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Competing Suppliers under Sales-Rebate Contract and Price Sensitive Demand in a Decentralized Supply Chain

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Abstract

This paper studies a competition in a decentralized supply chain between two independent suppliers who sell their products to a common retailer, under sales−rebate contract, and wherein the demand is sensitive to retail prices. The inventory related operational costs are included in the chain based on the economic order quantity model. The conditions of existence and uniqueness of the retailer's optimal solution are discussed and found to be satisfied under symmetric chain parameters. However, the optimal wholesale price is found to exist and unique under symmetric optimal demand rates. In addition, due to the non-linearity of the inventory related costs, it is difficult to obtain a close theoretical solution form. The optimal demand rates and wholesale prices of the model are calculated numerically. The profit functions of the retailers and the suppliers are evaluated and the total Nash profit is compared to that of the integrated system. Furthermore, the impact of the inventory related costs is investigated numerically.

Keywords
Supply chain management, competition, sale−rebate, Nash solution, optimization chain

1. Introduction
Competition in supply chain management has been reported extensively in the literature of economics, where different contracts and scenarios have been studied. Most of these publications have focused only on the competition between retailers who order their products from a single supplier and compete on different types of decision variables, such as retail price, lead-time, order quantity, time service,… etc. The contracts that have been studied in supply chain management are the wholesale price contracts, the buyback contracts, the revenue–sharing contracts, the quantity–flexibility contracts, the quantity–discount contracts, and the sale−rebate contracts (see Cachon [1]). The common conclusion, from the study of these contracts, is the favor that the supplier achieves than the competing retailers. In actual globalized and competitive market, however, the retailer has the possibility to provide his products from different suppliers, in order to maximize his profits and to compensate the spoiled choices of the consumers. For that, the impact of competition between suppliers should be studied with the same importance as in retailers' case. Actually, few studies were focused on this subject. The only recent study that discusses the competition between two suppliers under different contractual forms is reported by Cachon and Kök [2]. In this work, the authors have studied three types of contracts: wholesale−price contract, quantity−discount contract, and two−part−tariff contract in a decentralized supply chain where the demand is deterministic and sensitive to retail price. However, they did not study the sale-rebate contract and its impact on the different chain performances. For that, the current paper focuses on the study of a competition between two independent suppliers who sell their products to a common retailer in a decentralized supply chain, under sales−rebate contract, and wherein the demand is sensitive to retail price. In addition, retailer's inventory related operational costs are included based on economic order quantity (EOQ) model [3]. As described by Taylor [4], two forms of sale–rebate contracts exist. The first one is called linear, in which the supplier offers channel rebate to the retailer for any product sold. This form of sale rebate contract was used by Nissan Company and the market of hardware [4].The second form is more restrictive, in which the channel rebate is conditioned by the setting of a target. This last type of contract was studied in several publications [4-5]. Its main objective is to incite the retailer to make effort to order a quantity more than the target. It is used in the fields of hardware, software, and auto industries [4]. This form of sale rebate contract will be the essence of our paper. The conditions of existence and uniqueness of the retailer's optimal solution and that of the

suppliers are characterized, however, due to the non-linearity of the inventory costs, it was difficult to obtain a close theoretical solution form. The optimal demand rates and wholesale prices of the model are calculated numerically. The profit functions of the retailers and suppliers are evaluated and the total Nash profit is compared to that of the integrated system. Furthermore, the impact of the inventory related costs is investigated numerically.

This paper is organized as follows. Section 2 presents our model and the optimization problems for the two suppliers and the retailer. Section 3 explores the condition of existence and uniqueness of the retailer's optimal solution. Section 4 studies the optimal decision for the suppliers. Section 5 presents the numerical results and the discussion.

