An optimal Integrated Maintenance for Laundry Facility in Hospital Supply Chain

Dounnasr Fadila, Hajej Zied and Rezg Nidhal
LGIPM UFR-MIM
Metz, France
fadila. dounnasr @univ-lorraine.fr, zied.hajej@univ-lorraine.fr, nidhal.rezg@univ-lorraine.fr

Abstract

This paper discusses the supply chains system of health care facilities such as hospitals. This study discusses a jointly optimization problem of hospital supply chain that show great opportunity for reduicing cost and improving service level for laundry. In order to satify the random demand of hospital concerning the laundry system, the laundry facility system called another subcontractor. The principal Laundromat is subjected to a random failure. Consequently, an imperfect maintenance policy is proposed in order to reduce the average number of failure of the Laundromat. The objective of this study is to determine simultaneously the optimal plans of washing for the principal and subcontractor laundry facilities and the optimal maintenance strategy by minimzing the total cost of washing, storage and maintenance.

Keywords: Laundry plan, Imperfect maintenance, Subcontractor, Optimization, Service level, Supply chain.

1. Introduction

In recent years, the hospital supply chains propose a healthcare systems with a prime opportunity both to ease increases in expenses and help improve patient care. The transformation of the supply chains into a vital, collaborative, and strategic function is considered the greatest opportunity for the hospitals. In the other hand, the collaboration between administrators and clinicians will be interesting to anticipate and order all medical and non-medical items, such as drugs, diagnostic machines, washing, gloves, and sheets. Consequently, the supply chain would not only help control mounting costs but also boost efficiency and optimize clinical products.

Creating an advanced supply chain with great maturity involves the establishment of an inclusive governance structure, the implementation of robust processes for key functions such as procurement and materials, and integration and automation of information technology systems. In the same context, the different activities such as catering, laundry, procurement, transportation, etc. represent a consequent challenge for the hospital supply chains in terms of organization and budget. On the other hand, the improvement operationally and environmentally opportunities are necessary in order to obtain a significant cost savings for the laundry facilities as well as reduced environmental footprint. The term “supply chain management” can be divided into different areas such as: inventory management, distribution, maintenance and service level. Besides these different areas, there are several decisions to restructuring the processus, such as facility, material procurement and adapting to changes in the system and environment. In the context of healthcare supply chain, (Burns, 2002) considered the producers, the purchasers and the hospital systems integrated delivery networks a three major types of troupes. More precisely, the inventory management and supply chain has been a topic of extensive research where the objective is to reduce the cost of healthcare by improving the productivity of the system and the service level. In this context, several works treat the problem of just-in-time or stockless inventory system, subcontracting the inventory management system and the laundry service management, in order to improve scheduling decisions and creating better demand forcasts. (Meyer and Meyer, 2006) treat and discuss the problems of healthcare supply chains by including the high cost, wasteful performances and the complex requirements and regulations and propose the solution of these problem by increasing the collaboration between the various parties complicated (production, service, logistic, laundry service…), driven the demand and increasing visibility of practices and inventories. (Colletti, 1994) describes the problem of the health maintenance improvement for hospitals by studying the management of services costs and the various factors that affect costs and how the total

delivered cost of materials can be reduced strategically. (Norris, 1988) investigates reducing costs for hospitals by considering the total delivered cost of a product rather than just the unit cost.
On the other hand, the subcontracting is considered between the best solution to improve the service level of hospital system and reduce their costs. In this context, (Kamani, 2004) presented the problem of inventory management system of a hospital in the context of outsourcing. (Nicholson et al., 2004) proposed an analytical models in order to study the impact of outsourcing of inventory management decisions by comparing the inventory costs and service levels of non-critical inventory items of an in-house three-echelon distribution network to an outsourced two-echelon distribution network.
Concerning the demand management and service levels of hospital supply chains, (O'Neill et al., 2001) studies the problem of the inventory of green linens at the university of lowxa Hospitals and Clinics (UIHC) by examining the impact of implementing a Materials Requirement Planning (MRP) system in a health care setting.

In this paper, we study the case of laundry and maintenance management for an hospital laundry linen which uses subcontracting to meet a random demand. This work can consider as an intergrated maintenance in hospital supply Chain. In this context, we can cite the work of hajej et al. (2014) who treated the optimization problem of integrated maintenance for manufacturing system composed by several sectors. The aim of this work is to establish the economical production and maintenance plans for each sector by minimizing the total cost of production, maintenance and transport.

