A Novel Mathematical Programming Approach for Aggregate Proportioning: A Case Study for Highway Construction

Anil Kumar Agrawal
Mechanical Engineering Department, Indian Institute of Technology (BHU)
VARANASI – 221 005, INDIA
bhu.anil@gmail.com

Devendra Mohan
Civil Engineering Department, Indian Institute of Technology (BHU)
VARANASI – 221 005, INDIA
devendra.civ@iitbhu.ac.in

Abstract

Durability and serviceability of a highway can be ensured by use of proper quality materials and then, mixing them in right proportion. The present work is a case study on proportioning of aggregate materials for construction of surface course of highways. Most of the times, the aggregates available for mixing have particle size grading that, if used, will not meet all of the specifications. In such situations, it becomes very difficult to determine a right proportioning of aggregates that will have the minimum amount of deviation from such specifications. Existing methodologies for the proportioning do not ensure the best solution from this viewpoint. To overcome such difficulties, the aggregate proportioning problem has been formulated as a goal-programming model. The approach described in this paper is equally applicable for proportioning of the aggregates for some of the other layers of a highway.

Keywords
Goal-programming, Surface transportation, Highway construction and Aggregate proportioning.

1. Introduction

Development of infrastructure, particularly surface transportation, is one of the most important steps leading to overall progress and growth of any nation. Transportation contributes to economic and industrial development of any region. Transport facilities are required as passengers and various types of commodities need be transported from place to place. The transportation is also essential for rapid strategic movement in case of emergencies caused due to external or internal reasons and/or for effective maintenance and enforcement of law and order within the country. It contributes to national integration by restraining the narrow feelings of sectionalism and regionalism and checks the ugly development of slums by serving to avoid the concentration of population at low socio-economic echelon in and around the large urban areas. Government of India, having realized this fact, laid much greater emphasis on the development of surface transportation routes of high quality. Development of Golden Quadrilateral, and North-South and East-West corridors is the proof of the same.

Therefore, transportation facilities can be seen not only to affect economic growth of the nation, but also to have tremendous impact on social and cultural development of any country. India, being a vast country and very much underdeveloped when compared to other developed countries, has so far not been able to develop the road system upto the desired level. It is an established truth that greater industrial development is possible only if the road network is well planned, designed and developed. With efficient road network, industries even having small capital may get favourable conditions for their development. At present, all the major industries are concentrated along seaports or railheads where transport facilities for bulk carriers are available. Such places are getting crowded with the passage of time, creating biospheric pollution (Papacostas 1987, Chandola 2001) and deterioration in the living standards of its inhabitants. Motorways, known as expressways, can serve as an efficient means of fast communication between adjacent cities providing safety and comfort during the journeys.

In the process of overall development of the country, the introduction and increase in the number of motor vehicles have, on one hand, revolutionized our lives and brought comfort, pleasure and convenience. On the other hand, such a development has brought new evils such as traffic congestion, reduced level of road-travel safety and environmental hazards in the form of raised levels of biospheric pollution, which are also partly due to ill-planning of roads and highways as well as poor management of traffic.

Surface transportation involves a number of important national and international activities starting from the construction of village roads, other district roads, major district roads, state highways to national highways, and finally reaching the level of the execution of projects involving transnational highway construction. Since this kind of construction needs massive capital outlay, it is desired that they last for long and during that period, they provide high level of service.

1.1 Requirements for Highway Pavements

The surface of the road should be stable and non-yielding to allow the heavy wheel loads of road traffic to move with least possible rolling resistance. The road surface should also be even along the longitudinal profile to enable the fast vehicles to move safely and comfortably at the design speed. The unevenness and undulations of the surface, along the longitudinal profile of the highway, cause vertical oscillations in fast moving automobiles, increase in the fuel consumption and wear and tear of the vehicle components, all resulting in a considerable rise in the vehicle operation cost. Apart from this, uneven pavement surface causes discomfort and fatigue to the passengers of fast moving vehicles. Therefore, in order to provide a smooth and even surface for the traffic, the carriageway is provided with a suitably designed and constructed pavement structure.

Pavements are generally classified into two categories: flexible pavement and rigid pavement. A typical flexible pavement consists of four layers: subgrade, sub-base course, base course and surface course. The pavement carries the wheel loads and transfers the stresses on a wider soil sub-grade area below. The reduction in the wheel load stress, due to the pavement, depends both on its thickness and the characteristics of the pavement layers. There will be a small amount of temporary elastic deformation even on a good pavement surface when heavy wheel loads are applied. One of the objectives of a well-designed and constructed pavement is, therefore, to keep this deformation of the pavement within the permissible limits so that the pavement can sustain a large number of repeated load applications during its design life.

