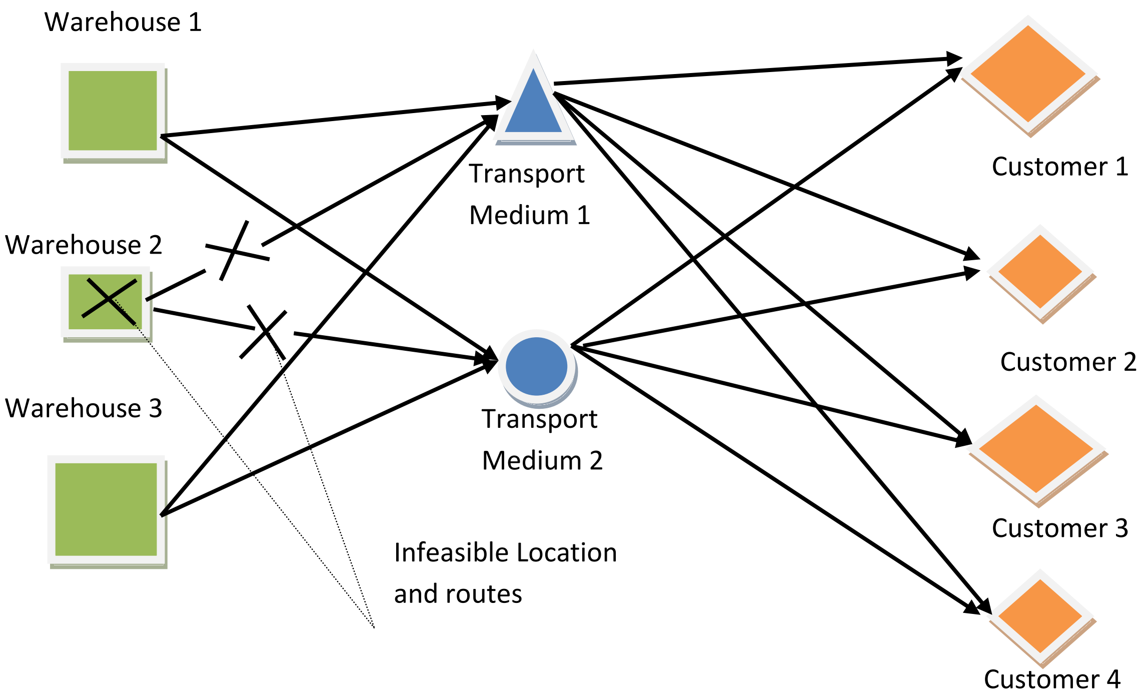

Figure 1. Schematic representation of FLP and FCSTP

2. Mathematical Formulation And Structure for FCSLTP

As indicated earlier, the FCSLTP is formulated as a mixed integer problem, with m- sources, n- destinations, k-conveyances. We have adapted the FCSTP model as presented by Sanei et al. (2017) to include the fixed cost of facility location which is seemingly not present in his formulation. The location cost is necessary to build an integrated model of various fixed cost planning horizons.

2.1 Assumptions for model development of FCSLTP

The following assumptions were considered in our model:

1. Deterministic input .
2. One stage or two echelon problem.
3. Fixed location cost and fixed chare route cost.
4. One Planning period and single item distribution problem.

a. Parameters For Model Formulation:

Below are the optimization parameters and variables used in the model formulation.

Deterministic parameters
i: Index for sources or location (warehouses, depots etc.)
j: Index for destinations (customers, other warehouses etc.)
r : Index for conveyances (or Transportation mediums)
m : Number of sources
n : Number of destinations
a : Number of conveyances
c_{ijr} : Unit cost of shipment on route (i, j) using conveyance r.

S_i : Capacity for source i
D_j : Demand for Destination j
T_r : Conveyance capacity based on the conveyance r
F_i : Fixed charge location cost
H_{ijr} : Fixed charge cost on shipping through route (i, j) based on the conveyance r .