This paper is organized as follows. The next section describes the problem description. In section 3, a mathematical modeling and a problem formulation are proposed as well as an analytical study is presented in order to easy the resolution of stochastic problem by proposing an equivalent deterministic model which minimize the total cost of washing, storage and maintenance. The section 4 presented a numerical example to illustrate the important of the proposed approach. Finally, the conclusion is given in Section 5.

2. Problem Description

2.1 Notation

The different notations used in this study are the following:

H. Δt: Finite laundry time horizon.
Δt: Laundry period.
*I(k):*the apount of inventory at each period k .
$u_L(k)$: rate of laundry capacity of principal Laundry facility at each period k.
$u_{LS}(k)$:rate of laundry capacity of subcontractor Laundry facility at each period k.
d(k): demand concerning the quantity of linen received from hospital at each period k.
δ: Percentage of received linen quantity after check for laundry.
u_L^{max}: Maximal rate of laundry of principal provider.
u_{LS}^{max}: Maximal rate of laundry of subcontractor provider.
θ: Probability index related to service level.
C_L: Washing unit cost of principal Laundry Facility.
C_{LS}: Washing unit cost of subcontractor Laundry Facility.
C_O: Inventory cost.
β_s: Availability of subcontractor Laundry Facility.
CcL: Unit preventive maintenance action cost of principal Laundromat.
CpL: Unit corrective maintenance action cost of principal Laundromat.

2.2 Problem Description

This study deals with case of hospital linen cleaning services characterized by laundry service composed by a Laundry facility. The laundry facility called upon a another laundry service subcontractor in order to satisfy the random demand of badly clean linen received from the hospitals under a given service level. The demand is assumed to be a stationary

random variable, being approximated by a normal distribution function with mean $\hat{d}$ and finite variance $V_d = \sigma_d^2 \geq 0$. After inspection, a part a linen $\delta.d$ will be washed in the principal and subcontractor Laundromat and the rest (1-δ).d will be rejected. After washing, the linen stored in the stock I and after will return to hospitals.
Point of view reliability, the principal laundromat is subjected to random failures, its failure rate $\lambda(t)$ increases with both time and according to the Laundromat using. An imperfect maintenance actions are performed in order to reduce the average number of failure. The only information concerning the subcontractor laundromat is the availability rate β_s . (figure 1)

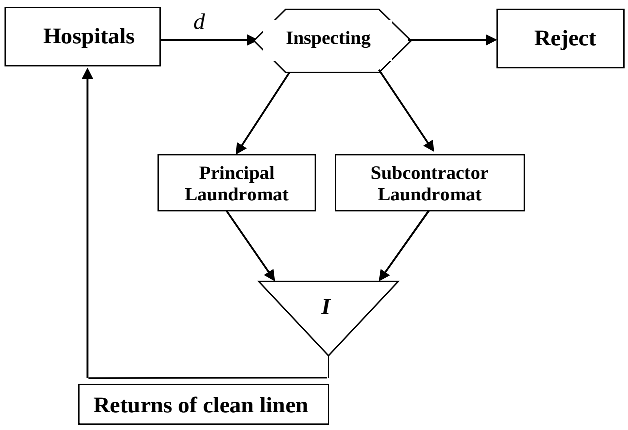

Figure 1: Problem Description

Our objective is to establish simultaneously the optimal plan of washing and the optimal imperfect maintenance strategy characterized by the optimal number of preventive maintenance actions. The aim is to minimize the total cost of washing, storage and maintenance.

3. Mathematical Modeling

3.1 Problem Formulation

The stochastic model of the problem characterized by the minimizing the total cost of laundry, storage, subcontractor and maintenance. The laundry planning problem described by the classical HMMS model with constraints on the decisions variables. In this section, the HMMS model (Holt et al., 1960) is modified in order to consider constraints on the storage, laundry, subcontractor and maintenance variables explicity in the formulation. The maintenance strategy adopted in this model is an imperfect maintenance policy with minimal repair.The imperfect maintenance strategy consists of a periodic operation that will reduce the failure rate during every maintenance period which mean that after a maintenance period, the machine will be younger but not as good as new. We will suppose that partial repair (e.g. replacement of badly worn components, lubrication, adjustment of others⋯.) will occur at maintenance period $i.T$ where $i \in \{1,2,\cdots\}$. For example, when $t=T$ after maintenance operation the systems reliability will decrease, however, it will be greater than its initial value at t_0.