Aggregates form the major portion of pavement structure and they are one of the prime materials used in pavement construction. Aggregates have to bear stresses induced due to the wheel loads on the pavement and on the surface course. They also have to resist wear due to abrasive action of traffic. Therefore, the properties of the aggregate mix used in road construction are of great significance to the highway engineers.

The aggregates are specified based on their size, shape, texture, etc. Size gradation is carried out by passing the aggregates through various sieves of successively decreasing sizes. The required aggregate sizes are chosen to fulfil the stipulated gradation. The grading for different type of roads has been specified by various agencies like American Society for Testing of Materials (ASTM), British Standards Institute (BSI), Bureau of Indian Standards (BIS) and Indian Road Congress (IRC). For the wearing course of a superior pavement, hard aggregates are preferred to resist the abrasive and crushing effects of heavy traffic loads and to resist adverse weathering effects (Khanna and Justo, 1991).

1.2 Bituminous Mixes for the Pavements

Desirable properties of a good bituminous mix are stability, durability, flexibility, skid resistance and workability (Khanna and Justo, 1991). Stability is a function mainly of friction and cohesion. If the volume of voids within the mix is restricted, its resulting strength properties improve. The minimum voids requirement for a given mix should be so selected as to provide space for necessary (i) densification that may develop under traffic movements and (ii) expansion of bitumen at high temperatures. In the absence of this, the bitumen bleeds over the surface and causes skidding.

Durability is defined as the resistance of the mix against weathering and abrasive actions. Flexibility is a property of the mix that measures the level of bending strength. Skid resistance is defined as the resistance of the finished pavement against skidding and is a function of surface texture and bitumen content. Workability is the ease with which the mix can be laid and compacted. It is the function of gradation of aggregates, their shape and texture, bitumen content and its type.

1.3 Design of the Bituminous Mixes

Mix design methods should aim at determining the right proportion of aggregates and bituminous material so as to impart the following properties to the mix.

1. Sufficient stability to satisfy the service requirements of the pavement and the traffic conditions without undue displacements
2. Durability by ensuring proper coating of the aggregates with sufficient bitumen for bonding of particles and waterproofing of mix
3. Sufficient voids in the compacted mix so as to provide a reservoir space for a slight additional compaction due to traffic and to avoid flushing, bleeding and loss of stability
4. Sufficient flexibility even in the coldest season to prevent cracking due to repeated application of traffic (wheel) loads
5. Sufficient workability for easy placing and effective compaction of the mix
6. Economy along with the required degrees of stability, durability and skid resistance of the pavement

The following steps need to be followed for a rational design of a bituminous mix (Khanna and Justo,1991; Wright and Dixcon, 2004):

1. <u>Selection of aggregates</u>: Aggregates that possess sufficient strength, hardness, toughness and soundness are chosen, keeping in view their availability and cost. Crushed aggregates and sharp or angular sands produce the mix of higher stability as compared to that of gravel and rounded sands.
2. <u>Determination of aggregate grading</u>: The properties of a bituminous mix, including the density and stability, are very much dependent on the aggregates and their grain size distribution. Most of the agencies and engineering organizations have specified the use of densely graded mixes. As higher size of aggregate gives higher stability, usually the biggest size aggregate is selected keeping in view the compacted thickness of the layer, provided all other factors are the same.
3. <u>Determination of specific gravity:</u> The specific gravity of aggregates is expressed as bulk, apparent or effective specific gravity.
4. <u>Proportioning of the aggregates:</u> One of the most important factors that strongly influences the structural properties of a bituminous pavement mix for highway construction is aggregate proportioning (Talbot and Richart 1923, Nijboer 1948, Fuller and Thompson 1969, Horsfield 1992, Sharma and Rao 1992). For this, the design grading is decided based on the type of construction work, thickness of the layer and availability of the aggregates. Then, the available aggregates are proportioned employing analytical, graphical or trial and error methods.
5. <u>Preparation of specimen:</u> The preparation of specimen depends on the stability test method used. Hence, the size of the specimen, compaction and other specifications should be followed as recommended in the selected

stability test method. The stability test methods, which are in common use for mix-design, are based on the procedures developed by Marshall, Hubbard-Field and Hveem (Khanna and Justo, 1991).