2. Model Formulation

A schematic illustration of our model is given in Fig.1. It consists of a common retailer who buys two products from two competing suppliers S_i ($i=1,2$). Each supplier announces his payment scheme by offering his whole-sale price w_i ($i=1,2$), and his channel rebate u_i ($i=1,2$) (i.e., the amount paid by the supplier to the retailer for each sold unit beyond a target t_i ($i=1,2$)). The supplier S_i has ample capacity to satisfy any retailer demand and produces products at a constant production cost rate c_i ($i=1,2$). To avoid trivial setting, it is assumed that $0<c_i<w_i<p_i$, $u_i \geq 0$, and $t_i \geq 0$. The demand rate vector $d=(d_i,d_j)$ depends on retail price for the pair of products. This study is restrained to the linear form of demand, which satisfies for all $i,j \in \{1,2\}$, $\partial d_i(p)/\partial p_i<0$ and $\partial d_j(p)/\partial p_j \geq 0$. It can be expressed as $d_i(p_i,p_j)=a_i-\alpha_i p_i+\delta_{ij}p_j$ where a_i, α_i, and δ_{ij} ($\min(\alpha_i,\alpha_j)>\delta_{ij}$) are the base market potential from the supplier i, the sensitivity of the demand to the product i, and the sensitivity of the demand to product j, respectively. This linear form of demand was largely used in management literature [6-10]. For simplicity, the inverse form of the demand is used as $p_i(d_i,d_j)=\theta_i-\beta_i d_i-\gamma_j d_j$ where $\theta_i=(\alpha_j a_i+\delta_{ij}a_j)/(\alpha_i\alpha_j-\delta_{ij}\delta_{ji})$, $\beta_i=\alpha_j/(\alpha_i\alpha_j-\delta_{ij}\delta_{ji})$, $\gamma_j=\delta_{ij}/(\alpha_i\alpha_j-\delta_{ij}\delta_{ji})$, and $\beta_i>\gamma_j>0$ for all $i,j\in\{1,2\}$ (Cachon and Kök [2]). Note that the demand rates d_i and d_j will be different when $\beta_i=\beta_j=\beta$, $\gamma_i=\gamma_j=\gamma$, and $\theta_i \neq \theta_j$. As the retailer will profit from the sale-rebate contract by ordering large quantity, he will face an increase in the inventory related operational costs. In this model, the inventory costs that exist in the economic order quantity (EOQ) model is adopted (Hax and Candea [3]). In such case, the retailer's inventory related operational costs are given by $G_i(d_i)=K_i d_i^{\lambda}$ where $K_i=\sqrt{2k_i h_i}\geq 0$, k_i, h_i, and $\lambda=0.5$ denote the economics of scale, the cost per order quantity, the holding cost, and a coefficient, respectively. Let $R_i(d_1,d_2)=p_i(d_1,d_2)d_i$ be the revenue of the retailer from the selling of product i without considering the sale-rebate contract. The total retailer profit function is expressed as

$$\pi(d_1,d_2)=\sum_{i=1}^{2}\left((\theta_i-\beta_i d_i-\gamma_{3-i}d_{3-i}-w_i)d_i-K_i d_i^{\lambda}+u_i\max((d_i-t_i),0)\right). \tag{1}$$

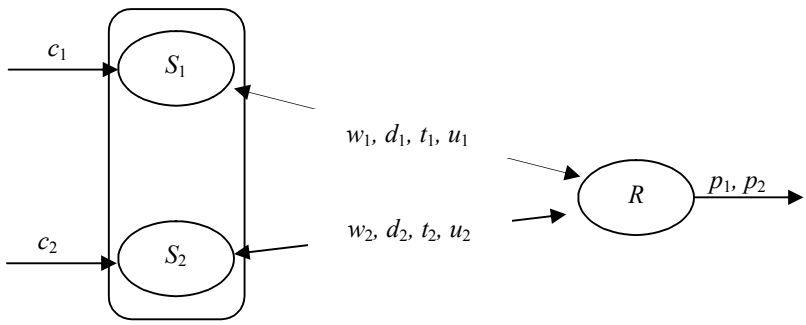

Figure 1: Model of a supply chain consisting of one retailer and two suppliers

This profit function can take four different forms based on the position of the demand rate from the two sides of the target. If a unique optimal solution exists, it will be localized in one of the four regions limited by the targets or at the boundaries. The position of the solution depends on the chain parameters. In the case where the two demand rates are less or equal to the targets given by the suppliers, the problem is equivalent to the wholesale contract. The

profit function of the supplier i, for $i \in \{1,2\}$, is given by

$$\Pi_i(d_1,d_2,w_i) = (w_i - c_i)d_i - u_i \max((d_i - t_i),0) \quad . \tag{2}$$

In the next section, the conditions of existence and uniqueness of the optimal solution are discussed by considering the concavity of the profit function of the retailer.