Decision Variables:
x_{ijr} : Quantity of products transported from source (i) to destination(j) using conveyance (k)
y_i : Location variable for selecting sources
y_{ijr} : Fixed charge variable in selecting which conveyance is utilized on the route (i)

Objective Function:
Original Problem (OP)
Min (OP):

$$\sum_{i=1}^{m} F_i\, y_i + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} c_{ijr}x_{ijr} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} H_{ijr}y_{ijr} \qquad (1)$$

Subject to

$$\sum_{j=1}^{n}\sum_{r=1}^{a} x_{ijr} \leq S_i y_i \qquad \forall\ i = 1 \dots m \qquad (2)$$

$$\sum_{i=1}^{m}\sum_{r=1}^{a} x_{ijr} = D_j \qquad \forall\ j = 1 \dots n \qquad (3)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijr} \leq T_r \qquad \forall\, r = 1 \dots a \qquad (4)$$

$$x_{ijr} \leq M_{ijr}\, y_{ijr} \qquad \forall\ i = 1 \dots m, \qquad \forall\ j = 1 \dots n,\ \ \forall\ r = 1 \dots a \qquad (5a)$$

M_{ijr} = min (S_i, D_j, T_r)

$$y_{ijr} = \begin{cases} 1 & x_{ijr} > 0 \\ 0 & Otherwise \end{cases} \qquad (5b)$$

$$x_{ijr} \geq 0 \qquad (6)$$

$$y_i = 0\ or\ 1\ , \quad y_{ijr} = 0\ or\ 1 \qquad (7)$$

Equation 1(a) is the objective function. The first term is the facility location cost, the second term is the route unit cost per conveyance type and the third term is the route fixed charge cost per conveyance type. Equation (2) is the supply capacity constraint of each location or sources. Equation (3) is the demand constraint to be met. Equation (4) is the conveyance capacity constraint. Equation (5a and 5b) route fixed charge requirement constraints. Equation (6) refers to the non-negativity constraint for the continuous variables and Equation(7) refer to the binary integer constraints.

b. Lower bound formulation of FCSLTP

As noted in earlier sections of this paper, the LRH works through the Lagrangian relaxation of some difficult constraints in the OP that makes it difficult to solve. Often, these constraints consist the integer constraints such as y_{ijr} and y_i in the OP above. These constraints make the optimization problem NP- hard in nature. We have selected the constraints (2) and (5a) to apply the Lagrangian multipliers ($\lambda_i\ i.e \sum_i^m \lambda_i$ and $\beta_{ijr}\ \ i.e \sum_{i=1}^m \sum_{j=1}^n \sum_{r=1}^a \beta_{ijr}$) respectively. These constraints have been shown by Cornuéjols et al. (1991) to give a strong lower bound though it may be computationally intensive. The Lagrangian multipliers used are such that each $\lambda_i \geq 0$ and each $\beta_{ijr} \geq 0$ in like fashion as Sanei et al. (2017). The Lagrangian Relaxation of the Original Problem (LR of OP) is given below:

LR of OP (λ, β) =
Minimize

$$\sum_{i=1}^{m} F_i\, y_i + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} c_{ijr} x_{ijr} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} H_{ijr} y_{ijr} + \sum_{i=1}^{m} \lambda_i \left(\sum_{j=1}^{n}\sum_{r=1}^{a} x_{ijr} - S_i y_i \right) +$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} \beta_{ijr} (x_{ijr} - M_{ijr}\, y_{ijr})$$

= Minimize

$$\sum_{i=1}^{m} (F_i - \lambda_i S_i) y_i + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} (c_{ijr} + \beta_{ijr})\, x_{ijr} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} (H_{ijr} - \beta_{ijr} M_{ijr}) y_{ijr} +$$

$$\sum_{i=1}^{m} (\lambda_i * \sum_{j=1}^{n}\sum_{r=1}^{a} x_{ijr}) \qquad (8)$$

Subject to
Constraints (3), (4),(5b) (6) and (7)

The Lagrangian relaxation of the Original problem i.e. LR of OP (λ, β) is decomposed into two sub-problems (SP1 and SP 2).