6. Determination of specific gravity of compacted specimen: The specific gravity of compacted specimen, as moulded, is determined.

7. Stability test on compacted specimens: Based on the selected design method, a stability test (Khanna and Justo, 1991) is carried out.

8. Selection of optimum bitumen content: The optimum bitumen content is selected based on the stability test method adopted and design requirements considered.

The present paper addresses the issues related to the problem mentioned earlier in Step (d) of bituminous mix design. The proportioning of the aggregates is carried out to satisfy the specifications laid out to meet the requirements in respect of economy, workability, strength and durability (Knight and Knight 1948, Hatherly and Leaver 1967, AASHO 1950).

2. The Proportioning Problem Formulation

The aggregate proportioning problem discussed here is based on a practical case of highway construction by a State Government agency of Uttar Pradesh in India. For the construction, four different aggregates were available. The problem was the determination of the proportion in which they were required to be mixed to meet the laid out specifications on particle size gradation as shown in Table 1.

For the above purpose, one samples drawn from each of the aggregates was subjected to sieving. Experimentally determined grading results were as shown in Table 2. From these results, it can be noticed that none of the tested samples could independently match the laid out specifications.

Table 1. Specified gradation for the aggregates (Khanna and Justo, 1991)

CLASS	SIEVE SIZE	% FINER RANGE
1	11.2 mm	88-100
2	5.6 mm	42-64
3	2.8 mm	22-38
4	710 µm	11-24
5	355 µm	7-18
6	180 µm	5-13
7	90 µm	3-9

Table 2. Gradation results for the four samples.

Class	% Finer				
	Range Specified	Obtained in the Sample			
		1	2	3	4
1	88-100	96.60	100.00	100.00	100.00
2	42-64	19.65	86.70	99.20	100.00
3	22-38	3.40	52.25	80.70	98.60
4	11-24	0.00	5.80	15.90	26.20
5	7-18	0.00	2.35	7.40	14.80
6	5-13	0.00	1.30	4.50	8.00
7	3-9	0.00	0.55	2.60	4.10

From the above information about size gradation of the four aggregate samples, a mathematical programming formulation of the proportioning problem was carried out to determine the proportion in which the four samples should be mixed to get a final aggregate-mix that will be closest to the standards shown in Table 1. Let,

X_1 = Proportional fraction of sample 1,
X_2 = Proportional fraction of sample 2,
X_3 = Proportional fraction of sample 3,
X_4 = Proportional fraction of sample 4,
U_i = Underachievement on lower side of the range specification on class i (i = 1, 2, …, 7),
V_i = Overachievement on higher side of the range specification on class i (i = 1, 2, …, 7).

Using the above notations, goal-programming formulation of the problem was carried out. Assigning unequal weights to the underachievement and overachievement on all the classes, the formulation will appear as given below.

Minimize $Z = (U_1 + U_2 + U_3 + U_4 + U_5 + U_6 + U_7) + (V_1 + V_2 + V_3 + V_4 + V_5 + V_6 + V_7)$ (1)

Subject to

$96.6X_1 + 100X_2 + 100X_3 + 100X_4 + U_1 \geq 88$ (2)
$96.6X_1 + 100X_2 + 100X_3 + 100X_4 - V_1 \leq 100$ (3)
$19.65X_1 + 86.7X_2 + 99.2X_3 + 100X_4 + U_2 \geq 42$ (4)
$19.65X_1 + 86.7X_2 + 99.2X_3 + 100X_4 - V_2 \leq 64$ (5)
$3.40X_1 + 52.25X_2 + 80.7X_3 + 98.6X_4 + U_3 \geq 22$ (6)
$3.40X_1 + 52.25X_2 + 80.7X_3 + 98.6X_4 - V_3 \leq 38$ (7)
$5.8X_2 + 15.9X_3 + 26.2X_4 + U_4 \geq 11$ (8)
$5.8X_2 + 15.9X_3 + 26.2X_4 - V_4 \leq 24$ (9)
$2.35X_2 + 7.4X_3 + 14.8X_4 + U_5 \geq 7$ (10)
$2.35X_2 + 7.4X_3 + 14.8X_4 - V_5 \leq 18$ (11)
$1.3X_2 + 4.5X_3 + 8X_4 + U_6 \geq 5$ (12)
$1.3X_2 + 4.5X_3 + 8X_4 - V_6 \leq 13$ (13)
$0.55X_2 + 2.6X_3 + 4.1X_4 + U_7 \geq 3$ (14)
$0.55X_2 + 2.6X_3 + 4.1X_4 - V_7 \leq 9$ (15)
$X_1 + X_2 + X_3 + X_4 = 1$ (16)
$X_1, X_2, X_3, X_4 \geq 0$ (17)
$U_1, U_2, U_3, U_4, U_5, U_6, U_7 \geq 0$ (18)
$V_1, V_2, V_3, V_4, V_5, V_6, V_7 \geq 0$ (19)