3. Retailer's Optimal Decision

Let $S_i(d_1,d_2)$ denote the first order derivative of the total retailer profit function with respect to d_i. For $i,j \in \{1,2\}$, it is expressed as

$$S_i(d_1,d_2) = \theta_i + u_i 1(d_i > t_i) - w_i - 2\beta_i d_i - \lambda K_i d_i^{\lambda-1} - (\gamma_i + \gamma_j)d_j, \tag{3}$$

where $1(A)$ denotes the indicator function which takes 1 if A is satisfied and 0 otherwise. Equation (3) depends on the on the sale-rebate u_i, wholesale price w_i, and independent of the targets t_i. Then, the position of the solution cannot be known from the two sides of the targets or at the boundaries. This random situation makes the problem difficult. In addition, the second order derivative of the retailer profit function is given by

$$\partial^2 \pi_i(d_1,d_2)/\partial d_i^2 = -2\beta_i + \lambda(1-\lambda)K_i d_i^{\lambda-2}. \tag{4}$$

It is worth to note that if $K_i > 0$, $\lim_{x\to 0^+} S_i(d_i,d_j) = \infty$. For that, it is optimal for the retailer and the system to carry both products. The condition of concavity of the profit function for the retailer is given by Lemma 1.

Lemma 1: For a given whole-sale price vector (w_i,w_j), $i,j \in \{1,2\}$, the retailer profit function is strictly concave on $\{(d_i,d_j), d_i.d_j > 0\}$ under the following conditions:

$$\beta_i = \beta_j = \beta\,,\ \gamma_i = \gamma_j = \gamma\,, \tag{5}$$

$$2R_i(d_1,d_2)/G_i(d_i) > [\beta_i/(\beta_i\beta_j - \gamma_i\gamma_j)]\, p_i/d_i\ . \tag{6}$$

Proof:

The retailer profit function depends on both d_i and d_j. Then, it is strictly concave if its Hessian is a negative definite matrix, which can be satisfied by the two following conditions:

(A) $\partial^2\pi/\partial d_i^2 < 0$ for $i \in \{1,2\}$,

(B) $\left|\partial^2\pi/\partial d_i^2\right| > \left|\partial^2\pi/\partial d_i \partial d_j\right|$ for $i,j \in \{1,2\}$ (i.e., the Hessian is strictly diagonally dominant).

First, $\partial^2\pi/\partial d_i^2 < 0$ can be translated to $2p_i d_i/G_i(d_i) > \lambda(1-\lambda)\beta_i^{-1} p_i/d_i$, which holds under (6) since $\beta(\beta^2-\gamma^2)^{-1} > \beta^{-1}$,and $\lambda^{-1}(1-\lambda)^{-1} = 4$. The condition given by (6) means that two times of the ratio between the revenue of the retailer without any contract and the inventory-related costs must be greater than the absolute value of the own price elasticity. This condition was developed by Bernstein and Federgruen [11] for decentralized retailers and then used by Cachon and Kök [2]. Second, as $\beta > \gamma$ in price equation, $\beta(\beta^2-\gamma^2)^{-1} > (2\beta - 2\gamma)^{-1}$ and $p_i d_i/G_i(d_i) > \lambda(1-\lambda)(2\beta-2\gamma)^{-1} p_i/d_i$ holds under (6).

The satisfaction of conditions given by (5) and (6) guarantees the existence of a uniqueness of the optimal demand rate (d_i^*, d_j^*) that maximizes the profit function of the retailer. However, this solution depends on the wholesale price. In addition, under positive economics of scale $K_i > 0$, the system of equation $S_i(d_1,d_2) = 0$ for $i \in \{1,2\}$ that gives (d_i^*, d_j^*) is non-linear, which requires a numerical resolution.