First Sub-problem i.e. SP1 of OP (λ, β) :
Minimize

$$\sum_{i=1}^{m} (F_i - \lambda_i S_i) y_i + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} (H_{ijr} - \beta_{ijr} M_{ijr}) y_{ijr} \qquad (9)$$

Subject to

$$y_i = 0 \text{ or } 1 \ , \quad y_{ijr} = 0 \text{ or } 1 \qquad (10)$$

Second Sub-problem i.e. SP2 of OP (λ, β):
Minimize

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} (c_{ijr} + \beta_{ijr})\, x_{ijr} + \sum_{i=1}^{m} (\lambda_i * \sum_{j=1}^{n}\sum_{r=1}^{a} x_{ijr}) \qquad (11)$$

Subject to

$$\sum_{i=1}^{m}\sum_{r=1}^{a} x_{ijr} = D_j \qquad \forall\ j = 1 \ldots n \qquad (12)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijr} \leq T_r \qquad \forall\, r = 1 \ldots a \qquad (13)$$

$$x_{ijr} \geq 0 \qquad (14)$$

We note that SP2 of OP is a linear programming (LP) problem and could be easily solved to optimality using a general purpose solver such as CPLEX .
For SP1 of OP which is a pure integer programming problem, we note two scenarios below which can lead to arriving at the optimal values $y_i^* \text{ and } y_{ijr}^*$

Scenario 1: When $(F_i - \lambda_i S_i) < 0$ and $(H_{ijr} - \beta_{ijr} M_{ijr}) < 0$, these imply negative values, therefore
The best minimum will be arrived at when $y_i^* = 1 \text{ and } y_{ijr}^* = 1$ respectively.
Scenario 2: When $(F_i - \lambda_i S_i) > 0$ and $(H_{ijr} - \beta_{ijr} M_{ijr}) > 0$, this imply positive values, therefore,
The best minimum will be arrived at when $y_i^* = 0 \text{ and } y_{ijr}^* = 0$ respectively

To arrive at the LRH Lower bound , we follow the procedure below ;

1. Compute SP2 of OP to generate the optimal x^*_{ijr} and $SP2^*$
2. For SP1 of OP, For all $i = 1 \dots m$, if $(F_i - \lambda_i S_i) < 0$ then $y^*_i = 1$
   Else $y^*_i = 0$
3. For SP1 of OP, For all $i = 1 \dots m, j = 1 .. n, r = 1 \dots a$, if $(H_{ijr} - \beta_{ijr} M_{ijr}) < 0$ then $y^*_{ijr} = 1$
   Else $y^*_{ijr} = 0$
4. Compute the optimal value for SP1 of OP i.e. $SP1^*$

   Where $SP1^* =$
   Minimize:
   $$\sum_{i=1}^{m} (F_i - \lambda_i S_i) y_i^* + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} (H_{ijr} - \beta_{ijr} M_{ijr}) \; y_{ijr}^* \qquad (15)$$

5. The LB is given as $SP1^* + SP2^*$

c. Upper bound formulation of FCSLTP

In the solution of SP1 of OP to arrive at $SP1^*$ according to equation 15, since the x^*_{ijr} values are not directly considered in selection of the y^*_i and y^*_{ijr} some possible contradictions or infeasibilities would have to be resolved or perturbed through. This will ensure feasibility as noted by Fisher (1981) and also Implemented by Sanei et al. (2017).The resolution of the contradictions are used in generating an upper bound to be used in the LRH.
These possible six (6) contradictions are identified and given below;

1. Given that $x^*_{ijr} > 0$, $y^*_i = 0$ and $y^*_{ijr} = 0$
2. Given that $x^*_{ijr} > 0$, $y^*_i = 0$ and $y^*_{ijr} = 1$
3. Given that $x^*_{ijr} > 0$, $y^*_i = 1$ and $y^*_{ijr} = 0$
4. Given that $x^*_{ijr} = 0$, $y^*_i = 1$ and $y^*_{ijr} = 1$
5. Given that $x^*_{ijr} = 0$, $y^*_i = 0$ and $y^*_{ijr} = 1$
6. Given that $x^*_{ijr} = 0$, $y^*_i = 1$ and $y^*_{ijr} = 0$

The following procedure can be used in resolving the contradictions

For all $i = 1 \dots m$, (y^*_i) and For all $i = 1 \dots m, j = 1 .. n, r = 1 \dots a$, (y^*_{ijr})