Objective function, expression (1) in the above formulation, aims at minimizing the sum of all the deviations from the specified specifications given in Table 1. Constraints (2) and (3) depict the requirements related to Class 1. Similarly, constraints (4) and (5) depict the requirements related to Class 2. These constraints, in pair, represent our requirements on percentage weight of the aggregate that should pass through the sieve corresponding to that class. For example, constraints (4) and (5) together would find deviation on the percentage weight of the aggregate that goes beyond 64 percent or less than 42 percent passing through the sieve of size 5.6 mm (Class 2). Constraints (6) through (15) similarly state the requirements as per Classes 3 to 7 of the specifications shown in Table 1. Equality (16) will help to determine proportions of the samples as a fraction of unit quantity. Non-negativity constraints (17), (18) and (19) simply state that no variable can take negative values.

3. The Solution

The above mathematical programming problem was solved on a personal computer using LINDO software. The results obtained are shown in Table 3.

Table 3. Solution to the linear programming formulation of the problem

VARIABLE	VALUE	VARIABLE	VALUE
X_1	0.637	X_3	0.000
X_2	0.000	X_4	0.363
U_1	0.000	V_1	0.000
U_2	0.000	V_2	0.000
U_3	0.000	V_3	0.000

U_4	1.485	V_4	0.000
U_5	1.625	V_5	0.000
U_6	2.095	V_6	0.000
U_7	1.511	V_7	0.000

From the solution given in the Table 3, it is obvious that the aggregate samples 2 and 3, for the present case, are of no use. The final product mix will require materials from samples 1 and 4 in the respective proportions of 63.68% (value of X_1) and 36.32% (value of X_4). Mixing samples 1 and 4 in the above proportion will result into a mix that will have aggregates in sizes as summarized in Table 4. From this table, it can be easily verified that the aggregate-mix determined would have particles of sizes and quantity quite close to those specified. Table 3 shows zero values for most of the deviational variables except for U_4, U_5, U_6 and U_7, which are at positive level. It will mean that the proportion–mix decided will have less of finer material and more of comparatively coarse aggregates. Since the large particles have a tendency to get reduced in size with the movement of traffic on the road, therefore, underachievement on higher classes, with particles of lower size, is not a so significant problem. With this in view, it is decided to prepare the aggregate-mix using the material from which samples 1 and 4 were drawn.

Table 4. Particle size gradation for optimal aggregate mix

CLASS	% FINER RANGE	
	Specified	**Obtained**
1	88-100	97.835
2	42-64	48.831
3	22-38	38.000
4	11-24	9.515
5	7-18	5.375
6	5-13	2.905
7	3-9	1.489

In case the specifications on the higher classes are to be strictly adhered to, appropriate additional quantities of some other aggregates in the corresponding size ranges may be taken to make up for the underachievement on the classes 4, 5, 6 and 7. It will help in attaining perfect compliance to the laid down specifications.

4. Sensitivity Analysis

The objective function, given by expression (1), does not differentiate between underachievements and overachievements while computing overall deviation from the given specifications. It was stated earlier that having of more content of the coarse particles is not all that serious as compared to that from finer ones. Assuming that underachievement is K% less prohibitive compared to overachievements, the expression (1) for the objective function is to be restated as

$$\text{Minimize } Z' = (1 - K/100)\,[(U_1 + U_2 + U_3 + U_4 + U_5 + U_6 + U_7)] + (V_1 + V_2 + V_3 + V_4 + V_5 + V_6 + V_7). \quad (20)$$

The constraints, in this new scenario, will not change and stay as such. Solving the problem with the modified objective function for varying values of K, the optimal solution was not found to change at all. Optimal mix and the deviations were found to be the same as are reported in Table 3. Looking at expressions for Z and Z', respectively from (1) and (21), it can be noticed that the coefficients associated with overachievements have become heavier compared to those associated with underachievements. Under this kind of change, with comparatively more weights, overachievement value will continue to remain as zero. With underachievements left in the expression, with each multiplied by a constant, is not going to change the nature of the objective function. Naturally, the optimal solution will remain to be the same.