4. Competing suppliers' Optimal Decision

In this section, a competition between two suppliers, who offer sale-rebate to a common retailer, is studied. It is worth to note that the optimal demand rate that the retailer searches for depends on the wholesale prices. Differentiating the profit function Π_i with respect of w_i gives

$$\partial\Pi_i(w_1,w_2)/\partial w_i = (w_i - c_i - u_i)\,\partial d_i(w_1,w_2)/\partial w_i + d_i(w_1,w_2) \tag{7}$$

Its second order derivative gives

$$\partial^2\Pi_i(w_1,w_2)/\partial w_i^2 = (w_i - c_i - u_i)\,\partial^2 d_i(w_1,w_2)/\partial w_i^2 + 2\,\partial d_i(w_1,w_2)/\partial w_i \ . \tag{8}$$

The concavity of profit function of the supplier *i* depends on the sign of the first and second orders derivative of the demand rate d_i of product *i* on the wholesale price w_i. Normally with increasing w_i, the demand rates d_i decreases and d_j increases simultaneously. This statement will be proved in the next part of this paper and the conditions of concavity of the supplier profit function will be discussed.

Lemma 2: Under the conditions given by (5), (6), and symmetric optimal demand rates solution $d^* = d_i^* = d_j^*$, there exists a Nash (w_1^*, w_2^*) that satisfies for $i \in \{1,2\}$

$$d_i^* + (w_i - c_i - u_i)\,\partial d_i^*/\partial w_i = 0 \ . \tag{9}$$

Proof:

Under the conditions given by (5) and (6), the unique (d_i^*, d_j^*) exists and satisfies $S_i(d_i^*, d_j^*) = S_j(d_i^*, d_j^*) = 0$. The first order derivative of $S_i(d_i^*, d_j^*)$ and $S_j(d_i^*, d_j^*)$ on the wholesale price w_i gives the following system of equations

$$\begin{cases} \partial S_i(d_i,d_j^*)/\partial w_i = -1 + (-2\beta_i + \lambda(1-\lambda)K_i d_i^{*\lambda-2})\,\partial d_i^*/\partial w_i - (\gamma_j + \gamma_i)\,\partial d_j^*/\partial w_i = 0 \\ \partial S_j(d_i,d_j^*)/\partial w_i = 0 + (-2\beta_j + \lambda(\lambda-1)K_j d_j^{*\lambda-2})\,\partial d_j^*/\partial w_i - (\gamma_j + \gamma_i)\,\partial d_i^*/\partial w_i = 0 \end{cases}.$$

Let $A_i = 2\beta_i - \lambda(1-\lambda)K_i d_i^{*\lambda-2} > 0$, $A_j = 2\beta_j - \lambda(1-\lambda)K_j d_j^{*\lambda-2} > 0$, and $B = \gamma_j + \gamma_i$. The impact of w_i on the two demand rates is given by

$$\begin{pmatrix} \partial d_i^*/\partial w_i \\ \partial d_j^*/\partial w_i \end{pmatrix} = \frac{1}{A_i A_j - B^2}\begin{pmatrix} -A_j \\ B \end{pmatrix}. \tag{10}$$

Note that $A_i A_j - B^2 > 0$ because the Hessian of retailer's profit function is strictly diagonally dominant. The demand functions d_i^* and d_j^* decreases and increases with the wholesale price w_i, respectively. Then, the sign of $\partial^2 d_i^*/\partial w_i^2$ must be evaluated in order to evaluate the sign of $\partial^2\Pi_i(w_1,w_2)/\partial w_i^2$. Using equation (10), $\partial B/\partial w_i = 0$, $\partial A_j/\partial w_i = (\partial A_j/\partial d_j^*)\,B(A_i A_j - B^2)^{-1}$, and $\partial A_i/\partial w_i = -(\partial A_i/\partial d_i^*)\,A_j(A_i A_j - B^2)^{-1}$. Therefore, $\partial^2 d_i^*/\partial w_i^2 = (A_i A_j - B^2)^{-3}\left[\partial A_j/\partial d_j^*\,B^3 - \partial A_i/\partial d_i^*\,A_j^3\right]$. As $A_i A_j - B^2 > 0$, the sign of $\partial^2 d_i^*/\partial w_i^2$ depends on the sign of $\partial A_j/\partial d_j^*\,B^3 - \partial A_i/\partial d_i^*\,A_j^3$, which is negative under symmetric optimal demand rates solution $d^* = d_i^* = d_j^*$.