1. If $x^*_{ijr} > 0$, and $y^*_i = 0$ and $y^*_{ijr} = 0$ then set $y^*_i = 1$ and $y^*_{ijr} = 1$
2. If $x^*_{ijr} > 0$, and $y^*_i = 0$ and $y^*_{ijr} = 1$ then set $y^*_i = 1$
3. If $x^*_{ijr} > 0$, and $y^*_i = 1$ and $y^*_{ijr} = 0$ then set $y^*_{ijr} = 1$
4. If $x^*_{ijr} = 0$, and $y^*_i = 1$ and $y^*_{ijr} = 1$ then set $y^*_i = 0$ and $y^*_{ijr} = 0$
5. If $x^*_{ijr} = 0$, and $y^*_i = 0$ and $y^*_{ijr} = 1$ then and $y^*_{ijr} = 0$
6. If $x^*_{ijr} = 0$, and $y^*_i = 1$ and $y^*_{ijr} = 0$ then set $y^*_i = 0$

Using the values of y^*_i , y^*_{ijr} and x^*_{ijr} in equation (1) above and after resolving any infeasibilities as indicated above , we arrive at our upper bound (UB)
UB of OP =
$$\sum_{i=1}^{m} F_i \, y^*_i + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} c_{ijr} x^*_{ijr} + \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{r=1}^{a} H_{ijr} y^*_{ijr} \qquad (16)$$

3 Lagrange Relaxation Heuristic Procedure

The UB of OP and LR of OP (λ, β) are the first requirements for the computation of this heuristic procedure. Following this is the sub gradient optimization method which is widely used in determining the necessary

Lagrange multipliers for the iterations. The parameters required for the termination procedure and sub gradient optimization are listed below;

1. A value (ε) that is user determined (or pre-specified) for algorithm termination. It is usually small sized positive number such that$(UB_{Best})-(LB_{Best}) \leq \varepsilon$. The term UB_{Best} refers to the best Upper Bound (UB) and while LB_{Best} is the best Lower Bound (LB).
2. Step size for Lagrange multipliers (λ *and* β) generation is given as θ^t. The symbol t refers to the iteration number.

$$\theta^t = \frac{\pi\,[(UB_{Best})-(LB_{Best})]}{\sum_i^m(\sum_{j=1}^n\sum_{r=1}^a x_{ijr}^t - S_i{y_i}^t)^2 + \sum_{i=1}^m\sum_{j=1}^n\sum_{r=1}^a(x_{ijr}^t - M_{ijr}\,y_{ijr}^t)^2}$$

3. A value (δ) is chosen from the interval (0, 2) as normally used in the sub-gradient procedure. When $t \geq t_{max}$, $= \delta/2$. The t_{max} is the number of maximum iterations allowed per δ used. We have in this paper also included a termination condition of $\delta = 0$ for the LRH.
4. $UB_{initial}$, $LB_{initial}$ are the initial values for the Upper Bound and Lower bound respectively.
5. λ_i^t and β_{ijr}^t refer to the Lagrange multipliers at the iteration number t. The initial Lagrange multiplier chosen are $\lambda_i^t = \lambda_i^*$ and $\beta_{ijr}^t = \beta_{ijr}^*$

Iterative steps for LRH:

Step 1 Initialize using the parameters (ε , t, t_{max} , $\lambda_i^t = \lambda_i^*$, $\beta_{ijr}^t = \beta_{ijr}^*$, $UB^t = UB_{initial}$, $LB^t = LB_{initial}$, $\delta = 2$)
$(LB\ of\ OP)^t = (LB\ of\ OP)^* = SP1^* + SP2^*$
$(UB\ of\ OP)^t = (UB\ of\ OP)^*$
Step 2 Solve $(LB\ of\ OP)^t = SP1^t + SP2^t$
$LB_{Best} = max\,[(LB\ of\ OP)^t,\ LB^t, 0\,)]$
Step 3 Find a feasible solution to the Upper bound from $(LB\ of\ OP)^t$ i.e. $(UB\ of\ OP)^t$
$UB_{Best} = min\,[(UB\ of\ OP)^t, \quad UB^t)]$
Step 4 if $(UB_{Best}) - (LB_{Best}) \leq \varepsilon$ Terminate the heuristic. Else
Step 5 Update the Lagrange Multipliers

$$\lambda_i^{t+1} = \lambda_i^t + \theta^t\left(\sum_{j=1}^n\sum_{r=1}^a x_{ijr}^t - S_i{y_i}^t\right) \qquad \forall\ i \in m$$