The model can be formulated mathematically as follows:

$$\mathrm{G}\left(U_L,U_{LS}\right)=\mathrm{C_o}\cdot \mathrm{E}\left\{I\left(\mathrm{H}\right)^2\right\}+\sum_{\mathrm{k=0}}^{\mathrm{H-1}}\left[\mathrm{C_o}\cdot \mathrm{E}\left\{I\left(\mathrm{k}\right)^2\right\}+\mathrm{C_L}\cdot \mathrm{u_L}\left(\mathrm{k}\right)^2+\mathrm{C_{Ls}}\cdot \beta_s^{\ 2}\cdot \mathrm{u_{Ls}}\left(\mathrm{k}\right)^2\right]$$
$$+\mathrm{C_{pL}}\cdot\left(\mathrm{N}-1\right)+\mathrm{C_{cL}}\cdot \varphi\left(u_{Lk},\mathrm{N}\right) \quad (1)$$

Subject to

$$I\left(\mathrm{k+1}\right)=I(k)+u_L(k)+\beta_s\cdot u_{LS}(k)-\delta\cdot d\left(k\right) \quad (2)$$

$$\Pr ob[I\left(k+1\right)\geq 0]\geq\theta \quad (3)$$

$$0\leq u_L\left(k\right)\leq u_L^{max} \quad (4)$$

$$0\leq u_{LS}\left(k\right)\leq u_{LS}^{max} \quad (5)$$

Where the equation (1) presents the total cost of washing, storage and maintenance during the finite horizon *H*. the balance equation of storage is describes by the relation (2) take into account the laundry cadences of principal and subcontractor laundry facilities. The service level for each period is given by the constraint (3). The upper and lower boundaries of laundry cadences of the principal and subcontractor Laundromats are given by the constraints (4) and (5).
For the part of maintenance cost is composed by the preventive maintence cost for each preventive maintenance action plus the corrective maintenance cost for the average number of failure. We assume that the preventive and corrective maintenance costs are known and constant as well as the durations of preventive and corrective maintenance actions are negligibles.

3.1 Analytical Study

In order to resolve the stochastic problem, the transformation to an equivalent deterministic problem is necessary. In this case, the deterministic formulation is given as following:

- Cost of laundry and storage

Since that $u_L\left(\mathrm{k}\right)$ and $u_{LS}\left(\mathrm{k}\right)$ are constants for each period Δt, we consider $u_L\left(\mathrm{k}\right)=\hat{u}_L\left(\mathrm{k}\right)$, $Var\left(u_L\left(\mathrm{k}\right)\right)=0$ and $u_{LS}\left(\mathrm{k}\right)=\hat{u}_{LS}\left(\mathrm{k}\right)$, $Var\left(u_{LS}\left(\mathrm{k}\right)\right)=0$.

For $d(k)=\hat{d}(k)$, the balance equation of storage state is given as following:

$$\hat{I}\left(\mathrm{k}+1\right)=\hat{\mathrm{I}}\left(\mathrm{k}\right)+u_L\left(k\right)+\beta_s\cdot u_{Ls}\left(k\right)-\delta\cdot\hat{\mathrm{d}}\left(\mathrm{k}\right) \quad (6)$$

And $E(I(k)^2)=k.\sigma_d^{\ 2}+\hat{I}(k)^2$

$\hat{I}(k)$: Average level of storage

Therefore , the deterministic total cost of laundry and storage is given by the following equation:

$$G(U_L, U_{LS}) = C_o \cdot \hat{I}(H)^2 + \sum_{k=0}^{H-1}\left[C_o \cdot \hat{I}(k)^2 + C_L \cdot u_L(k)^2 + C_{Ls} \cdot \beta_s^2 \cdot u_{Ls}(k)^2\right] + C_s \cdot \delta^2 \cdot (\sigma_d)^2 \cdot \frac{H(H+1)}{H} \quad (7)$$

- Service level constraint

The service level constraint is transformed in deterministic form by specifying a certainly cumulative minimal quantity of linen to laundry:

$$\Pr ob(I(k+1) \geq \theta => (U(k) \geq U_\theta(I(k),\theta)) \quad \text{k=0,1,…, H-1}$$

With:

$$U(k) = u_L(k) + \beta_s \cdot u_{LS}(k)$$

$U_\theta(\;)$: Cumulative minimal quantity of linen to laundry is defined by:

$$U_\theta(I(k),\theta) = \sqrt{k+1} \cdot \sigma_d \cdot \varphi^{-1}(\theta) + \delta \cdot \hat{d}(k) - \hat{I}(k) \quad \text{k=0,1,…, H-1} \quad (8)$$

$V_{d,k}$: variance of demand during the period k

φ: is a Gaussian cumulative distribution function with mean $\hat{d}(k)$ and variance.