When the economics of scale are zero $(K_i = 0)$, the optimal solution are given by

$$d_i = \frac{2\beta_j(\theta_i + u_i - w_i) - (\gamma_i + \gamma_j)(\theta_j + u_i - w_j)}{4\beta_i\beta_j - (\gamma_i + \gamma_j)^2}, \tag{11}$$

$$w_i = \frac{4\beta_j\beta_i(c_i + \theta_i + 2u_i) - 2\beta_i(\gamma_i + \gamma_j)(\theta_j + u_i)}{4\beta_j\beta_i - (\gamma_i + \gamma_j)^2}. \tag{12}$$

The condition of symmetry of the optimal demand rates limits the regions in which the optimal solution of the retailer is located, to only two regions or at the boundaries. These two regions are delimited by the targets and differ on the setting of the sale-rebate rate. Then, the retailer and suppliers profit functions will be discussed depending on the value of this parameter. When $u = u_i = u_j = 0$, the problem is restrained to a wholesale contract and the target has no meaning. However; if $u > 0$, the setting of the target will not affect the optimal demand rate or the wholesale price. It affects only the profit functions of the different actors of the chain. For the retailer, its profit function decreases linearly with increasing the target and will be limited by its maximum at a zero target and its minimum when the target is equal to the optimal demand rate solution. In addition, in contrast to the retailer, the supplier profit function increases linearly with decreasing its target. Its minimum will be obtained at a target equal to zero; however, its maximum will be achieved at a target equal to the optimal demand rate. In contrast to the supplier, the retailer

seems to achieve more profits when $u > 0$; however, this intuitive result will be not guaranteed and depends on the impact of the value of u on the optimal wholesale price and optimal demand rate.

5. Numerical results

This section presents the numerical results of the different equilibrium solutions under symmetric parameters ($\beta = \beta_i = \beta_j$, $\gamma = \gamma_i = \gamma_j$, $\theta = \theta_i = \theta_j$, $K = K_i = K_j$, $u = u_i = u_j$, $c = c_i = c_j$, $t = t_i = t_j$) and especially with a positive economic of scale $K > 0$. The following combination of parameters is used for the simulation ($\beta = 2$, $\gamma = 0.75$, $\theta = 100$, $K = \{1,2,3\}$, $u = \{0,5\}$, $c = 10$) a . To obtain the Nash solution, a non-linear system of equations is formed from the first order derivatives of the profit functions of the suppliers and the retailer. This system is given by

$$\begin{cases} g_1(d,w) = \theta + u - w - 2(\beta+\gamma)d - \lambda K d^{\lambda-1} = 0 \\ g_2(d,w) = ((\lambda(1-\lambda)Kd^{\lambda-2} - 2\beta)^2 - 4\gamma^2)d + (w-c-u)(\lambda(1-\lambda)Kd^{\lambda-2} - 2\beta) = 0 \end{cases} . \tag{13}$$

This system of equations is solved using the Newton method as expressed by

$$\begin{pmatrix} d^{k+1} \\ w^{k+1} \end{pmatrix} = \begin{pmatrix} d^k \\ w^k \end{pmatrix} - \begin{pmatrix} \partial g_1(d^k)/\partial d & \partial g_1(d^k)/\partial w \\ \partial g_2(w^k)/\partial d & \partial g_2(w^k)/\partial w \end{pmatrix}^{-1} \begin{pmatrix} g_1(d^k) \\ g_2(w^k) \end{pmatrix}. \tag{14}$$