$\lambda_i^{t+1} = Max\,(\lambda_i^{t+1}, 0)$
$\beta_{ijr}^{t+1} = \beta_{ijr}^t + \theta^t\,(x_{ijr}^t - M_{ijr}\,y_{ijr}^t) \qquad \forall\ i \in m,\ j \in n,\ r \in a$
$\beta_{ijr}^{t+1} = Max\,(\beta_{ijr}^{t+1}, 0)$
Step 6 if no improvement in LB_{Best} at $t \geq t_{max}$, then $\delta = \delta/2$
set $t = 0$ (termination and restart condition)

Step 7 if $\delta = 0$ (we terminate the Heuristic and select UB_{Best})
Step 8 Else $t = t + 1$ (Go to Step 2).

3.1 Numerical Computation

In order to show the workings of the solution method of this nature, data is usually randomly generated for various instances of problem sizes following some known probability distribution and coded using some programing languages. We however, present a hands-on example of the Lagrange relaxation heuristic to further explain the procedures and observations to note when performing such computations.
Our generic model problem size consists of (m) location binary variables, $(m \times n \times a)$ shipment continuous variables and $(m \times n \times a)$ fixed charge binary variables. Furthermore, we have used a standard general purpose optimizer software such as IBM CPLEX version 12.8 which is a enabled with a default Mixed Integer Linear Programming (MILP) solver to compare with the solution obtained with the LRH.

3.1.2 Data generation

The data parameters utilized are of two types. These are the parameters for the FCSLTP and that used in running the LRH. Part of our interest in this paper, lies in showing the workings of LRH for good comprehension of the user. Therefore we have included a small sized sample problem that fits the model parameters. The following values have been used for the Lagrange Heuristic parameters:

$\varepsilon = 0.01$, $UB_{initial} = +\infty$, $LB_{initial} = -\infty$, , $t_{max} = 5$, $t = 0$

3.1.3 Numerical example:

Number of sources $m\ (1 \dots i) = 3$
Number of destinations $n\ (1 \dots j) = 2$
Number of Conveyances $a\ (1 \dots r) = 2$

Table 1: Location fixed charges, supply and demand capacities, Unit cost per quantity shipped per conveyances

			r= 1		r= 2	
i	F_i	S_i	c_{ij1}		c_{ij2}	
1	150	25	1	3	3	2
2	250	30	2	2	2	1
3	200	40	2	1	1	3
D_j			20	15	20	15
T_r			10		25	

Table 2: Route fixed charges per conveyances

	r= 1		r= 2	
i	H_{ij1}		H_{ij2}	
1	6	4	8	6
2	8	6	6	8
3	6	8	6	4
j	1	2	1	2

3.1.4 Numerical solution

To begin the LRH solution, the initial values for the lagrange multipliers $\lambda_i^t = \lambda_i^*$ and $\beta_{ijr}^t = \beta_i^*$ need to be determined. From equation (15) and using our non-negativity constraints for the Lagrange multiplier, we observe that the values of λ_i^* to arrive at a minimum value of the objective function can either be zero or a positive value of magnitude sufficiently greater than $\frac{F_i}{S_i}$. For our numerical example computation, we used $\lambda_i = 0$ $(\forall i = 1 \dots 3)$ for the first LRH trial run and $\lambda_i^* = (4 * \frac{F_3}{S_3}) = 20$ $(\forall i = 1 \dots 3)$ for the second LRH trial run.

Similarly for the second set of Lagrange multipliers i.e. β_{ijr}^*, and also using equation (15), we observe that β_{ijr}^* could either be fixed at zero (0) or a positive number of magnitude greater than $\frac{H_{ijr}}{M_{ijr}}$. For both the First and Second trial runs, we have used $\beta_{ijr}^* = 1$ $(\forall i = 1 \dots 3,\ j = 1 \dots 2,\ r = 1 \dots 2)$. This is because all the $\frac{H_{ijr}}{M_{ijr}}$ in our numerical example computations are positive values between 0 and 1.