5. Conclusions

In a number of civil engineering applications, mixing of particles on the basis of their size gradation has been applied in various major areas like structural engineering for building construction, transportation engineering for making certain layers of flexible pavements, environmental engineering for preparing filtration systems, geotechnical engineering to erect embankments, and in hydraulics and water resources engineering to build earthen dams. In addition to the few applications mentioned above, there are many others involving the use of materials graded on the basis of size characteristics to achieve certain properties like surface-smoothness, durability, porosity, filterability, strength, compaction, drainage etc.

In the present paper, a case study regarding aggregate proportioning has been discussed with reference to bituminous mix design problem for highway construction. The approaches employed currently, like trial and error or graphical methods, consume more time and still do not necessarily yield the best solution. Many times, the solution may not be even possible from the application of these conventional techniques or these may yield the results showing large deviations from the specifications. Goal programming approach presented in this paper overcomes these difficulties and definitely ensures the best solution under the specified problem framework.

Versatility of this approach can be very effectively and successfully employed for solving a large number of problems of the similar nature in other areas also.

References

AASHO: American Association of State Highway Officials, Standard Specifications for Highway Materials and Methods of Sampling and Testing, Part II: Methods of Sampling and Testing, 6th Edition, p 262, 1950.
Chandola, S.P., *A Text Book of Transportation Engineering*, S Chand & Company Ltd., 2001.
Fuller, W.B. and Thompson, S.E., The laws of proportioning concrete, *Transactions of the American Society of Civil Engineers*, Vol. 59, p 67, 1969.
Hatherly, L.W. and Leaver, P.C., *Asphaltic Road Materials*, Edward Arnold Publishers Ltd., 1967.
Horsfield , H.T., The strength of asphalt mixes, *Journal of the Society of Chemical Industry*, 1992, p 107.
Khanna, S.K., and Justo, C.E.G., *Highway Engineering*, Nem Chand and Brothers, 1991.
Knight, B.H. and Knight, R.G., *Road Aggregates: Their Uses and Testing*, Edward Arnold Publishers Ltd., Vol 3**,** 1948.
Nijboer, L.W., *Plasticity as a Factor in the Design of Dense Bituminous Road Carpets*, Elsevier Publishing Company, 1948.
Papacostas, C.S., *Fundamentals of Transportation Engineering*, Prentice-Hall Inc., 1987.
Sharma, R.P., and Rao, S.K., Development of a Basis for the Design of Aggregate Gradings for Asphalt Paving Mixes, *Institution of Engineers (India) Journal- Civil Engineering*, Vol. 73, p 104, 1992.
Talbot, A.N., and Richart, F.E., *The strength of Concrete: Its Relation to Cement, Aggregate and Water*, University of Illinois Engineering Experiment Station Bulletin*,* Vol. 137, p 1, 1923.
Wright, P. H. and Dixon, K.K., *Highway Engineering*, John Wiley & Sons (Asia) Pvt., Ltd., Singapore, Seventh Edition, 2004.

Anil Kumar Agrawal is Professor serving the Department of Mechanical Engineering at Indian Institute of Technology (BHU) Varanasi, India. He earned his undergraduate degree in Mechanical Engineering from Moti Lal Nehru National Institute of Technology, Allahabad, India, and postgraduate and doctoral degrees from the Indian Institute of Technology Kanpur, India. He has papers published in journals, proceedings of the conferences and has edited books also. His areas of interest include Operations Research, Supply Chain Management, Simulation of Industrial Systems, and Lean and Six Sigma.

Devendra Mohan is Professor serving the Department of Civil Engineering at Indian Institute of Technology (BHU) Varanasi, India. He earned his undergraduate, postgraduate and doctoral degrees in Civil Engineering from the Indian Institute of Technology (BHU) Varanasi, India. He has papers published in journals and in proceedings of the

conferences. The main area of his interest include Air Pollution Control, Abatement of Automobile Exhaust Pollution, Microbiological Removal of Trace Toxins, Modelling of Environmental Systems and Environmental Ethics.