In this numerical study, the solutions of the wholesale price contract and the sale-rebate contract will be presented, compared, and their effect on the profits of the retailer, the suppliers, and the integrated system in presence of inventory related costs, will be discussed. For the optimization problem, the total profit function of the integrated system is obtained by excluding the endogenous parameters of the chain and it is given by

$$\pi_I = 2[(\theta - c - (\beta+\gamma)d)d - Kd^{\lambda}]. \tag{15}$$

The optimal solution of the profit function of the integrated system is obtained by solving its first order derivative, as given by

$$\partial \pi_I / \partial d = 2[\theta - c - 2(\beta+\gamma)d - \lambda K d^{\lambda-1}] = 0. \tag{16}$$

The different optimal results for the Nash equilibrium and for the integrated system are summarized in Table 1. In this simulation, the target takes two different values ($t = 0$ or $t = d$). For $0 < t < d$, the profit function of the retailer decrease linearly with the target. However, the profit of the supplier increases linearly in such target range. The case, in which the optimal demand rate is high than the target, is not studied here.

Table 1: Summary of Nash and optimal numerical results for the different parameters.

			K = 1	K = 2	K = 3		K = 1	K = 2	K = 3
Nash wholesale price (w)		Wholesale contract (u = 0)	44.499	44.383	44.267	Sale-rebate contract (u = 5)	49.499	49.383	49.267
Optimal demand rate (d)			10.062	10.055	10.047		10.062	10.055	10.047
Optimal retail price (p)			72.328	72.349	72.370		72.328	72.349	72.370
Target t = 0	Retailer profit function		553.709	549.699	545.695		553.709	549.699	545.695
	supplier profit function		347.147	345.718	344.289		347.147	345.718	344.289
Target t = d	Retailer profit function		553.709	549.699	545.695		453.085	449.151	445.223
	supplier profit function		347.147	345.718	344.289		397.459	395.992	394.525
Nash profit of the chain			1248.002	1241.136	1234.273		1248.002	1241.136	1234.273
Optimal demand for integrated system			16.341	16.319	16.296		16.341	16.319	16.296
Optimal price for integrated system			55.062	55.124	55.186		55.062	55.124	55.186
Profit function of integrated system			1766.950	1758.452	1749.959		1766.950	1758.452	1749.959
Nash profit / integrated profit			0.706	0.706	0.705		0.706	0.706	0.705

For the wholesale contract, the profit functions of the retailer and the suppliers are the same. However, in the sale-rebate contract, they increase and decrease with varying the target between $t = 0$ and $t = d$, respectively. The Nash wholesale price in the sale-rebate contract is higher than of that in the wholesale contact and the difference between

them is equal to the rebate rate (u = 5). However, the optimal demand rate and retail price are unchanged. This can be explained by the leadership of the supplier to take decision in the chain. Although the competition is between the suppliers, the retailer did not benefit from it in the sale rebate contract. It seems here that the decision variables depend on the leadership decision and not on the competition. The total profit of the chain under Nash equilibrium is independent of the contracts. It is explained by the linear change of profits with the target between $t = 0$ and $t = d$ (what is gained by a the retailer is lost by two suppliers together to make a compensation). The demand rate in the optimization problem (integrated system) is high than that of the Nash equilibrium, in contrast to the retail price. This can be explained by the remove of the leadership decision in the optimization problem, where the wholesale price is an endogenous parameter. The integrated system achieves around 30 % profit more than the Nash profit. Furthermore, increasing the economics of scale drops the different performances of the chain in the two forms of contract.

Finally, using sophisticated contract is proven to increase the profits of the system, in accordance with the results of Cachon and Kök [2]. However, using sale rebate contract increases the profit of the supplier, in contrast to the same result of Cachon and Kök [2].

5. Conclusion

In summary, the existence and uniqueness of optimal demand rate for the retailer in a target sale-rebate based decentralized supply chain was found to be conditioned by the symmetric of the chain parameters β_i and γ_i for $i \in \{1, 2\}$. The existence of the optimal wholesale price solution was found to be conditioned by the symmetric of the optimal demand rate solution. Further, the setting of the target affects only the profit functions of the supplier and the retailer and does not affect the optimal wholesale and demand rate. As important result, the optimal wholesale price; in the case of sale rebate contract; increases with approximately a difference equals to the channel rebate rate. The total profit of the chain under Nash equilibrium is independent of the contracts.

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