In all the first and second runs for the LRH we have fixed $t_{max} = 5$ i.e. the maximum iteration number. The results can be seen in Tables 3 and 4 below. The symbol u in Table 3 and 4 refers to the count of iterations done irrespective of restarting the heuristic ($\delta = \delta/2$) as per the heuristic procedure.

Table 3: LRH computation for initial Lagrange values $\lambda_i^* = 0 \; and \; \beta_{ijr}^* = 1$(First trial run)

$\boldsymbol{u}$	**t**	**δ**	**θ**	$(\boldsymbol{LB\ of\ OP})^t$	$\boldsymbol{LB_{Best}}$	$(\boldsymbol{UB\ of\ OP})^t$	$\boldsymbol{UB_{Best}}$	$(\boldsymbol{UB_{Best}}) - (\boldsymbol{LB_{Best}})$
1	0	2		-4	0	655	655	655
2	1		0.95855	-15	0	507	507	507
3	2		1.19294	-76.4707	0	465	465	465
4	3		3.38182	-77.4707	0	465	465	465
5	4		3.38182	-77.4707	0	465	465	465
6	5		3.38182	-77.4707	0	465	465	465
restart								
7	0	1	1.6909	-80.1177	0	465	465	465
restart								
8	0	0.5	0.84545	-80.1177	0	465	465	465
restart								
9	0	0.25	0.422725	-80.1177	0	465	465	465
restart								
10	0	0.125	0.211363	-80.1177	0	465	465	465
restart								
11	41	0.0625	0.105681	-80.1177	0	465	465	465

Table 4: LH computation for initial Lagrange values $\lambda_i^* = 20 \; and \; \beta_{ijr}^* = 1$ (Second trial run)

u	t	δ	θ	$(LB\ of\ OP)^t$	LB_{Best}	$(UB\ of\ OP)^t$	UB_{Best}	$(UB_{Best}) - (LB_{Best})$
1	0	2		-604	0	655	655	655
2	1		0.95273	-5	0	263	263	263
3	2		3.50667	-148.909	0	551	263	263
4	3		0.50095	-232	0	527	263	263
5	4		0.50095	-236.523	0	327	263	263
6	5		10.52	-1482.48	0	463	263	263
restart								
7	0	1	0.55368	-80.1177	0	465	263	263
restart								
8	0	0.5	0.27684	-80.1177	0	465	263	263
restart								
9	0	0.25	0.13842	-80.1177	0	465	263	263

We note in the LRH results shown in Table 3 above that $(LB\ of\ OP)^t$ and LB_{Best} failed to improve after the count $u = 6$, for which $t = 5$ (t_{max}). Hence we restarted the solution. Unfortunately, no better Lower bound solution was obtained on using the sub-gradient rule of $\delta = \frac{\delta}{2}$. Similarly in Table 4 above, LB_{Best} values failed to increase above zero(0) with the values of $(UB_{Best}) - (LB_{Best})$ becoming constant as both δ and θ reduced significantly, tending towards zero(0). The LRH search iteration was terminated in both the first and second trial runs using the condition that $\delta = 0$.

4 Discussion of Solutions Obtained

The Original Problem was coded into IBM CPLEX version 12.8 and solved with the default Mixed Integer Linear Programming (MILP) Solver. Table 5 below shows the results obtained for the decision variables under the different solution methods. The variables not shown were equal to zero (0) in the solution obtained. It can be well observed in Table 5 below that the LRH for the second trial run gave the lowest value of 263. The value (263) was obtained only due to the violation of one of the constraints used for the Lagrange Relaxation. This is constraint (5a) i.e. $x_{ijr} \leq M_{ijr}\, y_{ijr}$. The violation was at $x^*_{312} = 20$ and such that $x^*_{312} > (M_{312} = 15)$. This violation was most likely due to the non-inclusion of possible infeasibility contradiction of the term $x^*_{ijr} > M_{ijr}\, y_{ijr}$ for $x^*_{ijr} > 0$ under Upper bound formulation of FCSTLP of Section 2.4. By the infeasibility of constraint (5a), a lower bound was obtained. In addition, the different LRH values of λ^*_i and β^*_{ijr} used as the starting solution for the lagrange multipliers show how these values could help in the final lower bound or optimal solution reached by the heuristic.
A simple infeasibility resolution of the lower bound solution obtained by LRH (second trial) to satisfy constraint (5a) while also satisfying demand constraints would result into the value obtained by the CPLEX values.