- Total maintenance cost

The problem with perfect maintenance is that it is based on renewing completely failed components when the machine breaks down or at maintenance period. During this part, we will present a realistic model well known in industrial firms as preventive maintenance with minimal repair.

In order to set up partial renewal maintenance policy we need first to define the failure rate equation and from it we can determine the average number of failure.

- Failure rate estimation

The failure rate in the case of imperfect maintenance is expressed as follows:

$$\lambda_k(t) = \lambda_{k-1}(\Delta t)\left(1 - \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{T} \right\rfloor + 1\right)T} \right\rfloor\right) + \frac{u_L}{u_L^{max}}\,\lambda_n(t) + \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{T} \right\rfloor + 1\right)T} \right\rfloor \lambda(t=0)e^{\left\lfloor \frac{k}{T} \right\rfloor \alpha} \quad (9)$$

Where
$k \in \{1,2 \dots.\}$
T: Maintenance period;
$\lambda_n(t)$: Failure rate of the principal laundromat under nominal condition;

We mention that the failure rate equation can be expressed as follows as well:

$$\lambda_k(t) = \prod_{i=1}^{k}\left(1 - \left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{T} \right\rfloor + 1\right)T} \right\rfloor\right) \times \lambda_0 + \sum_{i=1}^{k-1}\left(\frac{u_{Li}}{u_L^{max}} \times \lambda_n(\Delta t)\right) + \frac{u_{Lk}}{u_L^{max}} \times \lambda_n(t) + \sum_{i=1}^{k}\left(\left\lfloor \frac{k-1}{\left(\left\lfloor \frac{k-2}{T} \right\rfloor + 1\right)T} \right\rfloor \times e^{\left\lfloor \frac{i}{T} \right\rfloor \times \alpha}\right) \times \lambda_0 \;\; \forall\, t \in \{0, \Delta t\} \tag{10}$$

We suppose we make a partial renewal at times T, 2T, 3T ... At each interval $I_k = [(k\text{-}1)\,T, kT]$, the failure rate will be equal to the failure rate of the previous interval I_{k-1} multiplied by a degradation factor e^{α} with a positive real number. For example, if the failure rate during I_1 is $\lambda(t)$ varies between $\lambda(0)$ and $\lambda(T)$, then during $I_2=[T,2T]$ after the partial replacement at $t = T$, the failure rate varies between $e^{\alpha}\,\lambda(0)$ and $e^{\alpha}\,\lambda(T)$. During the interval I_3 = [2T, 3T], the failure rate will vary between $\lambda(0)e^{2\alpha}$ and $\lambda(T)e^{2\alpha}$ etc.

The average number of failure is expressed as follows:

$$\varphi(u_{Lk}, \mathrm{N}) = \sum_{i=0}^{N-1}\left[\sum_{k=In\left(i+\frac{T}{\Delta t}\right)+1}^{In\left((i+1)\times\frac{T}{\Delta t}\right)} \int_0^{\Delta t} \lambda_k(t)\; dt\right] + \sum_{k=N\times T}^{H\Delta t} \int_0^{\Delta t} \lambda_k(t)\, dt \tag{11}$$

4. Numerical Exemple

A numerically study is presented in this section carakterized by a laundry service composed by a laundry facility that work during a finite horizon H=12 months with . Point of view reliability, we assume that the degradation law of the principal laundry unit follows the Weibull distribution with parameters shape γ=2 and Scale β=100. The other data of problem are presented as following

$\delta = 0.8$, $u_{Ls}^{max} = u_L^{max} = 18\ lu$, $\theta = 0.9$, $C_l = 3\,mu$, $C_{ls} = 8\,mu$, $\beta_s = 0.8$, $C_{cL} = 2500\,mu$, $C_{pL} = 1000\,mu$, $C_o = 2\,mu$ with mu: monetary unit and Lu: laundry unit

The demand are random a characterized by a Gaussian distribution with variance σ_d^2=2 and the average demands during the horizon H are given by the following table.