Table 5: Decision variables result (O.P.)

Method	Solver Characteristic	$Min\ (O.P.)$	x_{ijr}	y_i	y_{ijr}
CPLEX	Default MILP solver	**284**	$x_{311} = 5$ $x_{321} = 5$ $x_{312} = 15$ $x_{322} = 10$	$y_1 = 0$ $y_2 = 0$ $y_3 = 1$	$y_{311} = 1$ $y_{321} = 1$ $y_{312} = 1$ $y_{322} = 1$
LRH (First run) Lower bound	$\lambda^*_i = 0\ and\ \beta^*_{ijr} = 1$	**465**	$x_{111} = 10$ $x_{212} = 10$ $x_{222} = 15$	$y_1 = 1$ $y_2 = 1$ $y_3 = 0$	$y_{111} = 1$ $y_{212} = 1$ $y_{222} = 1$
LRH (Second trial) Lower bound	$\lambda^*_i = 20\ and\ \beta^*_{ijr} = 1$	**263**	$x_{321} = 10$ $x_{312} = 20$ $x_{322} = 5$	$y_1 = 0$ $y_2 = 0$ $y_3 = 1$	$y_{321} = 1$ $y_{312} = 1$ $y_{322} = 1$
Resolved LRH (Second trial) **Infeasibility**	$\lambda^*_i = 20\ and\ \beta^*_{ijr} = 1$	**284**	$x_{311} = 5$ $x_{321} = 5$ $x_{312} = 15$ $x_{322} = 10$	$y_1 = 0$ $y_2 = 0$ $y_3 = 1$	$y_{311} = 1$ $y_{321} = 1$ $y_{312} = 1$ $y_{322} = 1$

Values of x_{ijr}, y_{ijr} not shown in the table equals 0 in the solution.

5 Conclusion And Future Direction

The use of Lagrangian Relaxation Heuristic (LRH) in solving facility location and fixed charge solid transportation problem has been considered in this paper. The lower bound solution obtained for the LRH (second trial) was better compared to the value obtained for LRH (First trial) when the respective values obtained were compared to the CPLEX values. The results of the LRH could still result in a lower bound as was the case of the LRH(second trial) if possible infeasibility contradictions of the upper bound formulations are not well captured. For the LRH (second trial), a careful perturbation to satisfy one of the violated constraints would result in the CPLEX values obtained. In order to further test the performance of the LRH compared to the values obtainable from standard optimization software, extensive computations for various problem sizes and instances still have to be considered. It is also worth to note that a structured perturbation technique could be employed to resolve the possible LRH lower bound infeasibility. Furthermore during the LRH process, metaheuristics such as genetic algorithm, simulated annealing could be used to search for a better upper bound obtained, in case the solution fails to improve before the terminating conditions are reached.

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Biographies

Gbeminiyi John Oyewole : Is currently a doctoral student in the Department of Industrial and Systems Engineering at the University of Pretoria, South Africa. He obtained his Master's degree in Industrial and Production Engineering at the University of Ibadan, Nigeria. He has worked in the field of Logistics, transportation and as a supply chain officer at the Candel Agrochemical Plant in Lagos Nigeria. He is also an UNLEASH award winner in the category of sustainable production and consumption, whose team was able to design sustainable packaging materials for transportation using pallets. His research interests are in Facility location problems, applied optimization, operations research, supply chain and engineering Designs.

Olufemi Adetunji: is currently a senior lecturer at the Department of Industrial and Systems Engineering at the University of Pretoria, South Africa. He obtained his PhD at the same University. He has performed various research work with National Research Foundation (NRF) South Africa and with other leading governmental and private agencies within South Africa. He has also published a lot of articles in leading local and International Journals. His research Interests are in Supply Chain design and Engineering , Lean Manufacturing and applied optimization.