Table 1: Average Demands

$\hat{d}_0$	$\hat{d}_1$	$\hat{d}_2$	$\hat{d}_3$	$\hat{d}_4$	$\hat{d}_5$	$\hat{d}_6$	$\hat{d}_7$	$\hat{d}_8$	$\hat{d}_9$	$\hat{d}_{10}$	$\hat{d}_{11}$
17	19	17	17	17	16	18	16	18	15	17	16

Using the previews data, we illustrate the optimal laundry plan of principal and subcontractor Laundromats

Table 2: Optimal laundry plan of principal Laundromat

$u_L(0)$	$u_L(1)$	$u_L(2)$	$u_L(3)$	$u_L(4)$	$u_L(5)$	$u_L(6)$	$u_L(7)$	$u_L(8)$	$u_L(9)$	$u_L(10)$	$u_L(11)$
6	10	7	9	15	14	7	8	9	6	10	9

Table 3: Optimal Laundry plan of subcontractor Laundromat

$u_{Ls}(0)$	$u_{Ls}(1)$	$u_{Ls}(2)$	$u_{Ls}(3)$	$u_{Ls}(4)$	$u_{Ls}(5)$	$u_{Ls}(6)$	$u_{Ls}(7)$	$u_{Ls}(8)$	$u_{Ls}(9)$	$u_{Ls}(10)$	$u_{Ls}(11)$
6	8	6	10	7	7	6	12	6	16	10	8

From figure 2, the optimal maintenance plan for the principal Laundromat corresponds to the optimal number of preventive maintenance actions N*=3. The optimal interval between two preventive maintenance actions is T*=H/N*=4.Δt.

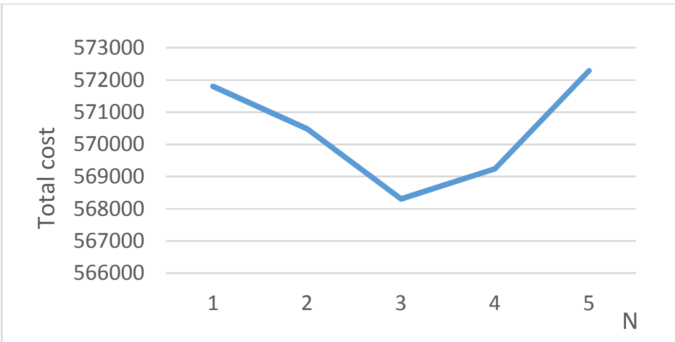

Figure 2: Total cost according to N

5 CONCLUSION

In this study, we have deal with hospital supply chain problematic where we have optimize the integrated maintenance for a laundry of linen system. In order to satisfy the demand of the hospitals under gien service level, the laundromat principal called to another subcontractor laundry facility. Firstly, we have proposed an optimal laundry plan that minimize the total cost of washing and storage. Secondly, by considering the effect of the laundry plan on the failure rate of the Laundromat as well as on the average number of failure, we have propose an imperfect maintenance policy that reduce the failure rate during every maintenance period in order to obtain the optimal number of preventive maintenance actions.

References

Burns, L.R., 2002, the healthcare Value Chain, San Francisico; Jossey-Bass.

Burns, L.R., Pauly, M.V., 2002. “Integrated Delivery Networks: A Detour on the Road to Intergarted Healthcare?” Health Affairs, pp 128-143.

Colletti, J., 1994. “Health Care Reform and the Hospital Supply Chain, “Hospital Materiel management Quarterly, vol. 15, no. 4, pp 28-35.

C. C. Holt, F. Modigliani, J.F. Muth, and H.A. Simon , Planning, production, inventories and work forces, *Prentice-Hall, Englewood Cliffs*, N. J., 1960.

Meyer, A., Meyer, D., 2006. "Roundtable Proceedings, Transforming the Healthcare Supply Chain." M. Singh, ed., MIT Center for Transportation & Loogistics, http://ctl.mit.edu/public.

Kamani, P.,2004. "Hospital upply Chain Savings," ASCET, vol.6, pp 62-65.

Nicholson, L., Vakharia, A.J., Erenguc, S.S, 2004. "Outsourcing Inventory Management Decisions in Healthcare: Models and Application," European Journal of Operational Research, vol. 154, no. 1, pp 271-290.

Norris,H.E., 1988. " Remaining Competitive by Controlling Delivered Costs," Hospital Materiel Management Quarterly, vol.9, no.3,pp 57-62.

O'Neill, L., Murphy, M., Gray, D., Stoner, T., 2001, " An MRP System for Surgical Linen Management at a Large Hospital, " Journal of Medical Systems, vol. 25, no.1, pp 63-71.

Zied, H., Nidhal, R., Cormier, G, 2014. Integrated transportation and maintenance problem. IEEE 11th International Conference on Networking, Sensing and Control (ICNSC),pp 474 – 479, 14330875, DOI: 10.1109/ICNSC.2014.6819672.